

Quantum Frontiers in Fundamental Physics: From Sensing Rare Decays to Solving Collider Challenges



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Image: Science News

Plan

- Introduction
 - What is a quantum computer?
 - Why quantum computing?
 - Simple examples
- Applications in particle physics
 - Quantum optimization
 - Qubits as sensors for radiative decays of neutrinos and dark matter particles
 - Quantum network for HEP?
- Summary

Disclaimer: I am a theoretical particle physicist who is excited about AI/ML and quantum computing.

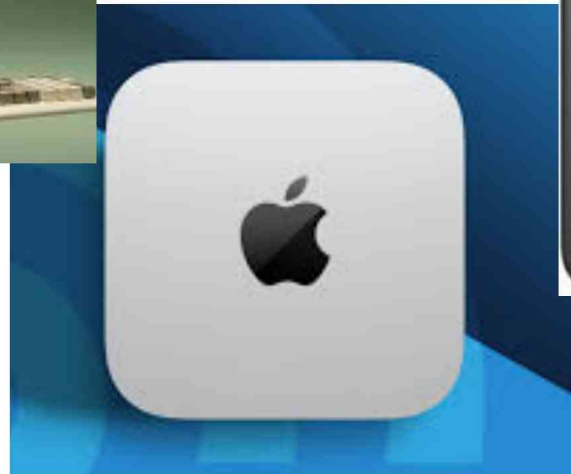
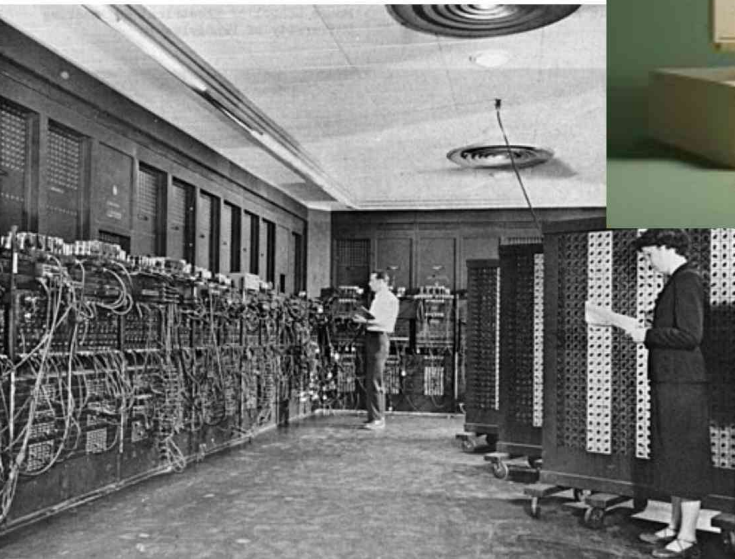
Quantum Revolution

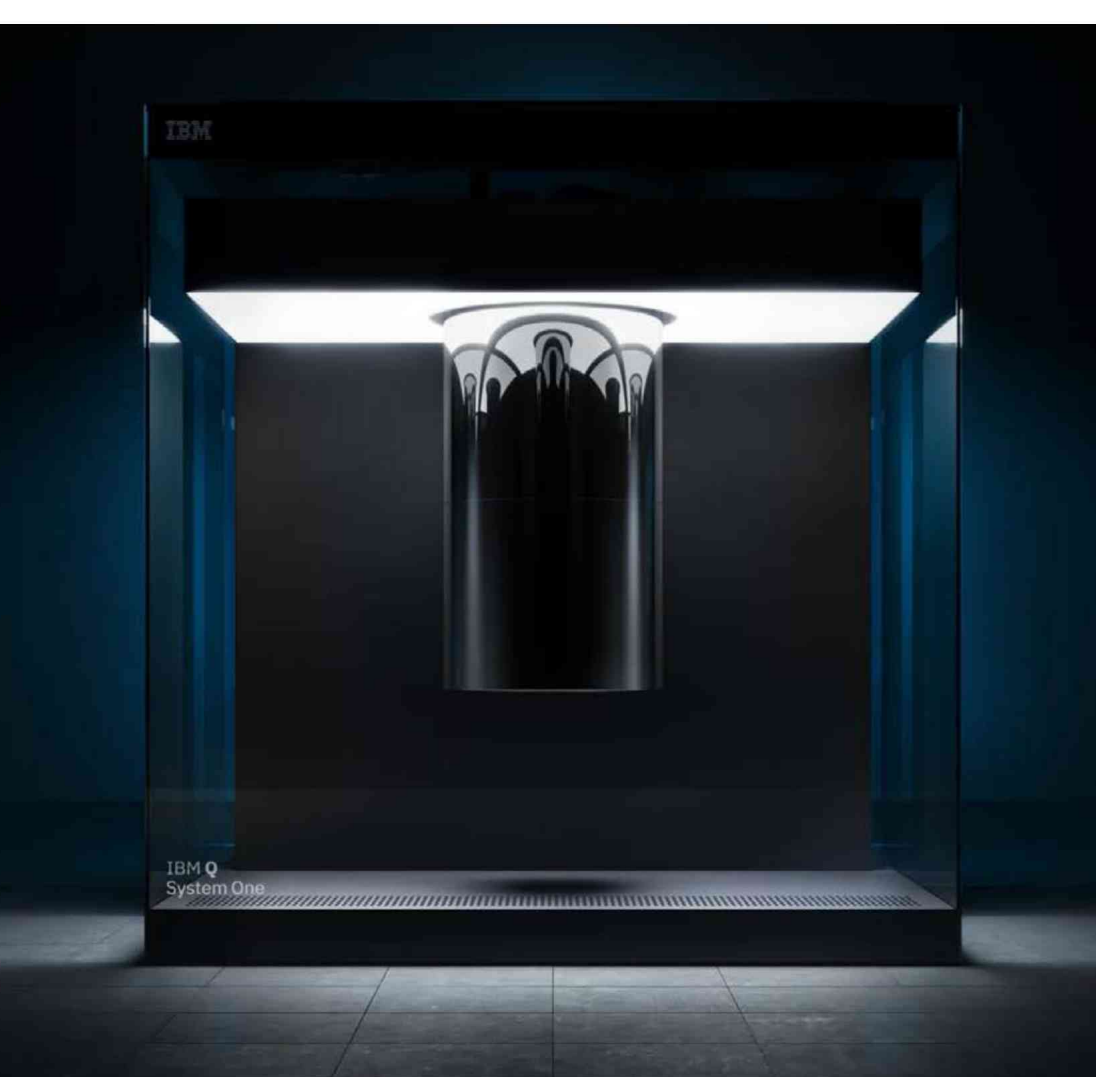
- **Quantum 1.0:** The **first** quantum revolution is the era of discovery where quantum mechanics' principles were first understood and led to technologies like transistors, lasers, and GPS.
 - Transistors, lasers, semiconductors, MRI, and GPS. These technologies apply quantum effects but don't fully leverage advanced quantum properties like entanglement.
- **Quantum 2.0:** The **second** quantum revolution is the current and emerging era focused on actively controlling quantum phenomena like entanglement and superposition to build new technologies, including quantum computing, sensors, and secure communication.
 - **Quantum computing:** Harnessing superposition and entanglement for powerful computation.
 - **Quantum sensing:** Developing highly sensitive sensors for medical imaging, navigation (quantum GPS), and other applications.
 - **Quantum simulation:** Using quantum systems to simulate complex molecules and materials.
 - **Quantum communication:** Using entanglement to create unhackable communication networks.

What is a quantum computer?

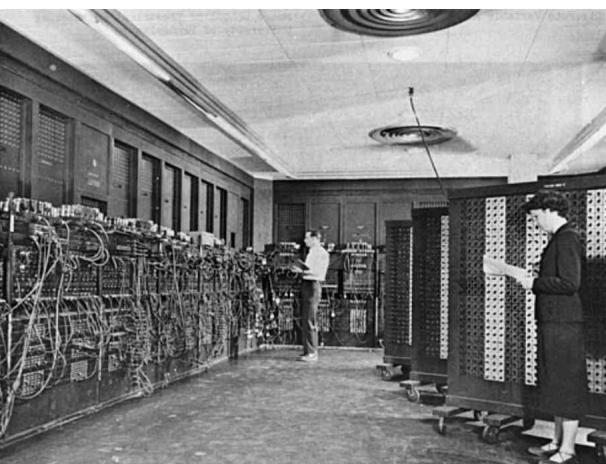
- A quantum computer is a new kind of computer that's based on the laws of **quantum physics**.
- It can do **certain things** faster than normal computers because it follows a different set of rules.
 - **Classical speed** is about how fast each operation happens (billions/sec).
 - **Quantum speed** is about **how many possibilities are explored at once due to quantum phenomena**, leading to vastly fewer steps for specific problems.

ENIAC (US Army, 1945)



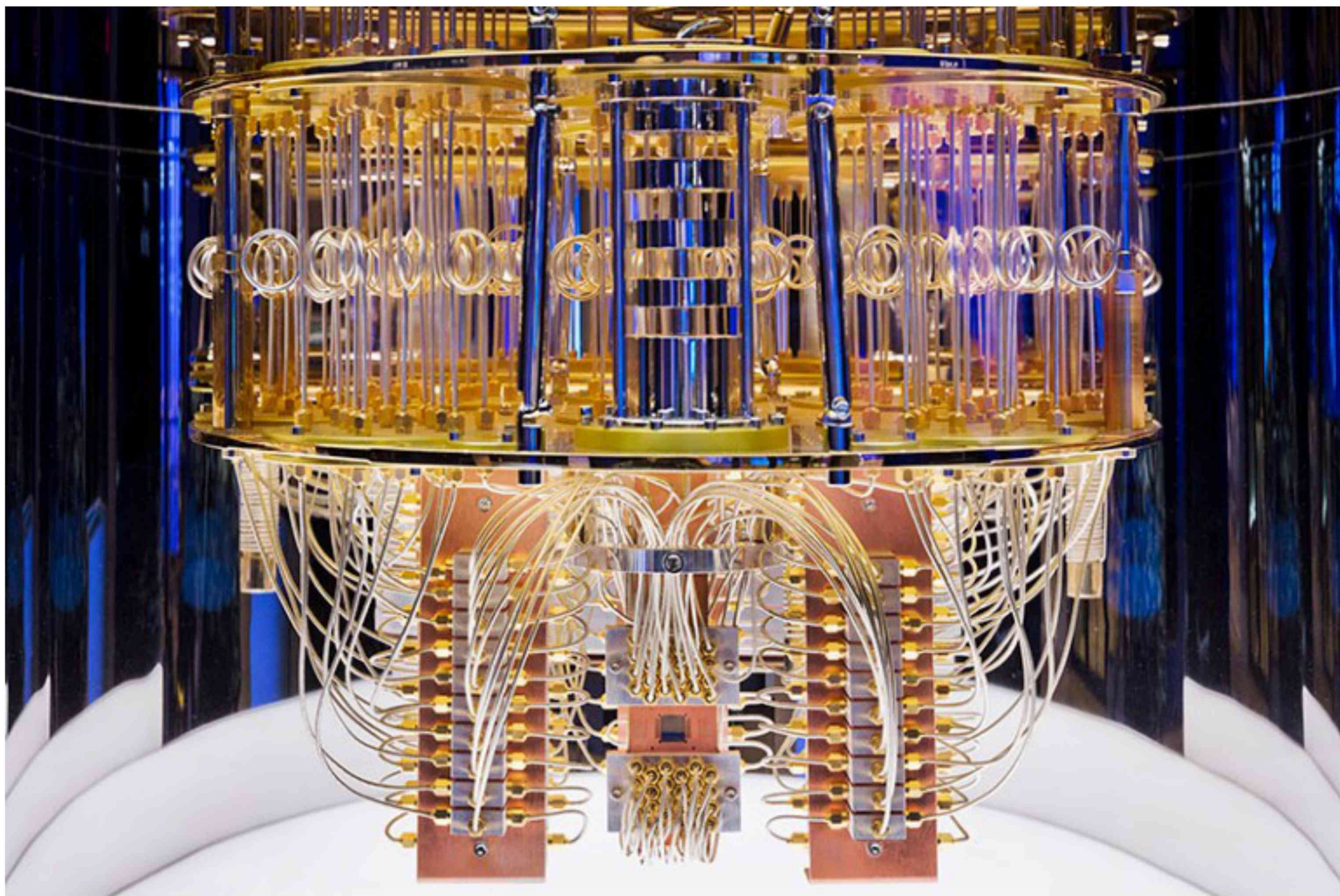


Eagle 127 qubit
IBM Quantum computer
Tens of million dollars
(depending on service contract)



ENIAC (US Army),
~\$500k (**Equivalent
to ~\$7million**)





IBM cloud service charges

Subscription time at National Energy Research Scientific Computing Center (NERSC) supported by US Department of Energy



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For getting started

Open Plan

Access quantum computers for up to 10 minutes of runtime per month.

Free

Sign upView features

For small-scale work

Pay As You Go Plan

Pay for the quantum computer access you use. Billed per second of usage.

Starts at \$96 USD / minute

Sign upView features

For project-based work

Flex Plan

Pre-purchase time for projects that require execution flexibility or bursts of sustained usage.

Starts at \$72 USD / minute

Contact usView features

For work at scale

Premium Plan

Subscribe for large-scale capacity with Quantum Allocation Units (QAUs). Use up to 1600 minutes per QAU every 28 days.

Starts at \$48 USD / minute

Contact usView features

For unparalleled access

Dedicated Service

Access to a dedicated on-prem quantum system that is serviced and maintained by IBM Quantum.

Price requires quote

Contact usView features



Quantum Computing: A Soccer Analogy

By Thomas Wong



- If the rules were changed so that players could use their hands, soccer would look very different. In some cases, it would be faster or easier for athletes to use their hands to hold, throw, or catch the ball. In other cases, however, it would still be better to kick the ball. For example, most soccer players can kick the ball across the field faster than they can throw the ball or run with it. So, to get the ball across the field as quickly as possible, it may still be better to use one's feet.
- Analogously, the essence of quantum computing is to change the rules so that a computer can now use its “hands.” That is, the rules of the game are changed from the laws of classical physics to the laws of quantum physics. As a result, a quantum computer can solve some problems faster by using its “hands.” For other problems, using one's “feet” is better, so a quantum computer is no faster for these problems.
- Quantum computers are NOT expected to replace traditional computers for everyday use. Quantum computers excel at specific types of calculations that are intractable for classical computers, but they are not a general-purpose replacement. Classical computers will remain essential for most tasks, while quantum computers will be used for specialized applications.
- Quantum Junction at the International Conference for High Performance Computing.

Digital Computing vs Quantum Computing

Use binaries to perform calculations and solve problems

Use QM / qubits to solve problems

Bit: $x \in \{0,1\}$

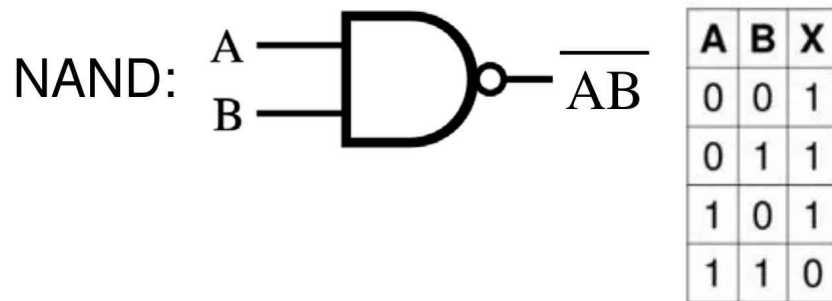
Qubit: $|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} \end{pmatrix}$

Digital computation with n bits:
 $\{0,1\}^n \longrightarrow \{0,1\}^m, m \leq n$

Quantum computation with n qubits: $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{2^n} \end{pmatrix} \in \mathbb{C}^{2^n}$

Classical gates:

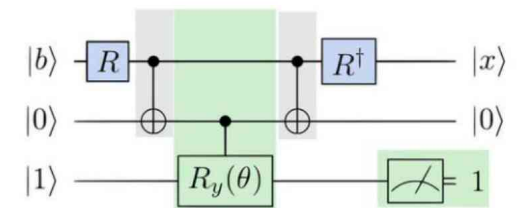
Quantum gates: Unitary transformation
Measurement



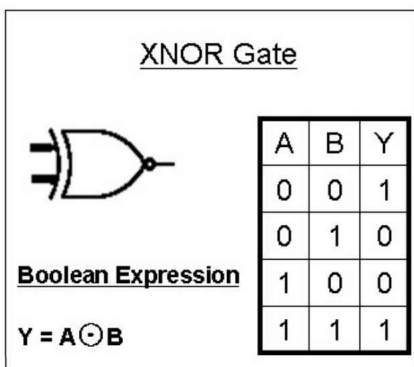
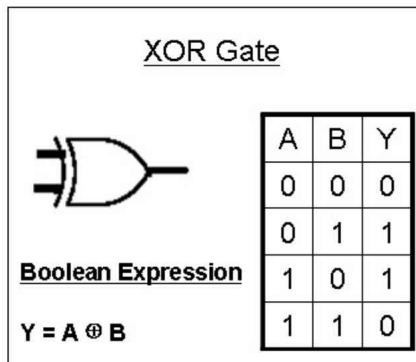
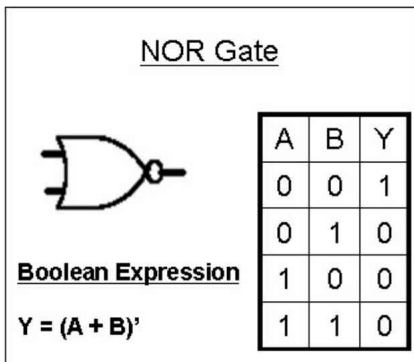
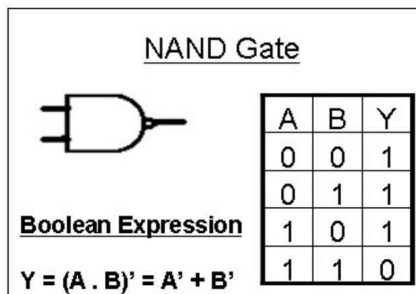
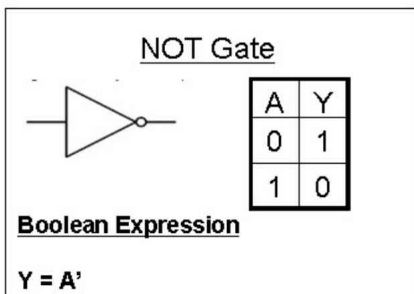
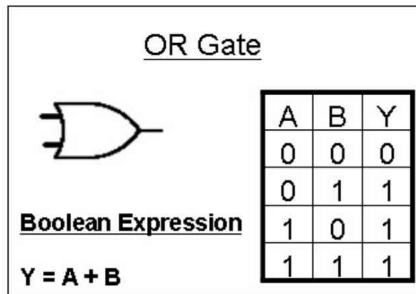
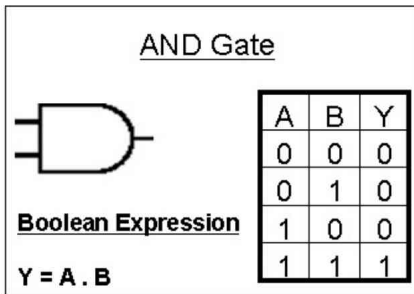
Rotation around x-axis: $R(\theta) = \exp \left(-i\theta \frac{\sigma_x}{2} \right)$

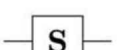
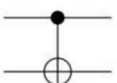
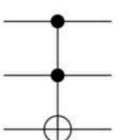
Algorithms

Quantum Algorithms



Classical gates Quantum gates

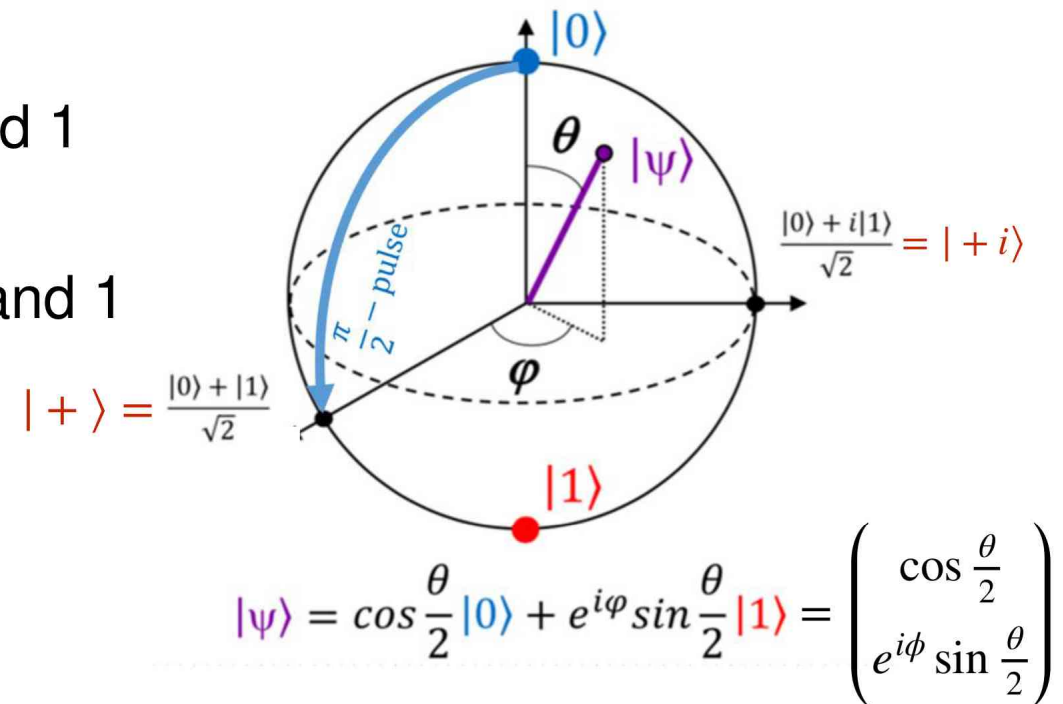


Operator	Gate(s)	Matrix
Pauli-X (X)	 	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

These gates are platform-dependent.

Quantum Bits (Qubits): Unit-normalized two dimensional complex vector

- **Qubit** - quantum bit
- **Superposition** - combination of 0 and 1 ($|0\rangle$ and $|1\rangle$)
- **Amplitude** - coefficient in front of 0 and 1 ($|0\rangle$ and $|1\rangle$)
- **Measurement** - get $|0\rangle$ or $|1\rangle$ with probability = $|\text{amplitude}|^2$
 - **Qubit collapses** to $|0\rangle$ or $|1\rangle$
- **Normalized** - total probability is 1.
- **Entanglement** - outcome two qubits are intertwined. (Entangled vs separable)

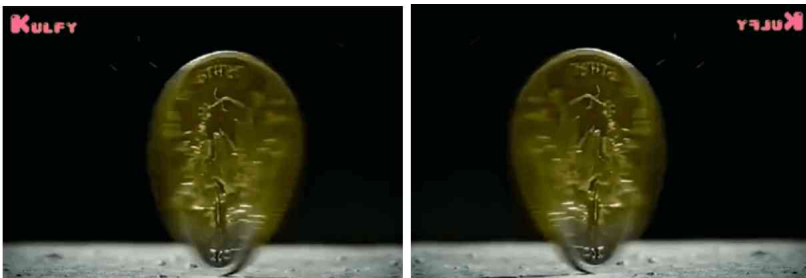


$$|\pm\rangle = \frac{1}{\sqrt{2}}|0\rangle \pm \frac{1}{\sqrt{2}}|1\rangle$$

$$|\pm i\rangle = \frac{1}{\sqrt{2}}|0\rangle \pm i\frac{1}{\sqrt{2}}|1\rangle$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

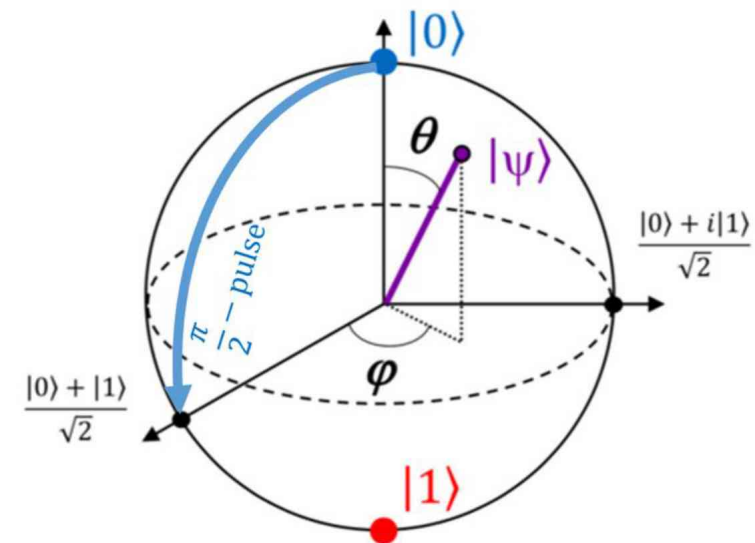


Why Quantum Computing?

- Cryptography: factoring, **hidden subgroup program**, **discrete logarithm problem**
- Optimization
- Search algorithm
- Quantum simulation
- Quantum Machine Learning
 - Quantum Advantages?
 - Learns better with small # of data
 - Faster convergence
 - Less # of parameters
- What are the interesting problems?

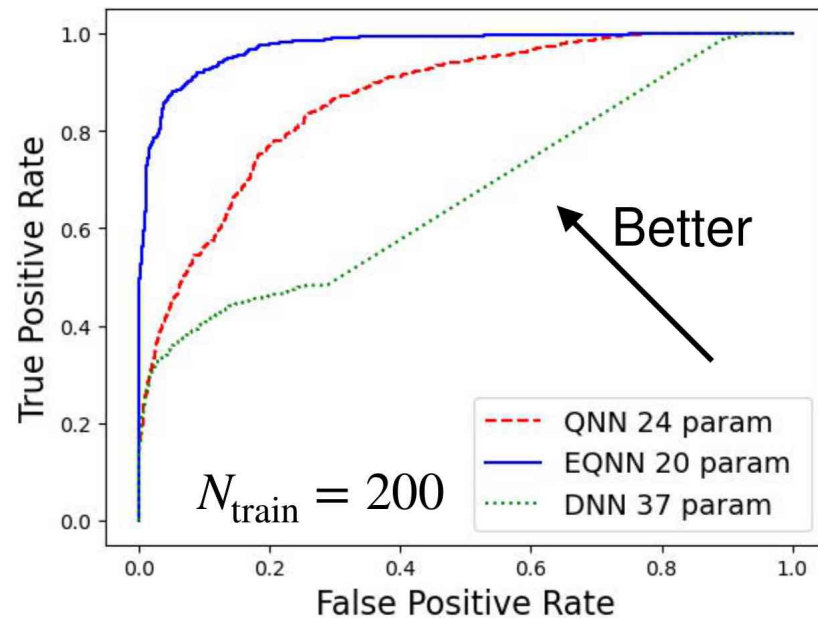
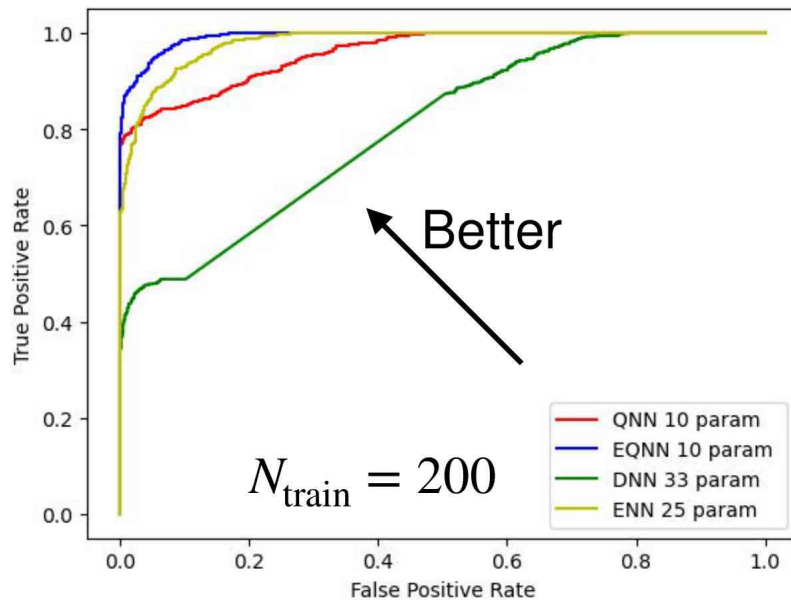
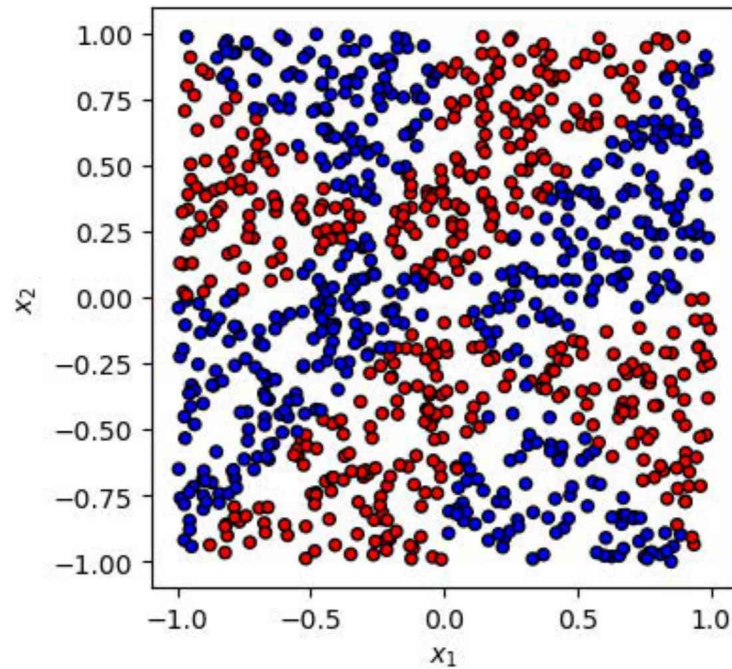
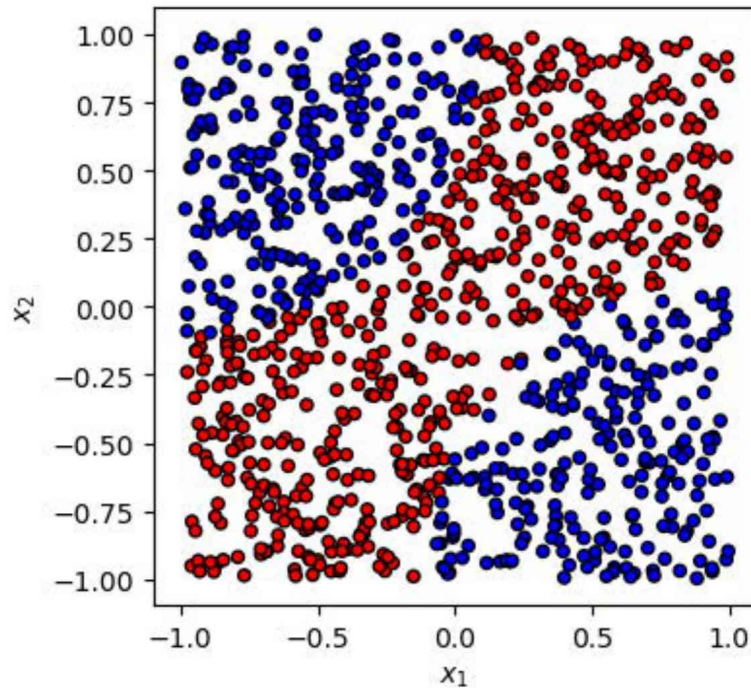
Find the generators of the subgroup $H < G$, that is implicitly defined by a function f on G such that f is constant and takes distinct values on each coset of H .

Find the discrete logarithm of $y \in \mathbb{Z}_p^*$ with respect to base b such that $b^x = y \pmod{p}$

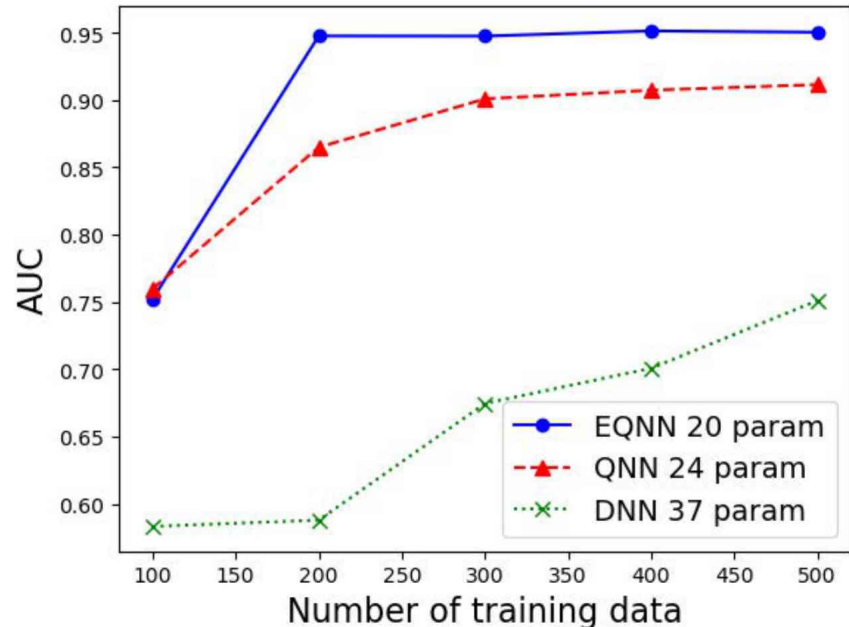
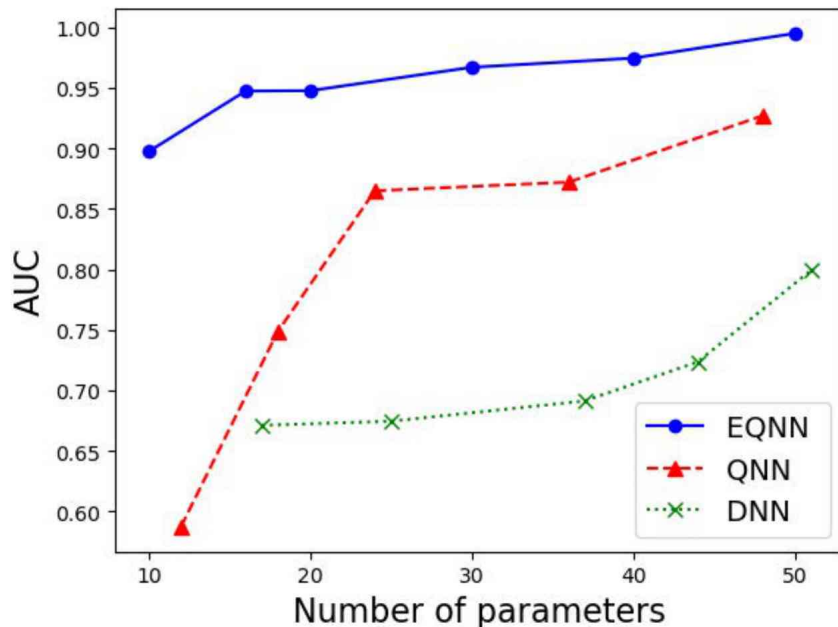
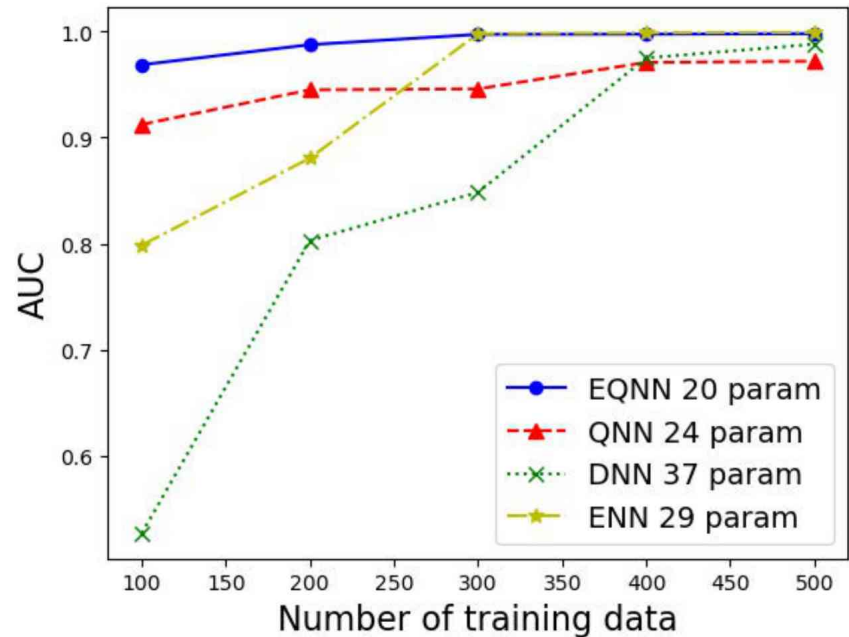
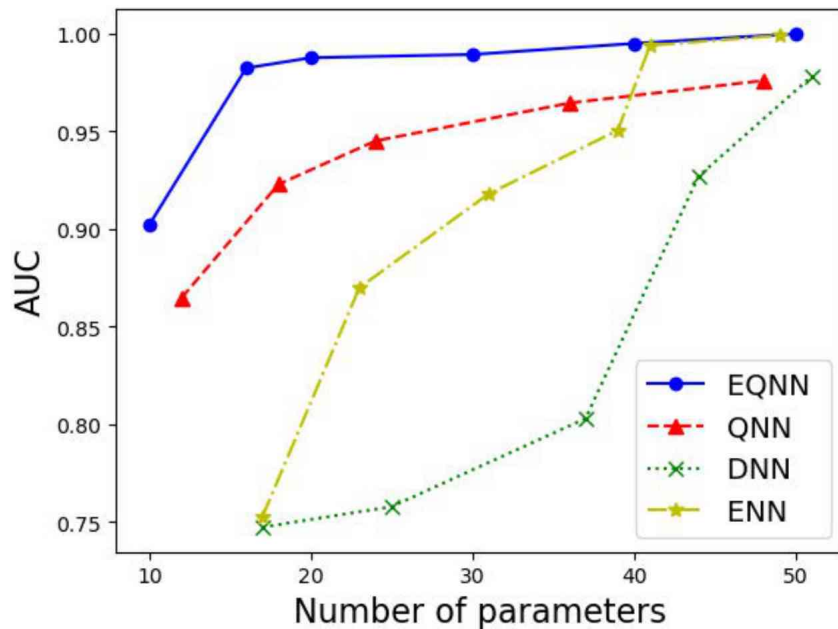


$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle$$

QML: Equivariant Quantum Neural Networks



QML: Equivariant Quantum Neural Networks



Example: Inner product

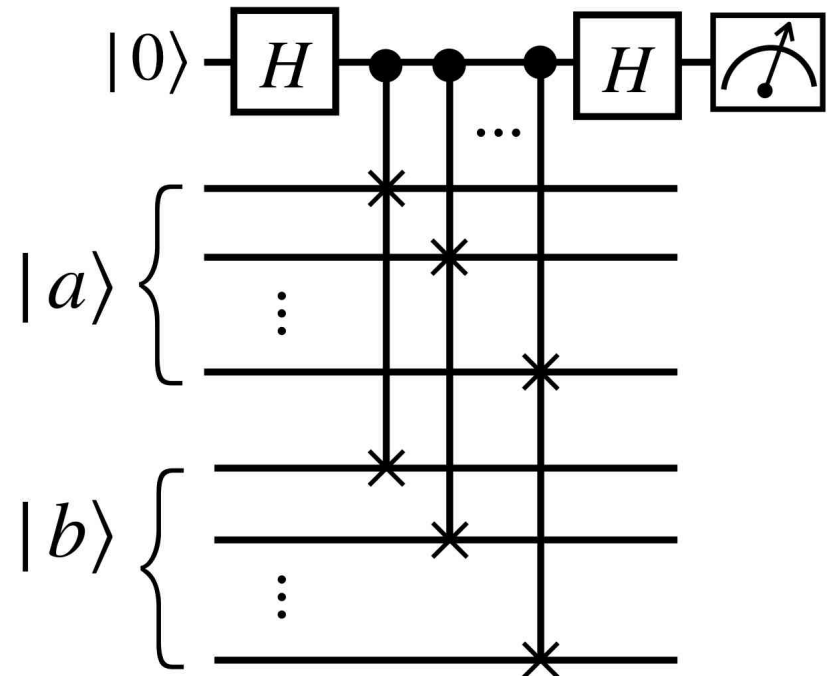
$$\vec{a} = (a_1, a_2, \dots, a_{1024}) = |a\rangle$$

$$\vec{b} = (b_1, b_2, \dots, b_{1024}) = |b\rangle$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \dots + a_{1024} b_{1024}$$

How many operations

= 1024 multiplications + 1023 additions



$|a\rangle$: 10 qubits are needed to embed $\vec{a} = (a_1, a_2, \dots, a_{1024})$

$|b\rangle$: 10 qubits are needed to embed $\vec{b} = (b_1, b_2, \dots, b_{1024})$

$|0\rangle$: 1 additional qubit as a registry

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

12 operations are needed.

$$2^{10} = 1024$$

$$\text{Prob}(0) - \text{Prob}(1) = |\langle a | b \rangle|^2 = |\vec{a} \cdot \vec{b}|^2$$

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

Proof: Inner product

$$|a\rangle \otimes |b\rangle = |a\rangle |b\rangle$$

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

$$(1) \quad H(|0\rangle |a\rangle |b\rangle) = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |a\rangle |b\rangle$$

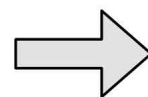
$$(2) \quad SWAP H(|0\rangle |a\rangle |b\rangle) = \frac{1}{\sqrt{2}} SWAP(|0\rangle + |1\rangle) |a\rangle |b\rangle = \frac{1}{\sqrt{2}} (|0\rangle |a\rangle |b\rangle + |1\rangle |b\rangle |a\rangle)$$

$$(3) \quad \begin{aligned} |\psi\rangle &= H SWAP H(|0\rangle |a\rangle |b\rangle) = \frac{1}{\sqrt{2}} H(|0\rangle |a\rangle |b\rangle + |1\rangle |b\rangle |a\rangle) \\ &= \frac{1}{2} |0\rangle (|a\rangle |b\rangle + |b\rangle |a\rangle) + \frac{1}{2} |1\rangle (|a\rangle |b\rangle - |b\rangle |a\rangle) \end{aligned}$$

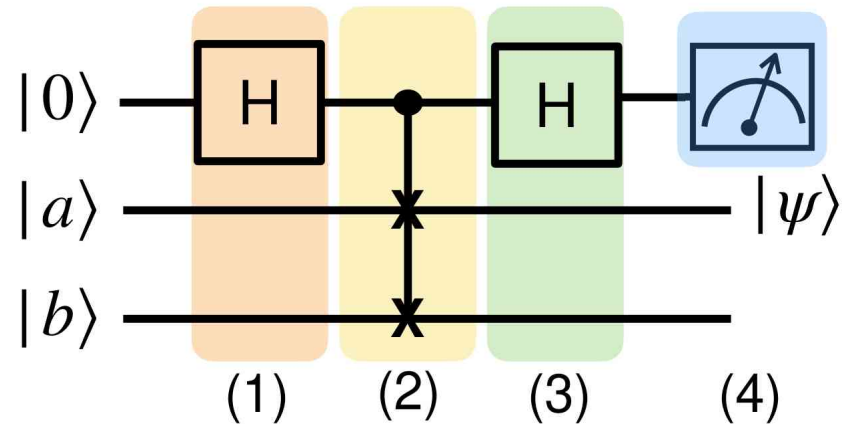
sum of the “unswapped” and the “swapped”

difference

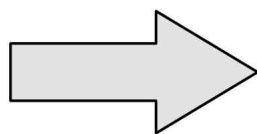
$$(4) \quad \begin{aligned} P(0) &= |(\langle 0 \otimes I) |\psi\rangle|^2 = \frac{1}{2} - \frac{1}{2} |\langle a|b\rangle|^2 \\ P(1) &= |(\langle 1 \otimes I) |\psi\rangle|^2 = \frac{1}{2} + \frac{1}{2} |\langle a|b\rangle|^2 \end{aligned}$$



$$P(1) - P(0) = |\langle a|b\rangle|^2$$



Using vector notation



Using Dirac bracket notation

$$\vec{a} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

$$\vec{b} = (1, 0)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \frac{1}{\sqrt{2}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 \\ &= \frac{1}{\sqrt{2}} \approx 0.707 \end{aligned}$$

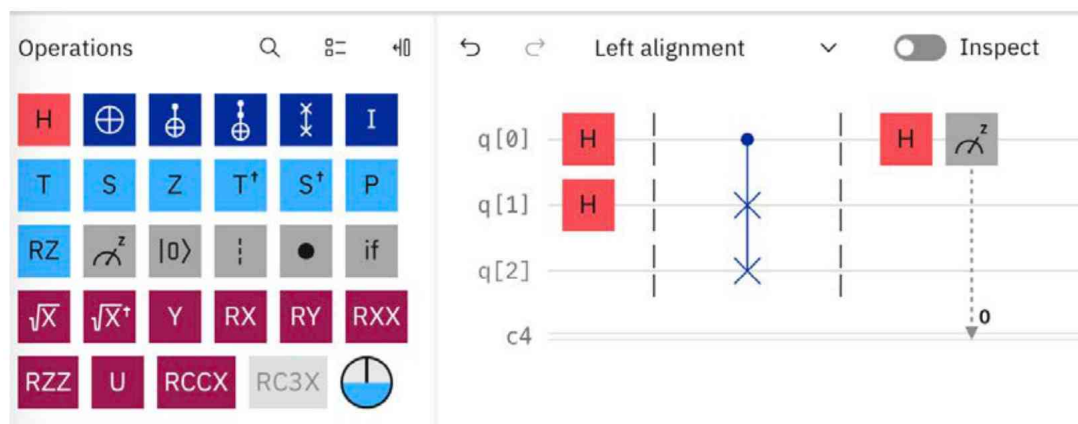
$$|\vec{a} \cdot \vec{b}|^2 = 0.5$$

$$|a\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = H|0\rangle$$

$$|b\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{aligned} \langle a | b \rangle &= \frac{1}{\sqrt{2}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 \\ &= \frac{1}{\sqrt{2}} \approx 0.707 \end{aligned}$$

$$|\langle a | b \rangle|^2 = P(1) - P(0) = 0.5$$



IBM Quantum
Composer

Example: Inner product

- Let $|\psi\rangle, |\phi\rangle \in \mathbb{C}^{2^n}$ be two $N = 2^n$ dimensional vectors. How to compute the magnitude of the inner product $|\langle\phi|\psi\rangle|^2$?

- Digital

- $N = 2^n$ multiplication and additions
- Decompose multiplications and additions as NAND gates

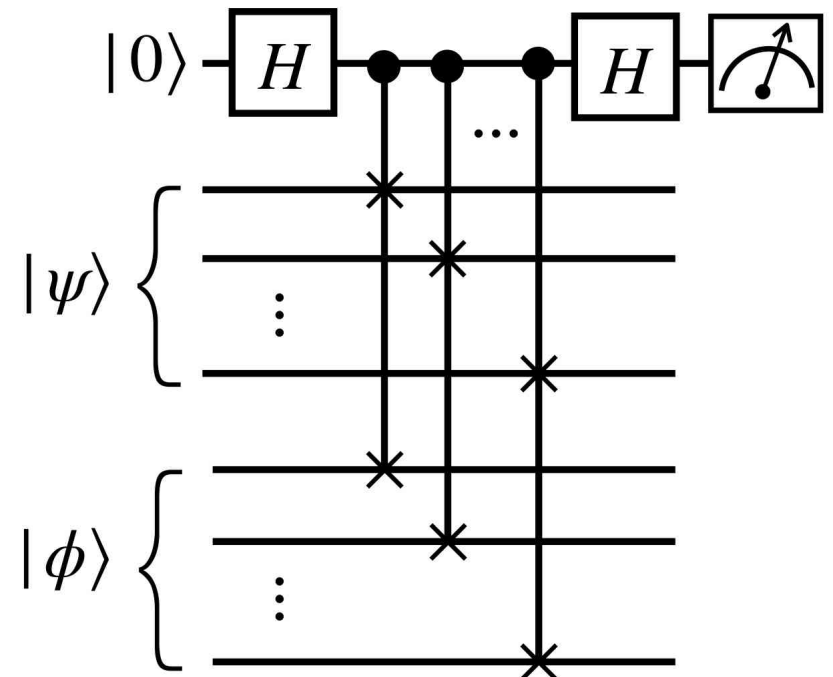
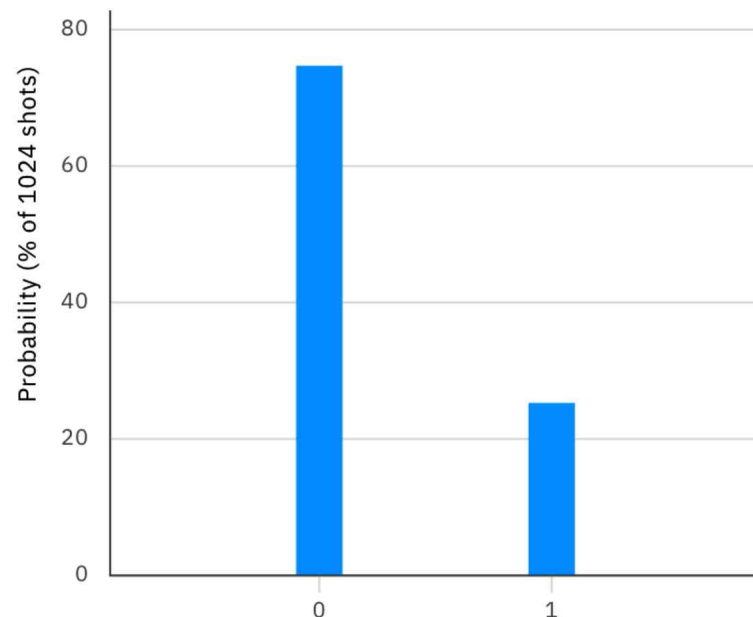
$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Quantum

- Run the following circuits with $2n + 1$ qubits and $n + 2$ gates

- $\text{Prob}(0) - \text{Prob}(1) = |\langle\phi|\psi\rangle|^2$



Example: MathMagic

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

2	3	6	7	10	11	14	15
18	19	22	23	26	27	30	31
34	35	38	39	42	43	46	47
50	51	54	55	58	59	62	63

4	5	6	7	12	13	14	15
20	21	22	23	28	29	30	31
36	37	38	39	44	45	46	47
52	53	54	55	60	61	62	63

8	9	10	11	12	13	14	15
24	25	26	27	28	29	30	31
40	41	42	43	44	45	46	47
56	57	58	59	60	61	62	63

16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

32	33	34	35	36	37	38	39
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XXXXXX1

1	3	5	7	9	11	13	15
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XXXXX1X

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XXX1XX

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XX1XXX

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X1XXXX

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1XXXXXX

Suppose your number was 43

1	3	5	7	9	11	13	15
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40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

In base 2: 101011

Suppose your number was 43

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
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8	9	10	11	12	13	14	15
24	25	26	27	28	29	30	31
40	41	42	43	44	45	46	47
56	57	58	59	60	61	62	63

16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

In base 2: $101011 = 100000 + 001000 + 000010 + 000001$

Suppose your number was 43

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

2	3	6	7	10	11	14	15
18	19	22	23	26	27	30	31
34	35	38	39	42	43	46	47
50	51	54	55	58	59	62	63

4	5	6	7	12	13	14	15
20	21	22	23	28	29	30	31
36	37	38	39	44	45	46	47
52	53	54	55	60	61	62	63

8	9	10	11	12	13	14	15
24	25	26	27	28	29	30	31
40	41	42	43	44	45	46	47
56	57	58	59	60	61	62	63

16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

In base 2: $101011 = 100000 + 001000 + 000010 + 000001$

In base 10:

$$= 32 + 8 + 2 + 1$$

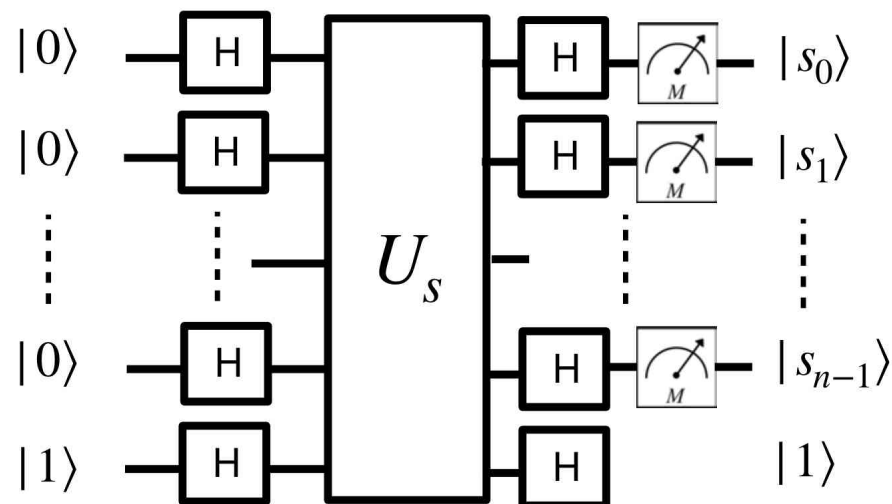
$$= 43$$

Is there a way to find out the answer by asking less than 6 questions?

- Core components of Quantum Information Science and Technology
 - What problems do we want to solve? ([domain specific research knowledge](#))
 - Need different set of rules and a novel algorithm must be developed ([quantum algorithms](#))
 - Need a device that can execute the algorithm ([quantum devices](#))
 - Suitable materials are required to build such a device ([quantum materials](#))

Bernstein-Vazirani Algorithm

- A n -bit function $f : \{0,1\}^{\otimes n} \longrightarrow \{0,1\}$, which outputs a single bit, is guaranteed to be of the form $f_s(x) = x \cdot s$, where s is an unknown n -bit string and $x \cdot s = x_0s_0 + \dots + x_{n-1}s_{n-1} = \sum_{i=0}^{n-1} x_i s_i \pmod{2}$. Find the unknown string $s = (s_0s_1 \dots s_{n-1})$.
- Best classical algorithm uses $\mathcal{O}(n)$ calls to $f_s(x) = x \cdot s \pmod{2}$. **Each query gives one bit of information of s (because f outputs one bit).**
- **How do we find s with less than n queries? $\rightarrow$ Use superposition (over all possible input bit strings)**

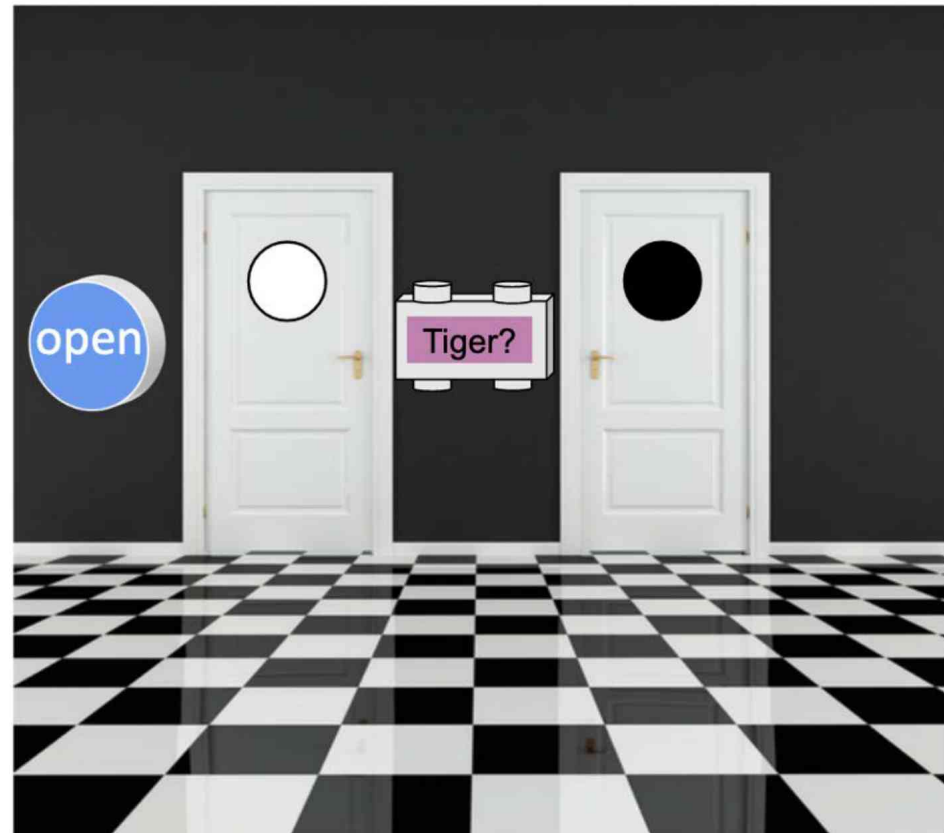


\$ vs Tiger

- Teaching quantum information science to high-school and early undergraduate students by Sophia Economou, Terry Rudolph, Edwin Barnes, 2005.07874
- You encounter two doors:
Money behind at least one door



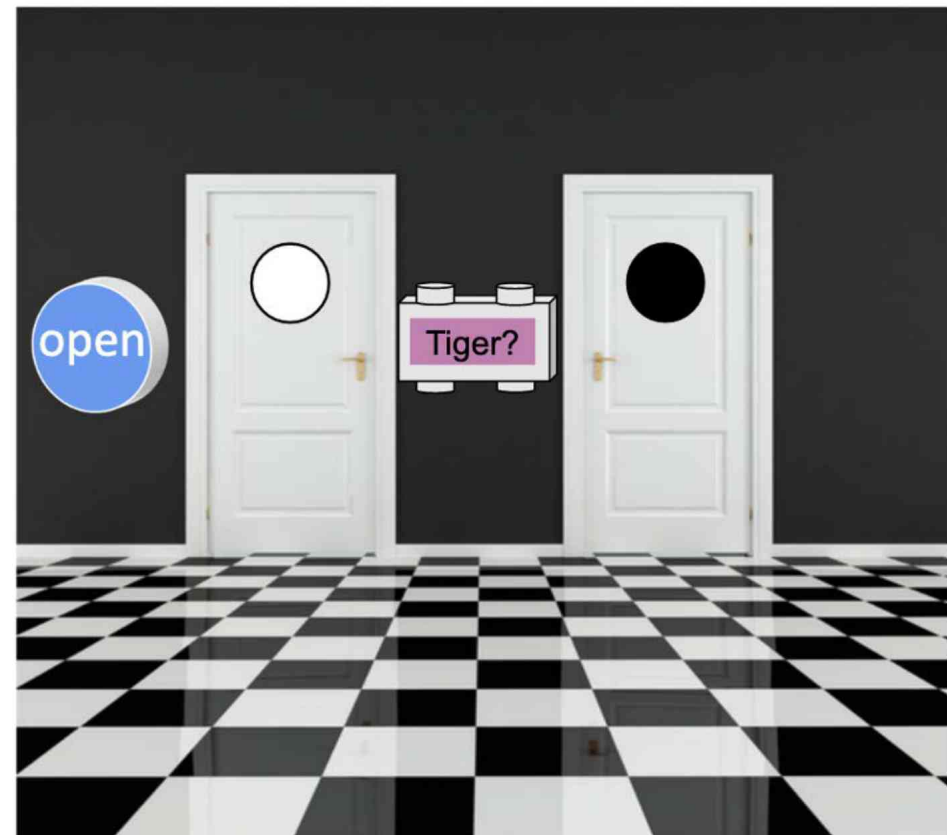
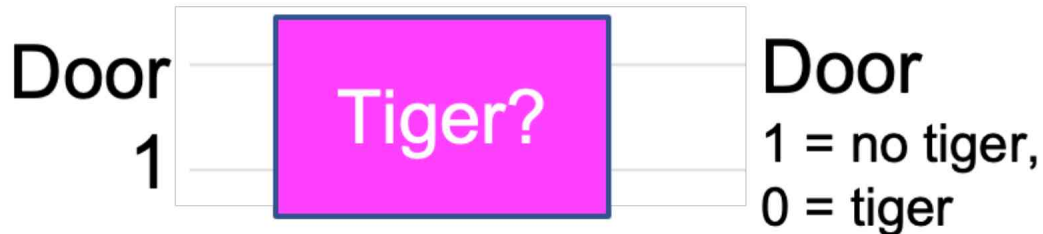
- Tiger might be lurking behind one door





\$ vs Tiger

- The button on the left opens both doors
- YOU WANT TO BE SURE THERE'S NO TIGER BEHIND EITHER DOOR BEFORE YOU PUSH THE "OPEN" BUTTON
- The device in the middle will tell you if there is a tiger behind the door that you ask about – but you only get to use it once



\$ vs Tiger

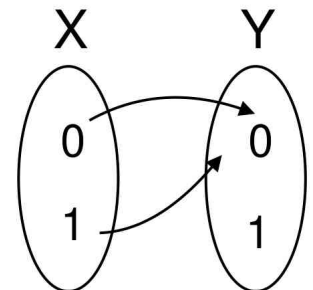
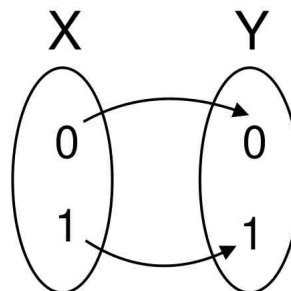
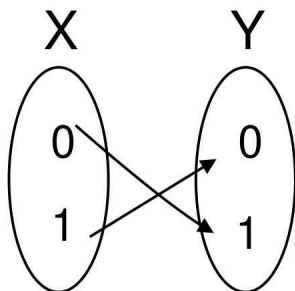
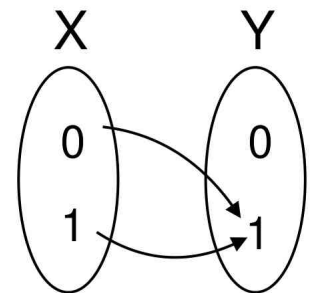
- List the three different scenarios for what's behind the doors:



Deutsch Algorithm

1985

- We want to find out whether a particular function, with one input bit and one output bit is constant or balanced. **Classically, we need to evaluate the function twice** (i.e., for input = 0 and input = 1), but remarkably, **we only need to evaluate the function once using quantum algorithm**, by using Deutsch's algorithm.
- Consider a simple function, $f(x) : \{0,1\} \longrightarrow \{0,1\}$
- For possible functions
 - Identity: $f(0) = 0$ and $f(1) = 1$
 - Bit-flip function: $f(0) = 1$ and $f(1) = 0$
 - Constant function: $f(x) = 0$ or $f(x) = 1$



The First Wave of Quantum Machine Learning?

PRL **103**, 150502 (2009)

PHYSICAL REVIEW LETTERS

week ending
9 OCTOBER 2009



Quantum Algorithm for Linear Systems of Equations

Aram W. Harrow,¹ Avinatan Hassidim,² and Seth Lloyd³

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(Received 5 July 2009; published 7 October 2009)

Solving linear systems of equations is a common problem that arises both on its own and as a subroutine in more complex problems: given a matrix A and a vector $\vec{b}$, find a vector $\vec{x}$ such that $A\vec{x} = \vec{b}$. We consider the case where one does not need to know the solution $\vec{x}$ itself, but rather an approximation of the expectation value of some operator associated with $\vec{x}$, e.g., $\vec{x}^\dagger M \vec{x}$ for some matrix M . In this case, when A is sparse, $N \times N$ and has condition number κ , the fastest known classical algorithms can find $\vec{x}$ and estimate $\vec{x}^\dagger M \vec{x}$ in time scaling roughly as $N\sqrt{\kappa}$. Here, we exhibit a quantum algorithm for estimating $\vec{x}^\dagger M \vec{x}$ whose runtime is a polynomial of $\log(N)$ and κ . Indeed, for small values of κ [i.e., poly $\log(N)$], we prove (using some common complexity-theoretic assumptions) that any classical algorithm for this problem generically requires exponentially more time than our quantum algorithm.

$$Ax = b$$

Complexity of inversion of a regular matrix= $O(N^3)$

Complexity of inversion of a sparse matrix= $O(N)$

The First Wave of Quantum Machine Learning?

PRL **103**, 150502 (2009)

PHYSICAL REVIEW

Citations per year

09



Quantum Algorithm for Linear Systems

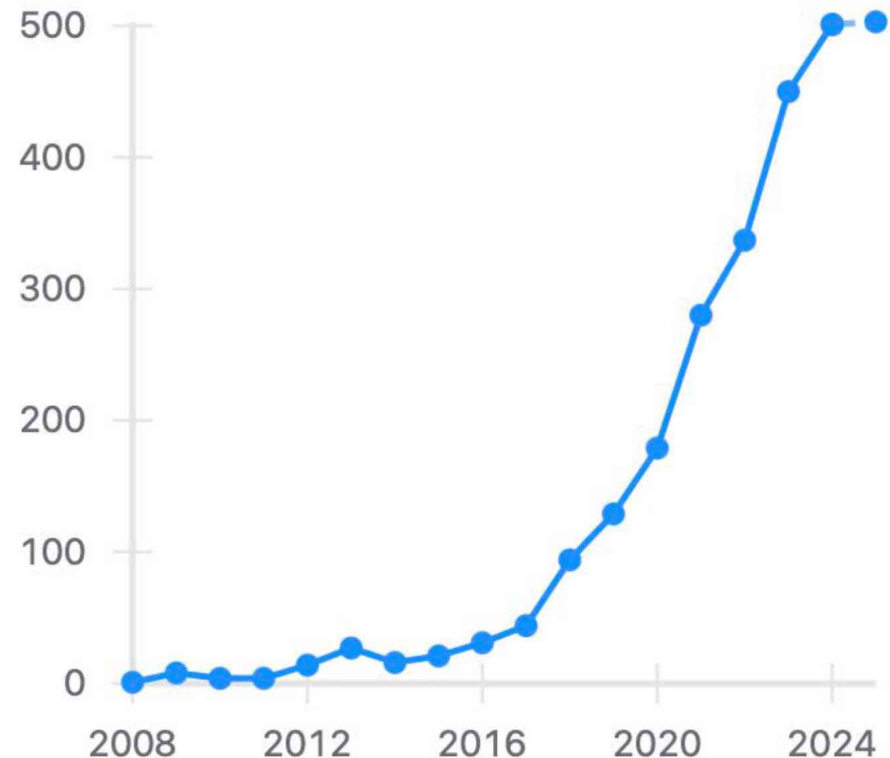
Aram W. Harrow,¹ Avinatan Hassidim

¹*Department of Mathematics, University of Bristol*

²*Research Laboratory for Electronics, MIT, Cambridge, MA*

³*Research Laboratory for Electronics and Department of Mechanical Engineering, MIT, Cambridge, MA*
(Received 5 July 2009; published 15 September 2009)

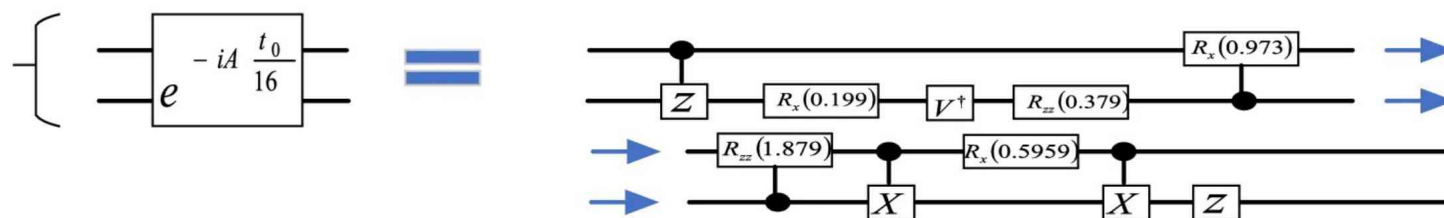
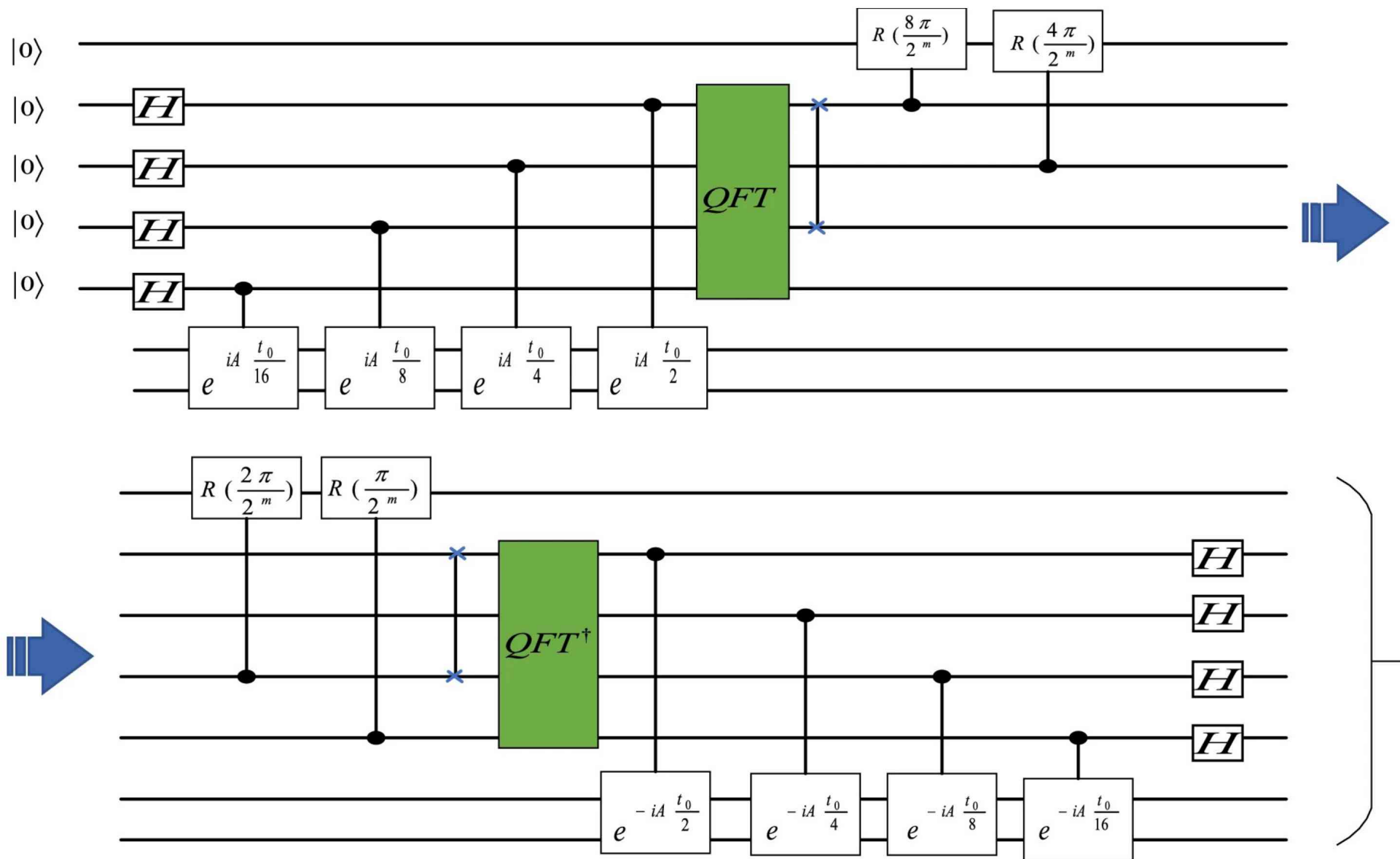
Solving linear systems of equations is a common problem in more complex problems: given a matrix A and a vector b , the case where one does not need to know the solution. The expectation value of some operator associated with $\vec{x}$, e.g. $\vec{x}^\dagger M \vec{x}$ where M is sparse, $N \times N$ and has condition number κ , the fastest classical estimate $\vec{x}^\dagger M \vec{x}$ in time scaling roughly as $N\sqrt{\kappa}$. Here, we present a quantum algorithm to estimate $\vec{x}^\dagger M \vec{x}$ whose runtime is a polynomial of $\log(N)$ and κ . Improve (using some common complexity-theoretic assumptions) the problem generically requires exponentially more time than our quantum algorithm.

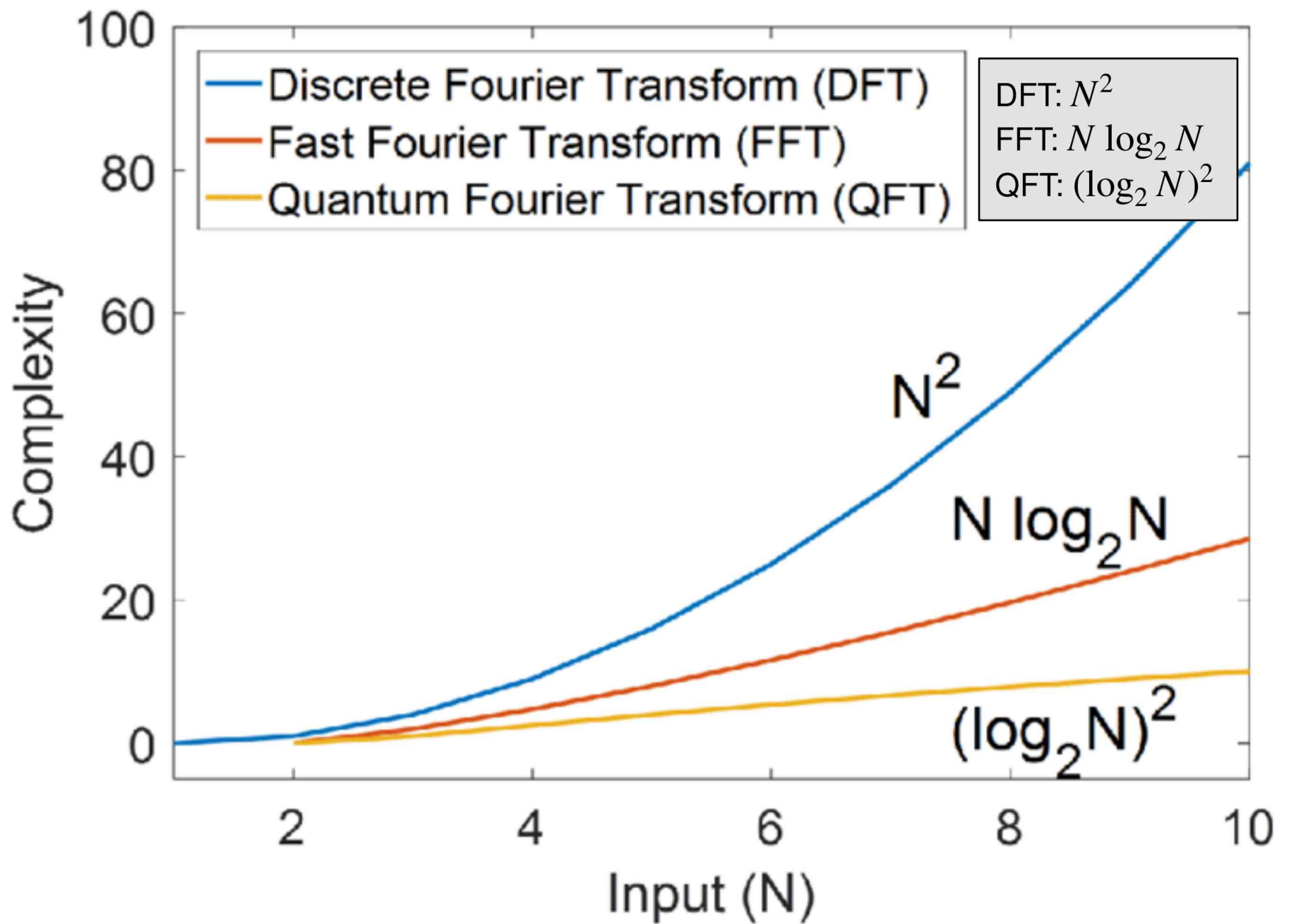


$$Ax = b$$

Complexity of inversion of a regular matrix= $O(N^3)$

Complexity of inversion of a sparse matrix= $O(N)$





Experimental realization

PRL **110**, 230501 (2013)

PHYSICAL REVIEW LETTERS

week ending
7 JUNE 2013



Experimental Quantum Computing to Solve Systems of Linear Equations

X.-D. Cai,¹ C. Weedbrook,² Z.-E. Su,¹ M.-C. Chen,¹ Mile Gu,^{3,4} M.-J. Zhu,¹ Li Li,^{1,*} Nai-Le Liu,^{1,†}
Chao-Yang Lu,^{1,‡} and Jian-Wei Pan¹

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(Received 6 March 2013; published 6 June 2013)

Solving linear systems of equations is ubiquitous in all areas of science and engineering. With rapidly growing data sets, such a task can be intractable for classical computers, as the best known classical algorithms require a time proportional to the number of variables N . A recently proposed quantum algorithm shows that quantum computers could solve linear systems in a time scale of order $\log(N)$, giving an exponential speedup over classical computers. Here we realize the simplest instance of this algorithm, solving 2×2 linear equations for various input vectors on a quantum computer. We use four quantum bits and four controlled logic gates to implement every subroutine required, demonstrating the working principle of this algorithm.

["A two-qubit photonic quantum processor and its application to solving systems of linear equations". Scientific Reports. 4: 6115.](#)
["Experimental realization of quantum algorithm for solving linear systems of equations". Physical Review A. 89 \(2\): 022313](#)

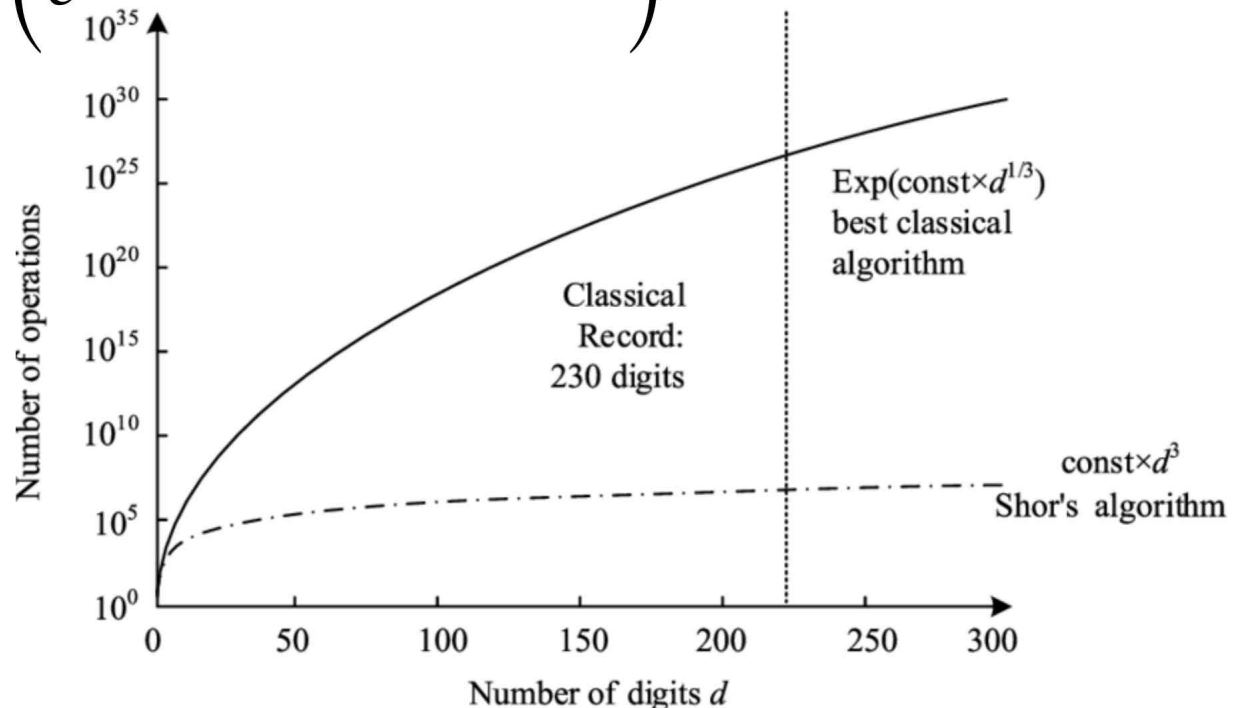
Quantum algorithms for systems of linear equations inspired by adiabatic quantum computing, Phys.Rev.Lett. 122 (2019) 6, 060504
==> ["Experimental realization of quantum algorithms for a linear system inspired by adiabatic quantum computing". Phys. Rev. A 99, 012320.](#) 8 dimensional linear equation.

Shor's factoring algorithm

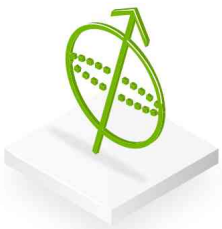
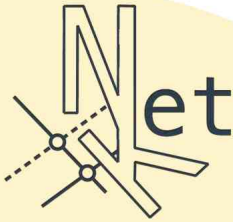
- To factor an integer N , Shor's algorithm runs in polynomial time, meaning the time taken is polynomial in $\log N$, **the size of the integer** given as input. Specifically, it takes quantum gates of order $O((\log N)^2(\log \log N)(\log \log \log N))$.
- This is significantly faster than the most efficient known classical factoring algorithm, the general number field sieve, which works in sub-exponential time: $O(e^{1.9(\log N)^{1/3}(\log \log N)^{2/3}})$

RSA-250 =
641352894770715802787901901705773
890848250147429434472081168596320
245323446302386235987526683477087
37661925585694639798853367
 ×
 333720275949781565562260106053551
 142279407603447675546667845209870
 238417292100370802574486732968818
 77565718986258036932062711

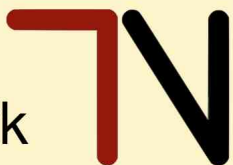
RSA factoring challenge
 (Product of exactly two primes)



A few popular tools for quantum simulation (Classical Quantum Computers)

- Qiskit (IBM)  **Qiskit**
 - IBM Quantum Composer
 - IBM Quantum Platform
 - PennyLane and Strawberry Fields (Xanadu)  PENNYLANE
 - TensorFlowQuantum (Google)  TensorFlow Quantum
 - CUDA Quantum (NVIDIA) 
 - TensorCircuit 
- 

NetKet



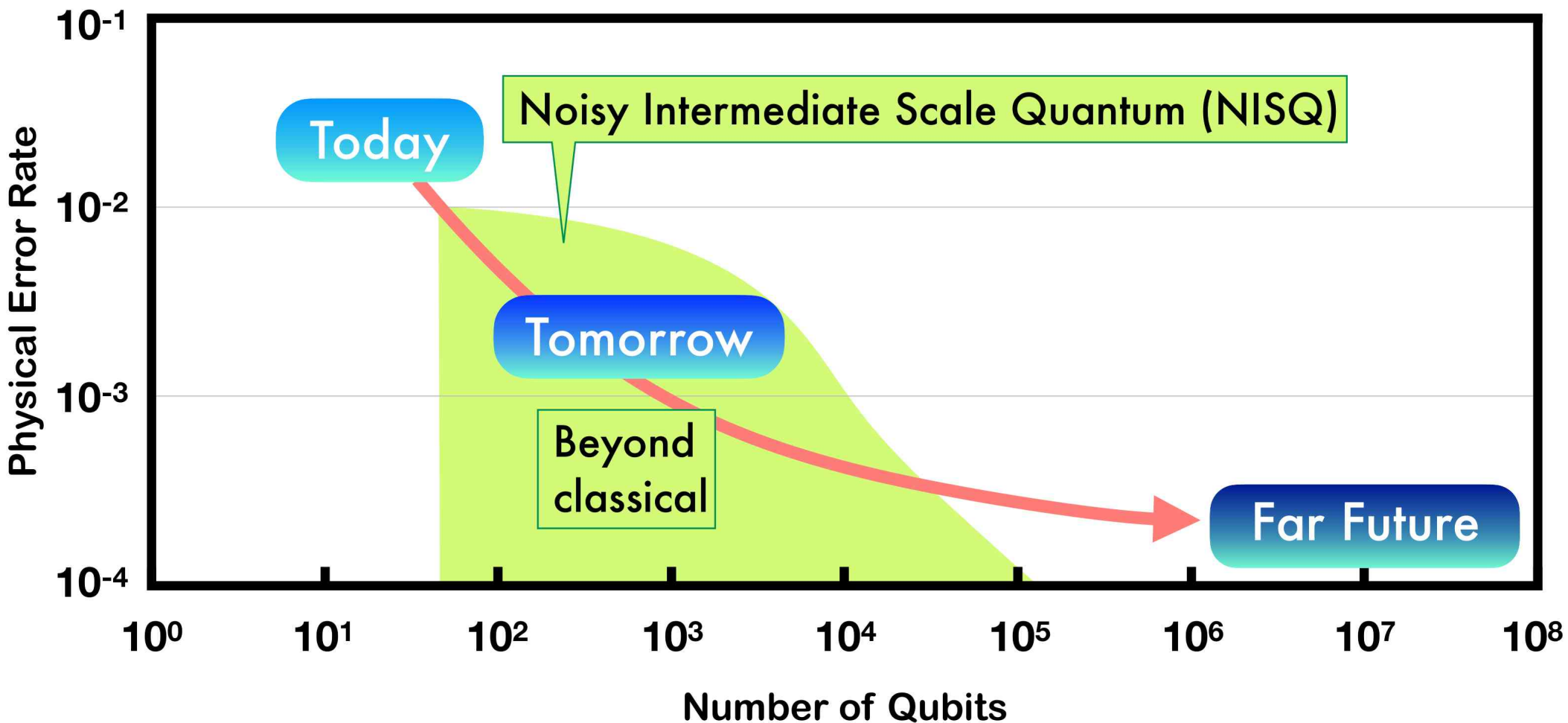
Tensor Network

Quantum computers are hard to build

- Qubits, unlike classical bits, need to interact strongly among themselves to form entangled states, which in turn form the basis for computation in quantum computers. But to achieve this experimentally is incredibly hard.

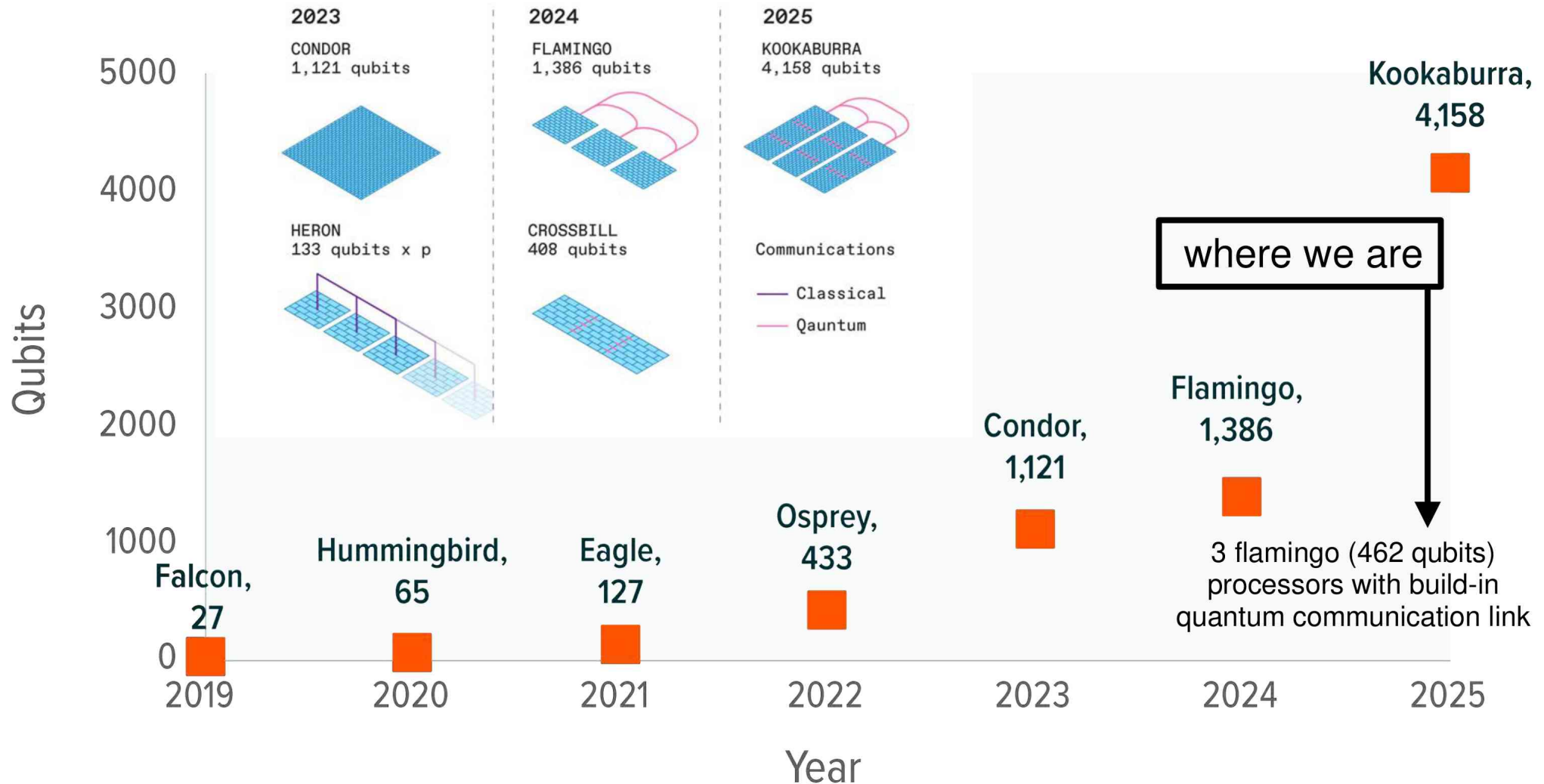


Quantum Hardware Roadmap



IBM QUANTUM PROCESSORS ROADMAP

Sources: Global X analysis of information derived from: IBM. (2022). Our new 2022 development roadmap.



Note: 2022 onwards includes planned processor launches.

IBM has been following a quantum-computing road map that roughly doubles the number of qubits every year.

A quantum computing partnership with the University of Chicago and the University of Tokyo

100,000-qubit system by 2033

The commitment, to be signed May 21 on the sidelines of the G7 and U.S.-Japan Leaders' Meeting, aims to advance development of a fault-tolerant quantum computer by supporting research, entrepreneurship and workforce training.

May 17, 2023 · 2 min read

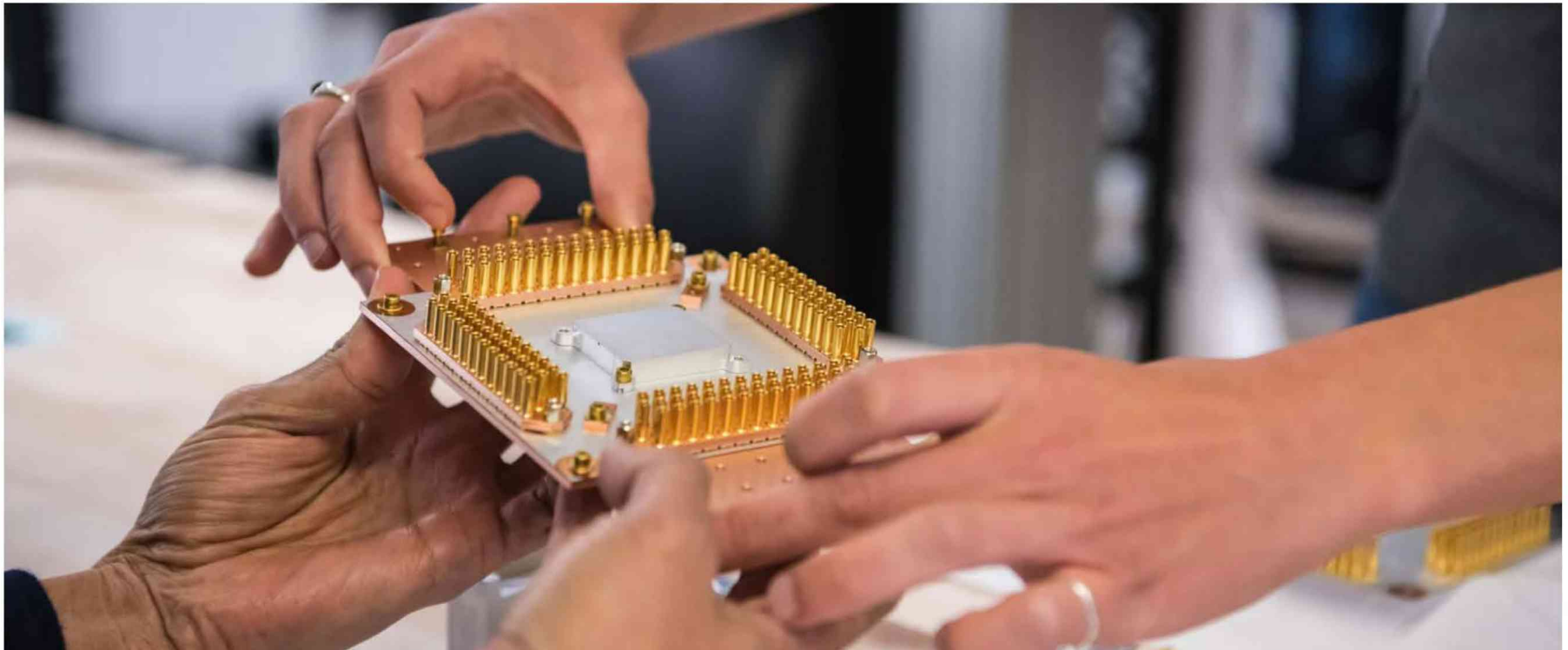


Charina Chou
Director and COO,
Google Quantum AI



Hartmut Neven
VP, Google Quantum AI

 Share



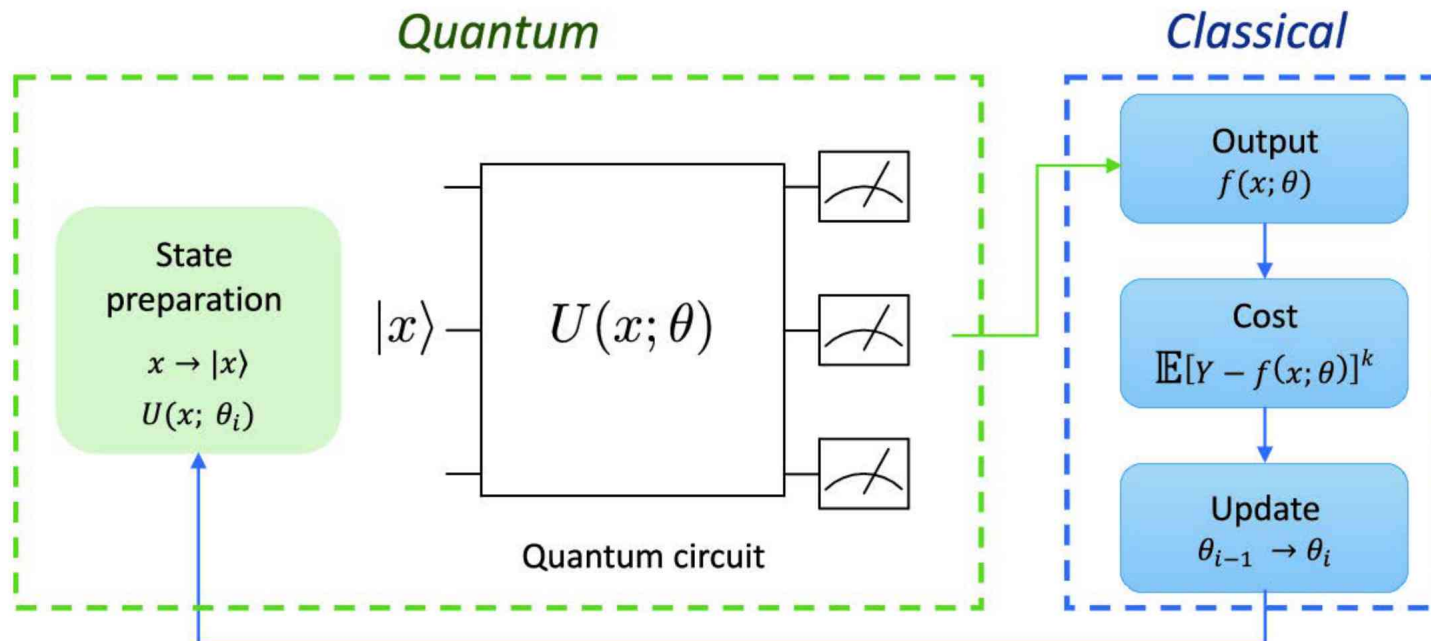
Where quantum computers are useful

- **Quantum optimization: combinatorial problems at the LHC** [2410.22417](#)
 - Finance, physics, materials discovery, science [2508.09139](#)
- **Quantum devices (qubits) as sensors: detection of dark matter and neutrino decays**
 - Dark matter detection, quantum magnetometry + electrometry, atomic clocks
- Quantum simulation/Hamiltonian simulation
 - Physics, chemistry
- **Quantum machine learning** [2510.10501](#), [2411.15315](#), [2412.21082](#), [2411.13520](#)
 - Classification, generative models, anomaly detection
- Quantum cryptography and security
 - Quantum key distribution, post-quantum cryptography
- Quantum linear algebra and numerical methods
 - HHL algorithms, eigenvalue problems
- Quantum communication and networks
 - **Entanglement and violation of Bell's inequality** [2407.01663](#), [2407.07147](#), [2305.07075](#)
 - Teleportation, quantum internet
- Quantum sampling
 - Supremacy tasks, random circuits

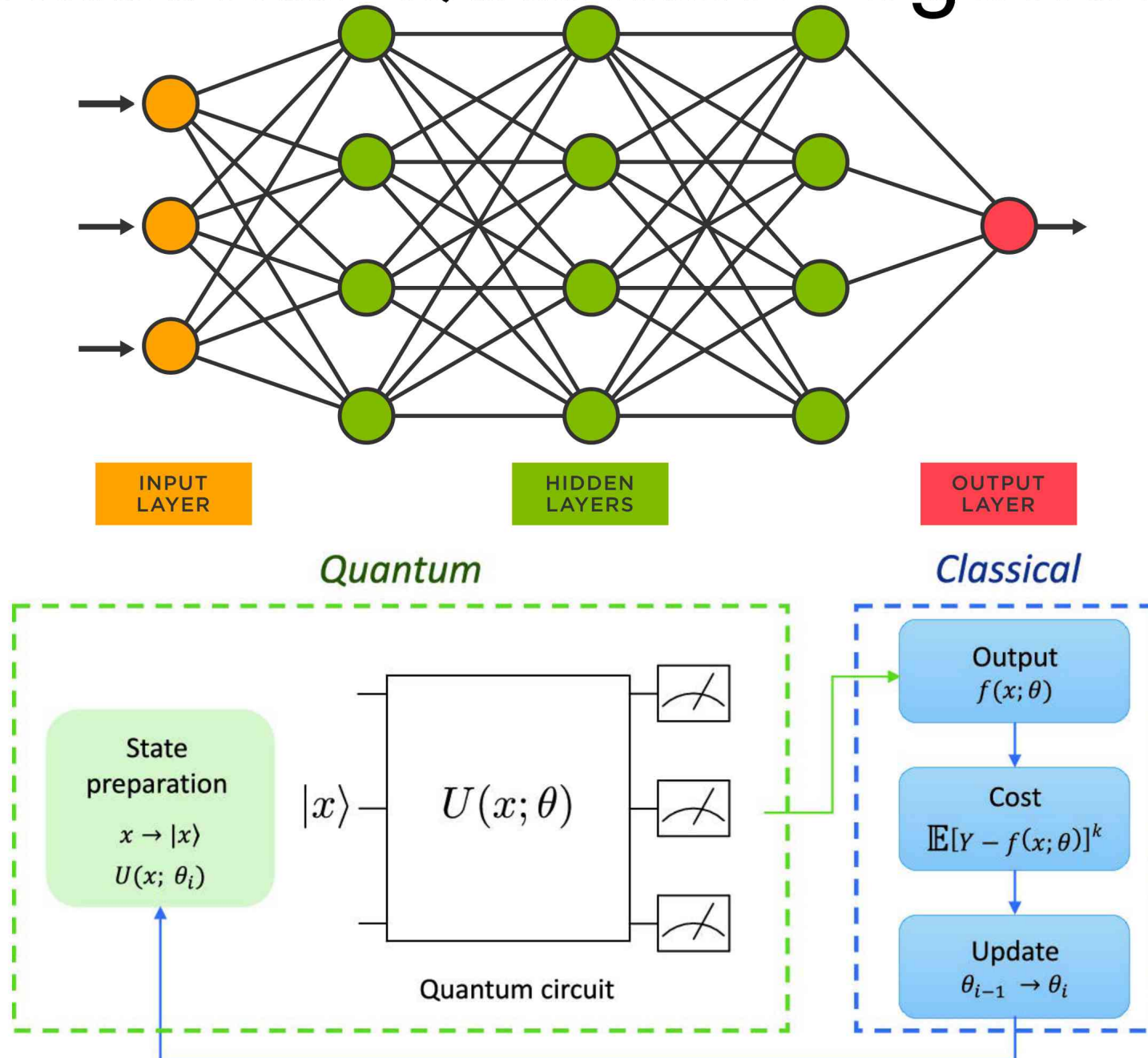
Variational Quantum Algorithms

- Hybrid quantum-classical model is suggested to circumvent the issue of going slow with quantum annealer as well as implementing Hamiltonian in the available hardware.
- Quantum: parameterize wave function
- Classical: minimize/maximize the expectation value of H in the problem.

$$E(\vec{\theta}) = \langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle$$

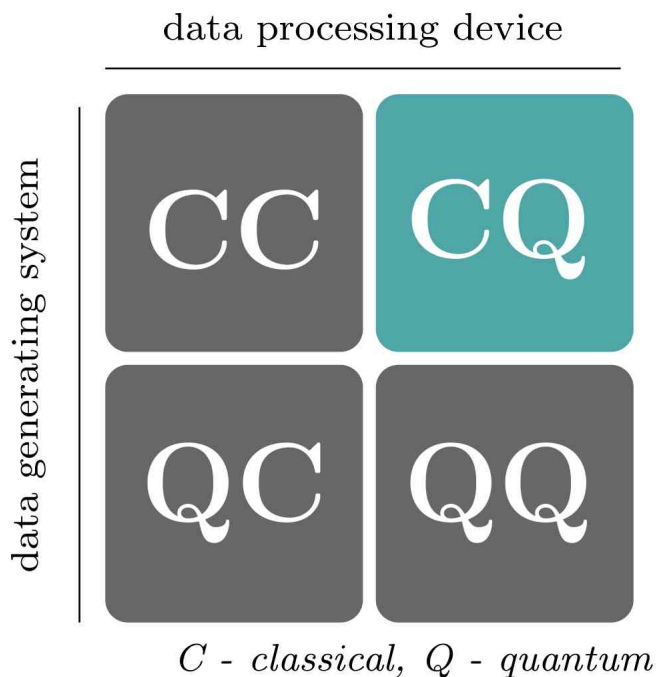


Variational Quantum Algorithms



Quantum Machine Learning

- Artificial Intelligence: Statistical prediction
- Machine Learning: Learn from data
- Quantum Machine Learning: Learn from data with quantum algorithms
 - Subdiscipline of quantum computing and quantum information science



- CC: classical data being processed classically
- QC: how machine learning can help with quantum computing
- CQ: classical data fed into quantum computer for analysis (quantum machine learning)
- QQ: quantum data being processed by quantum computer (ex: Quantum simulation)

Quantum optimization

Quantum Optimization

- Optimization problems are everywhere.
 - Continuous optimization
 - Discrete optimization: combinatorial optimization
 - Quadratic Unconstrained Binary Optimization (QUBO)
 - NP hard problem
 - Quadratic function might have several local minima
 - Close connection to Ising model
- Apply quantum algorithms to solve optimization problem
 - Gate model: use universal gates (Pauli's)
 - Non-gate model (quantum annealer): relies on adiabatic theorem to find a minimum energy of Hamiltonian corresponding to the minimum value of some cost function.

Combinatorial problems at the LHC

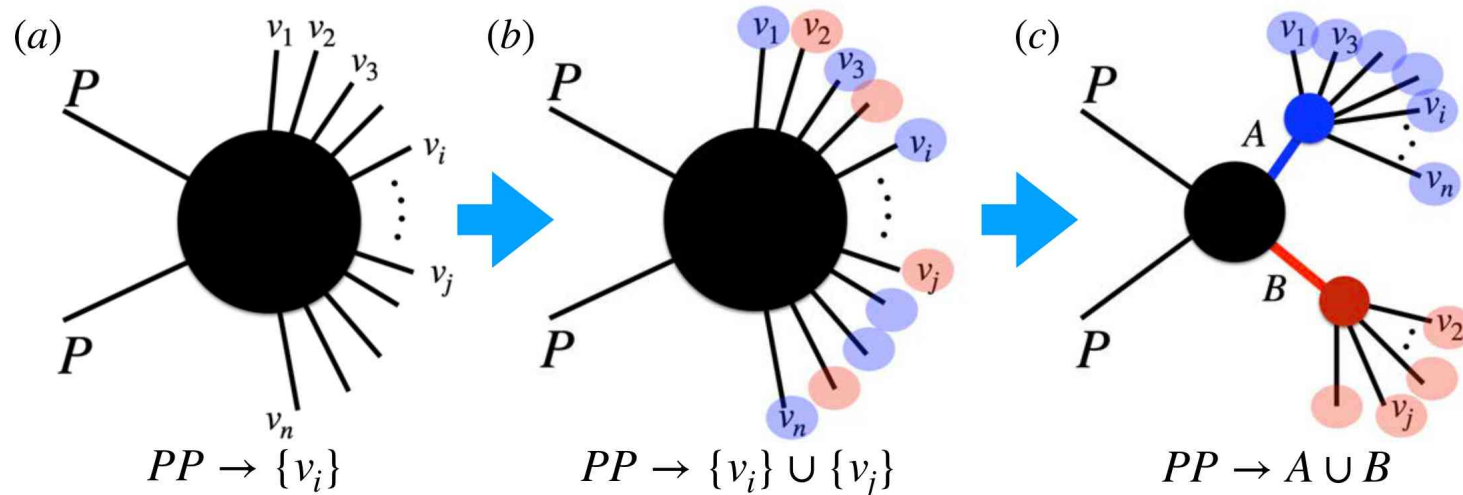
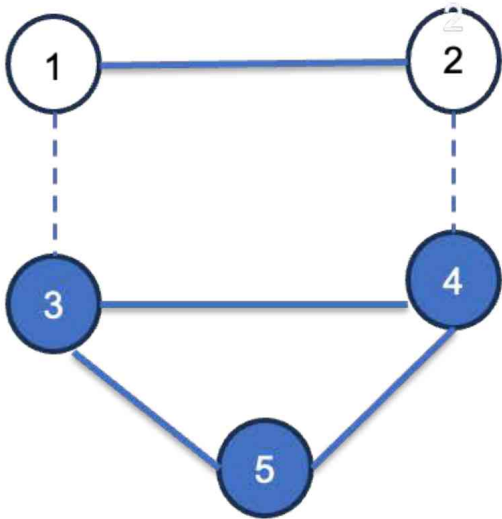


FIG. 1. (a) n -observed particles (b) Dividing n particles into two groups for $2 \rightarrow 2$ process (c) Identified event-topology with A and B .

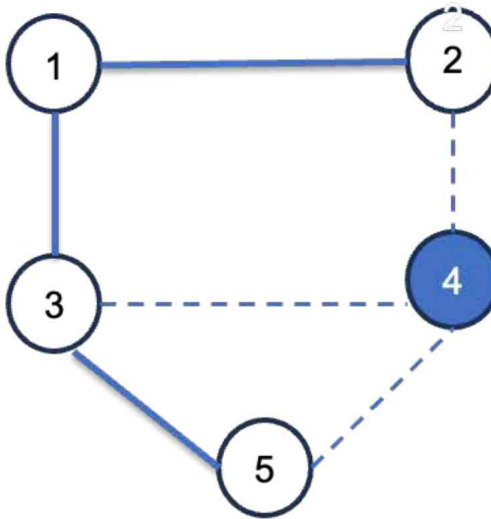
- Assuming $2 \rightarrow 2$ production with subsequent decays, identification of an event-topology becomes a binary classification, with 2^{n-1} possibilities.
- **Combinatorial problem: What would be an efficient way of assigning all observed particles in two decay chains?**

MaxCut problem

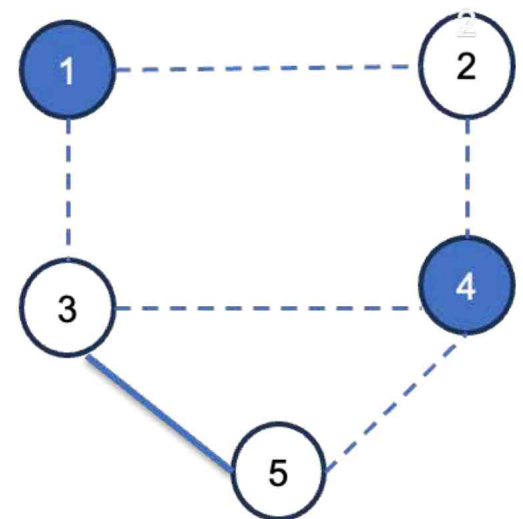
- Objective:
 - Maximize the number of cut edges in a graph when split into 2 parts
 - Divides the set of nodes in the graph into two subsets so that we have as many edges as possible that go between the two sets



Cut size = 2



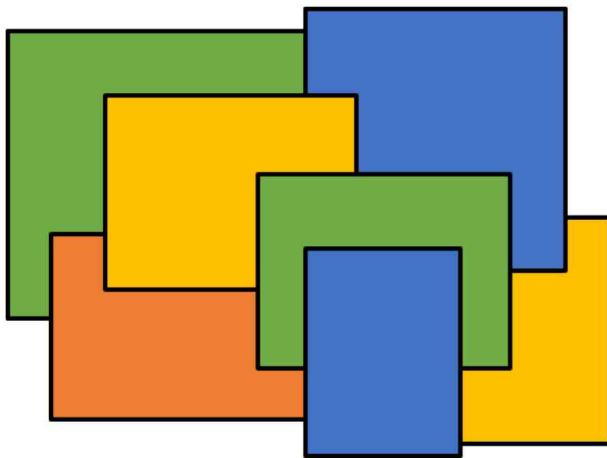
Cut size = 3



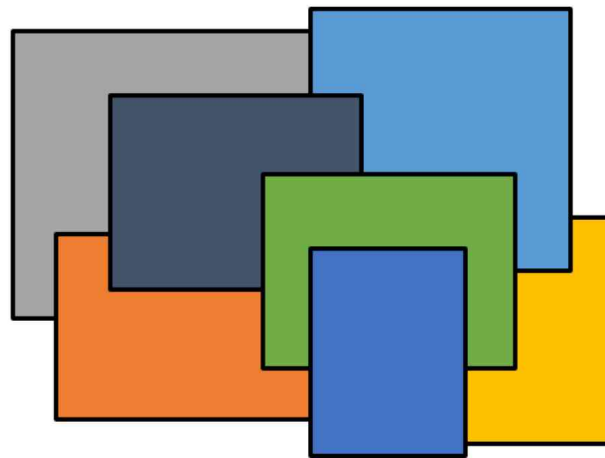
Cut size = 5

Map Coloring

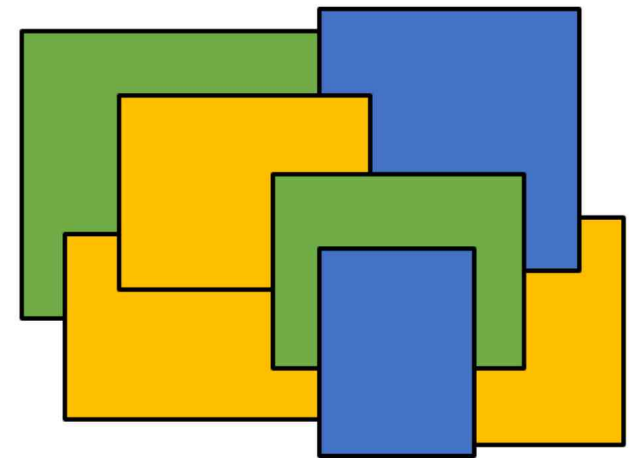
- Problem: Color the regions of a given map such that
 - at most four colors are used
 - no two adjacent* regions have the same color



Valid four-coloring



Invalid; more than 4 colors

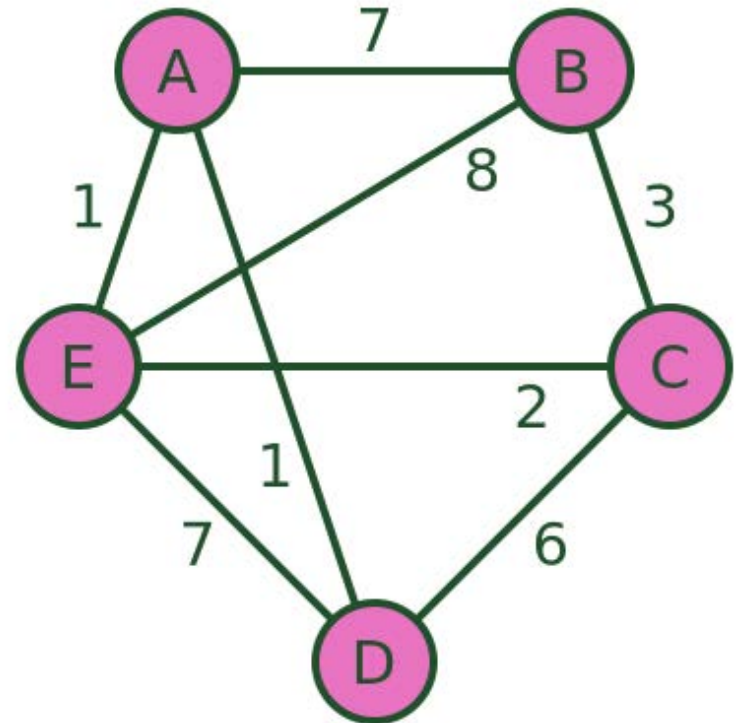


Invalid; yellows touch

* "adjacent" means "shares an edge"; sharing a corner is OK

Traveling salesman problem (TSP)

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?
- It is an NP-hard problem in combinatorial optimization, important in theoretical computer science and operations research.



Traveling salesman problem (TSP)

- Label vertices $1, \dots, N$ and the edge set (uv) to be directed. (For undirected, add (uv) and (vu)).
- Introduce N^2 bits $x_{v,i}$, where v represents the vertex and i represents its order to the edge set.
- Every vertex can only appear once in a cycle, and that there must be a j -th node in the cycle for each j .
- For the nodes in our prospective ordering, if $x_{u,j}$ and $x_{u,j+1}$ are both 1, then there should be an energy penalty if $(uv) \notin E$.

$$H_A = A \sum_{v=1}^n \left(1 - \sum_{j=1}^N x_{v,j} \right)^2 + A \sum_{j=1}^n \left(1 - \sum_{v=1}^N x_{v,j} \right)^2 + A \sum_{(uv) \notin E} \sum_{j=1}^N x_{u,j} x_{v,j+1}$$

- $A > 0$ is a constant. The ground state of this system has $H = 0$, only if we have an ordering of vertices where each vertex is only included once, and adjacent vertices in the cycle have edges on the graph – i.e., we have a **Hamiltonian cycle**.

- The traveling salesman also needs to minimize the total traveling distance (weight)

$$H_B = B \sum_{(uv) \in E} W_{uv} \sum_{j=1}^N x_{u,j} x_{v,j+1}$$

- B should be small enough that it is never favorable to violate the constraints of H_A . $0 < B \max(W_{uv}) < A$

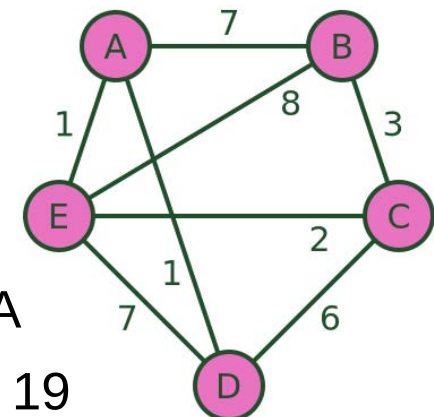
$$H_{\text{full}} = H_P = H_A + H_B$$

	1	2	3	4	5
A	1				
B			1		
C				1	
D					
E		1			1

DAECB

AEBCDA

$$1+8+3+6+1 = 19$$



Breaking limitation of quantum annealer in solving optimization problems under constraints

Masayuki Ohzeki^{1,2,3*}

2002.05298

[Nature, Scientific Reports volume 10, Article number: 3126 \(2020\)](#)

Quantum annealing is a generic solver for optimization problems that uses fictitious quantum fluctuation. The most ground-breaking progress in the research field of quantum annealing is its hardware implementation, i.e., the so-called quantum annealer, using artificial spins. However, the connectivity between the artificial spins is sparse and limited on a special network known as the chimera graph. Several embedding techniques have been proposed, but the number of logical spins, which represents the optimization problems to be solved, is drastically reduced. In particular, an optimization problem including fully or even partly connected spins suffers from low embeddable size on the chimera graph. In the present study, we propose an alternative approach to solve a large-scale optimization problem on the chimera graph via a well-known method in statistical mechanics called the Hubbard-Stratonovich transformation or its variants. The proposed method can be used to deal with a fully connected Ising model without embedding on the chimera graph and leads to nontrivial results of the optimization problem. We tested the proposed method with a number of partition problems involving solving linear equations and the traffic flow optimization problem in Sendai and Kyoto cities in Japan.

Supply chain logistics

Route Optimization

Portfolio management

Nurse scheduling problem

Image recognition

Remote sensing imagery

Classifier optimization

Drug discovery

Material science

Seismic inversion

Quantum simulation

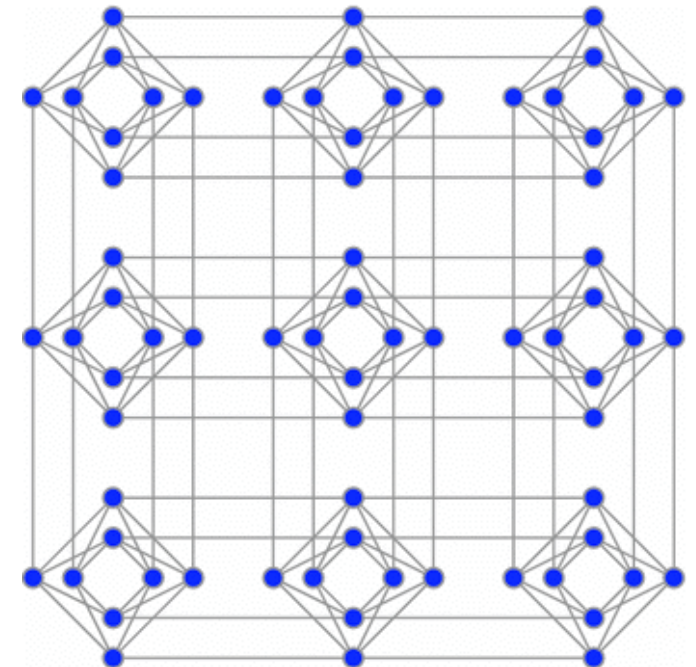
Financial modeling

Cryptography

Better batteries

Cleaner fertilization

Electronic material discovery



Quadratic Unconstrained Binary Optimization (QUBO)

- QUBO: combinatorial optimization problem with a wide range of applications from finance to ML (partitioning, graph coloring, task allocation, max-sat, max-cut etc)

$$f: \mathbb{Z}_2^n \longrightarrow \mathbb{R}$$

Quadratic polynomial over binary variable

$$f(x) = \sum_{i=1}^n \sum_{j=1}^i q_{ij} x_i x_j + \sum_{i=1}^n h_i x_i$$

$$x_i \in \mathbb{Z}_2 = \{0,1\}, \quad h_i, q_{ij} \in \mathbb{R}$$

$$x = (x_n x_{n-1} \cdots x_2 x_1)$$

- Find a binary vector x^* which minimizes f (binary strings of n-bits)

$$x^* = \underset{x \in \mathbb{Z}_2^n}{\operatorname{argmin}} f(x)$$

- In matrix notation, $f(x) = x^T Q x$, where $Q \in \mathbb{R}^{n \times n}$
- NP-hard problem: no polynomial-time algorithms are known.

Ising formulations of many NP problems

1 Introduction

- 1.1 Quantum Adiabatic Optimization
- 1.2 Ising Spin Glasses
- 1.3 The Goal of This Paper
- 1.4 What Problems Are Easy (to Embed) on Experimental AQO Devices?

Andrew Lucas

1302.5843

2 Partitioning Problems

- 2.1 Number Partitioning
- 2.2 Graph Partitioning
- 2.3 Cliques
- 2.4 Reducing N to $\log N$ Spins in Some Constraints

$$H = - \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z$$

Fully connected (mean field) Ising model

3 Binary Integer Linear Programming

4 Covering and Packing Problems

- 4.1 Exact Cover
- 4.2 Set Packing
- 4.3 Vertex Cover
- 4.4 Satisfiability
- 4.5 Minimal Maximal Matching

5 Problems with Inequalities

- 5.1 Set Cover
- 5.2 Knapsack with Integer Weights

6 Coloring Problems

- 6.1 Graph Coloring
- 6.2 Clique Cover
- 6.3 Job Sequencing with Integer Lengths

7 Hamiltonian Cycles

- 7.1 Hamiltonian Cycles and Paths
- 7.2 Traveling Salesman

8 Tree Problems

- 8.1 Minimal Spanning Tree with a Maximal Degree Constraint
- 8.2 Steiner Trees
- 8.3 Directed Feedback Vertex Set
- 8.4 Undirected Feedback Vertex Set
- 8.5 Feedback Edge Set

9 Graph Isomorphisms

10 Conclusions

- 1. Quantum annealers
- 2. Universal Quantum Computers

Portfolio optimization

Markowitz model

$$F = \sum_{ij} C_{ij} n_i n_j - \zeta \sum_i R_i n_i$$

Diagram illustrating the Markowitz model equation for portfolio optimization:

- F : risk minus returns
- C_{ij} : correlations between assets
- $n_i n_j$: no. of assets of type- i
- ζ : risk tolerance
- R_i : return on i 'th asset

Spin glasses

Sherrington-Kirkpatrick model

$$H = \sum_{ij} J_{ij} s_i s_j + \sum_i h_i s_i$$

Diagram illustrating the Sherrington-Kirkpatrick model equation for spin glasses:

- H : Hamiltonian
- J_{ij} : couplings between sites
- $s_i s_j$: spin at site i
- h_i : external magnetic field at i 'th site

Neural networks

Hopfield model

$$E = \sum_{ij} w_{ij} v_i v_j + \sum_i \theta_i v_i$$

Diagram illustrating the Hopfield model equation for neural networks:

- E : energy function
- w_{ij} : weights of the learning rule
- $v_i v_j$: state of neuron i
- θ_i : activation threshold for i 'th neuron

P vs NP

- In Theoretical Computer Science, the two most basic classes of problems are P and NP.
- **P** includes all problems that **can be solved “efficiently”**.
 - For example: add two numbers. The formal definition of "efficiently" is in **time that's polynomial in the input's size**.
- **NP** (nondeterministic polynomial (time)) includes all problems that given a solution, one can efficiently verify that the solution is correct.
 - An example is the following problem: given a bunch of numbers, can they be split into two groups such that the sum of one group is the same as the other. Clearly, if one is given a solution (two groups of numbers), it's simple to verify that the sum is the same. (This is a partitioning problem).
- **What's unknown is whether problems such as the one above have an efficient algorithm for finding the solution.** This is the (in)famous (unsolved) **P = NP problem**, and the common conjecture is that no such algorithm exists.
- Now, **NP hard problems** are such problems that were shown that if they can be efficiently solved (which, as mentioned, is believed to not be the case), then each and every problem in NP (each and every problem whose results can be efficiently verified) can be efficiently solved. **In other words, if you're up to showing that P=NP, you might want to take a stand at any of those NP-hard problems since they are "equivalent" in some way to all others.**

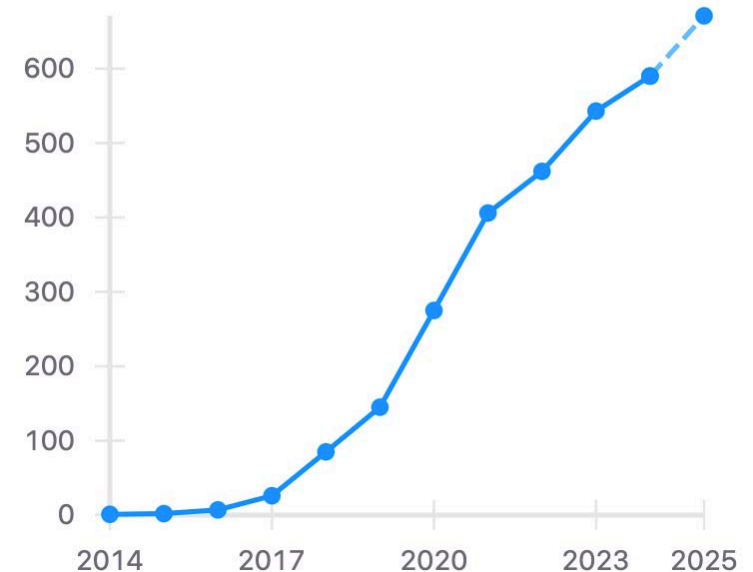
Quantum Approximate Optimization Algorithm (QAOA)

- Abstract:** We introduce a quantum algorithm that produces approximate solutions for combinatorial optimization problems. The algorithm depends on a positive integer p and the quality of the approximation improves as p is increased. The quantum circuit that implements the algorithm consists of unitary gates whose locality is at most the locality of the objective function whose optimum is sought. The depth of the circuit grows linearly with p times (at worst) the number of constraints. If p is fixed, that is, independent of the input size, the algorithm makes use of efficient classical preprocessing. If p grows with the input size a different strategy is proposed. We study the algorithm as applied to MaxCut on regular graphs and analyze its performance on 2-regular and 3-regular graphs for fixed p . For $p = 1$, on 3-regular graphs the quantum algorithm always finds a cut that is at least 0.6924 times the size of the optimal cut.

1411.4028

E. Farhi,
J. Goldstone,
S. Gutmann

Citations per year



Quantum Approximate Optimization Algorithm (QAOA)

Farhi et al 2014

$$H_P = C(s) = H_C = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right) : \text{Problem Hamiltonian}$$

(i, j) : the edge index

i : vertex index

$$H_M = B = H_B = \sum_j \sigma_j^X : \text{Mixer Hamiltonian (Initial Hamiltonian)}$$

Full Hamiltonian: $H(t) = \left(1 - \frac{t}{T} \right) H_M + \frac{t}{T} H_P$

$H(0) = H_M = \text{initial Hamiltonian}$

$H(T) = H_P = \text{problem Hamiltonian}$

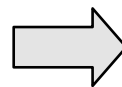
$$|\psi\rangle = \exp \left[-i \int_0^t H(t') dt' \right] |\psi_0\rangle = \exp \left[-i \sum_{j=1}^p H(j\Delta t) \Delta t \right] |\psi_0\rangle$$

$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left[\left(1 - \frac{j\Delta t}{T} \right) H_M + \frac{j\Delta t}{T} H_P \right] \right] |\psi_0\rangle$$

$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left(1 - \frac{j\Delta t}{T} \right) H_M \right] \exp \left[-i\Delta t \frac{j\Delta t}{T} H_P \right] |\psi_0\rangle$$

Works in the adiabatic limit
or $p \rightarrow \infty$

$$= \prod_{j=1}^p \underbrace{\exp \left[-i\beta_j H_M \right]}_{U(H_M, \beta_j)} \underbrace{\exp \left[-i\gamma_j H_P \right]}_{U(H_P, \gamma_j)} |\psi_0\rangle$$

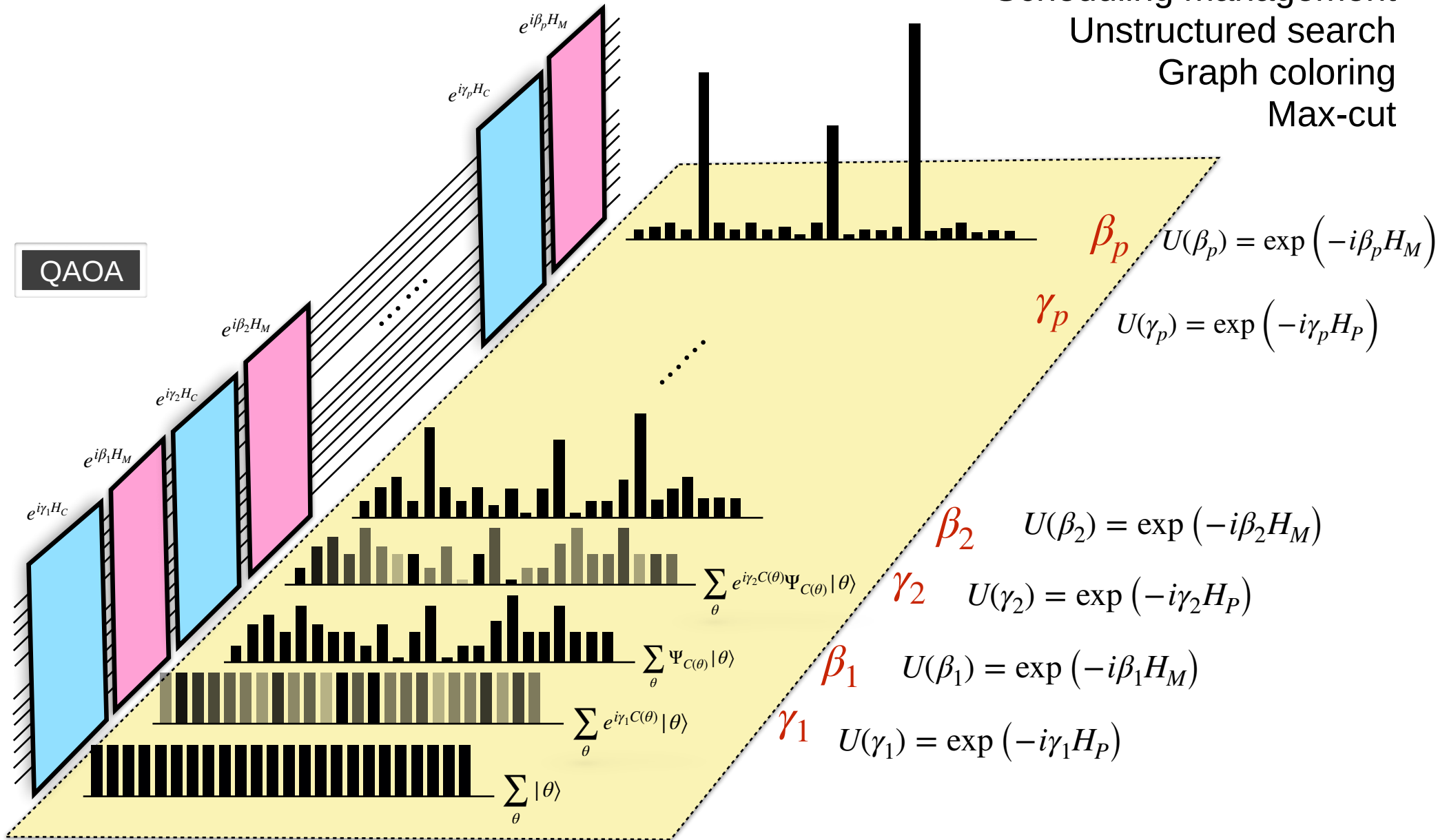


$$|\gamma, \beta\rangle = \prod_{j=1}^p U(H_M, \beta_j) U(H_P, \gamma_j) |+\rangle^{\otimes n}$$

QAOA

QAOA

Maximum Likelihood detection
Traveling salesman problem
Scheduling management
Unstructured search
Graph coloring
Max-cut



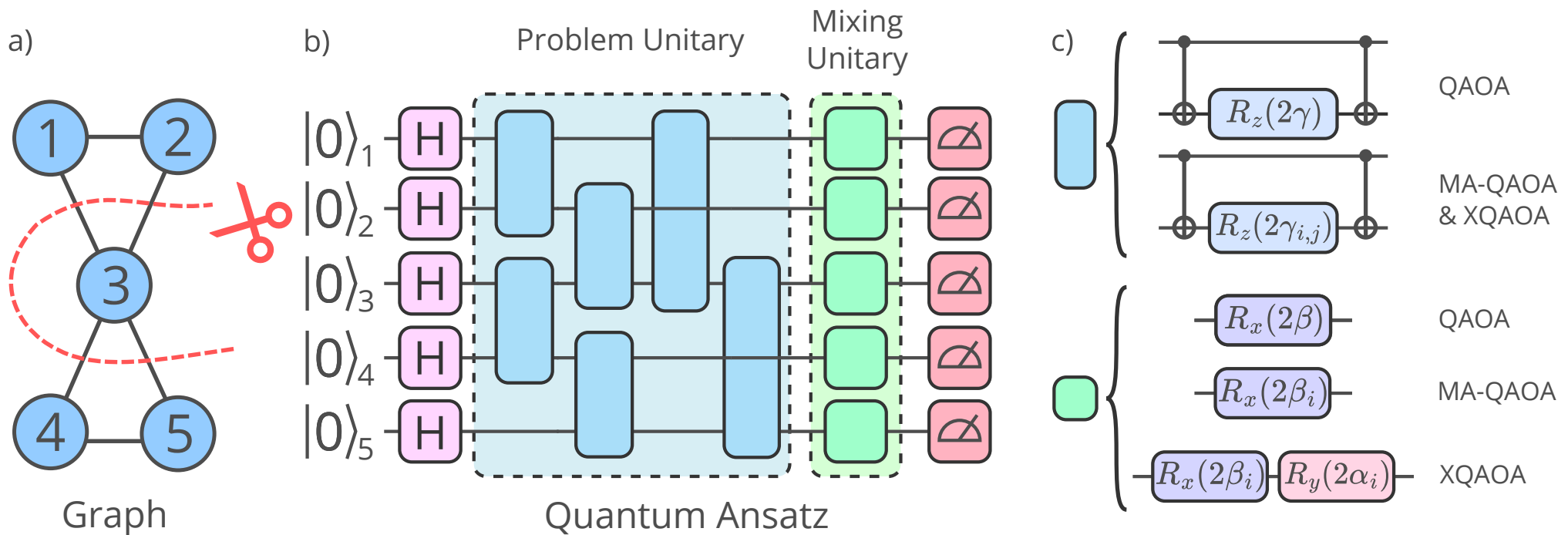


FIG. 1: a) A specific instance of a graph for which we want to identify a set of vertices that maximises the number of edges that are cut. b) A quantum circuit with a single iteration of a quantum ansatz applied to it. The quantum ansatz consists of a unitary operation specific to the problem being solved and a problem-independent mixing unitary. c) Decomposing the problem and mixing unitaries for QAOA, MA-QAOA, and XQAOA into CNOT and single-qubit rotation gates.

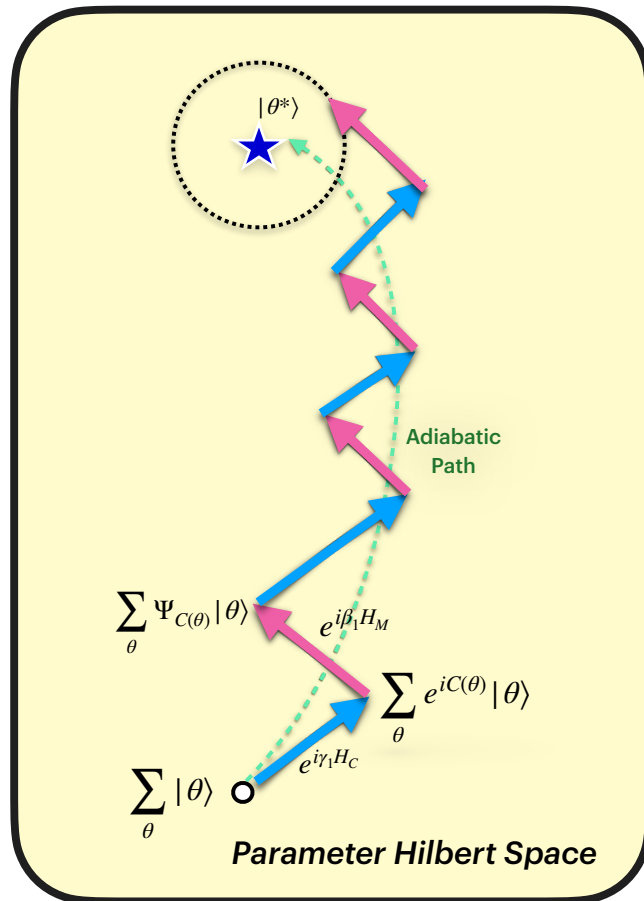
- QAOA: one parameter for each mixer layer
- ma-QAOA: n_q parameters for each mixer layer
- XQAOA: $2n_q$ parameters for each mixer layer

Multi-angle Quantum Approximate Optimization Algorithm: 2109.11455

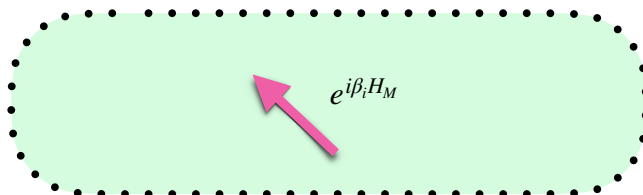
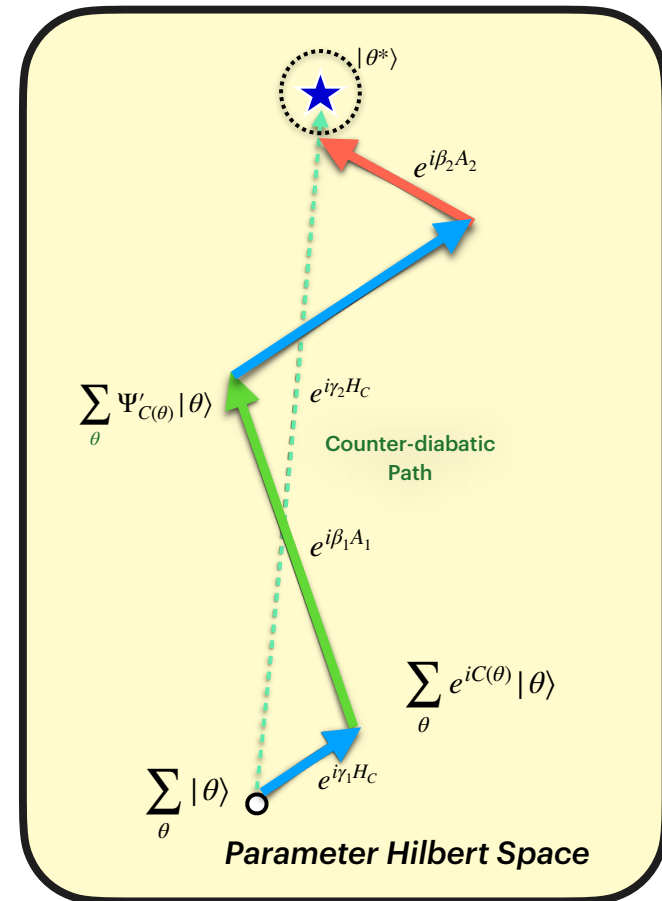
An Expressive Ansatz for Low-Depth Quantum Approximate Optimisation: 2302.04479

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)

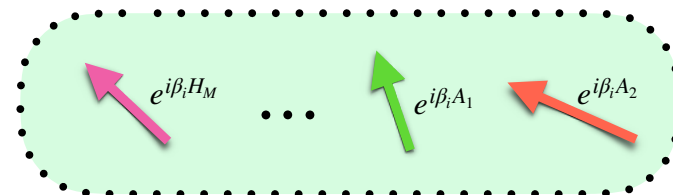
QAOA



Adaptive QAOA



Mixers Pool



Feedback-based ALgorithm Quantum Optimization (FALQON)

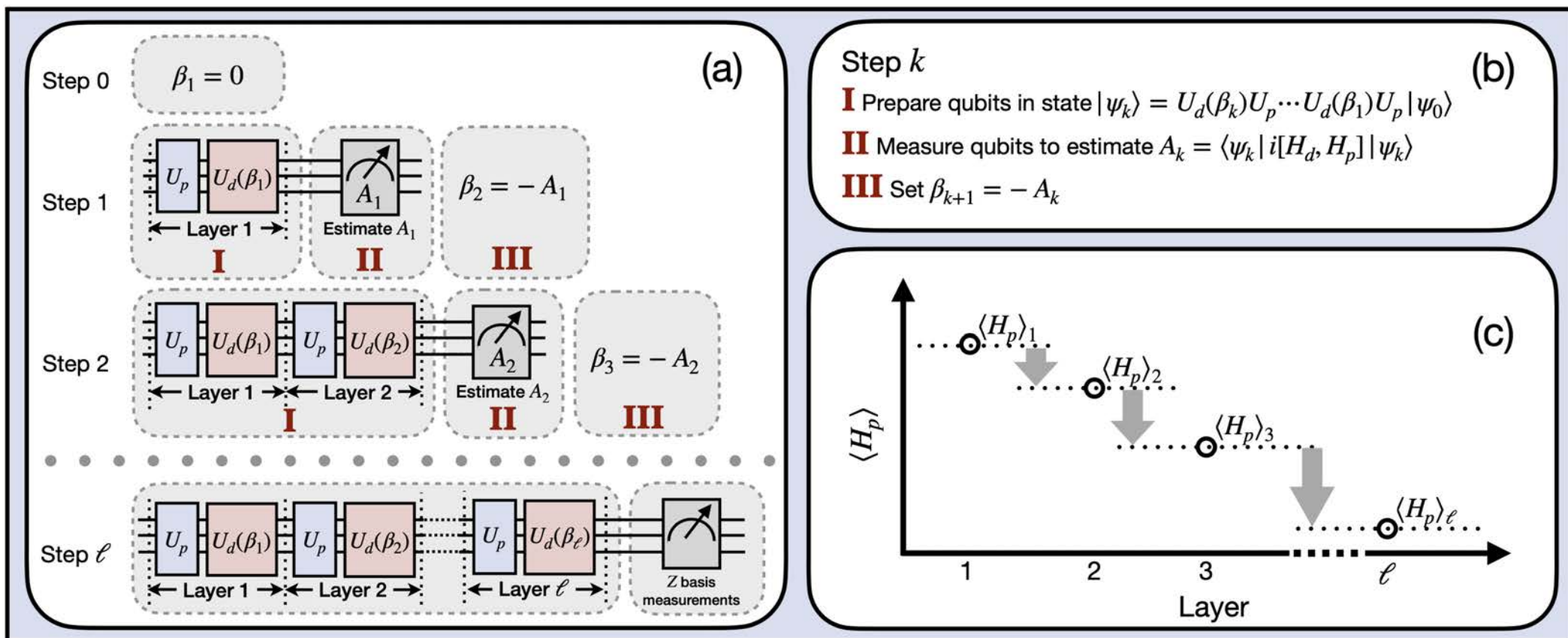


Figure 1. (a) The procedure for implementing FALQON. The initial step is to seed the procedure by setting $\beta_1 = 0$. The qubits are then initialized in the state $|\psi_0\rangle$, and a single FALQON layer is implemented to prepare $|\psi_1\rangle = U_d(\beta_1)U_p|\psi_0\rangle$. The qubits are then measured to estimate A_1 , whose result is fed back to set $\beta_2 = -A_1$, up to sampling error. For subsequent steps $k = 2, \dots, \ell$, the same procedure is repeated, as shown in (b): the qubits are initialized as $|\psi_0\rangle$, after which k layers are applied to obtain $|\psi_k\rangle = U_d(\beta_k)U_p \cdots U_d(\beta_1)U_p|\psi_0\rangle$, and then the qubits are measured to estimate A_k , and the result is fed back to set the value of β_{k+1} . This procedure causes $\langle H_p \rangle$ to decrease layer-by-layer as per $\langle \psi_1 | H_p | \psi_1 \rangle \geq \langle \psi_2 | H_p | \psi_2 \rangle \geq \dots \geq \langle \psi_\ell | H_p | \psi_\ell \rangle$, as shown in (c), such that the quality of the solution to the combinatorial optimization problem monotonically improves with circuit depth. The protocol can be terminated when the value of $\langle H_p \rangle$ converges or a threshold number of layers ℓ is reached. Then, after the final step, Z basis measurements on $|\psi_\ell\rangle$ can be used to determine a best candidate solution to the combinatorial optimization problem of interest, by repeatedly sampling from the probability distribution over bit strings induced by $|\psi_\ell\rangle$ and selecting the outcome associated with the best solution.

Variational Quantum Imaginary Time Evolution (VarQITE)

- State time evolution under a Hamiltonian: $|\psi(t)\rangle = \sum_m c_m \exp\left(-iE_m \frac{t}{\hbar}\right) |m\rangle$
- Imaginary time, $t \longrightarrow -i\beta$

$$|\psi(\beta)\rangle = \sum_m c_m \exp\left(-E_m \frac{\beta}{\hbar}\right) |m\rangle \underset{\beta \rightarrow \infty}{\sim} c_0 \exp\left(-E_0 \frac{\beta}{\hbar}\right) |0\rangle$$

All coefficients decay $\nearrow$

$$|0\rangle = \lim_{\beta \rightarrow \infty} \frac{|\psi(\beta)\rangle}{\| |\psi(\beta)\rangle \|}$$

$$|\psi_{n+1}\rangle = \frac{e^{-\Delta\tau H \hbar} |\psi_n\rangle}{\| e^{-\Delta\tau H \hbar} |\psi_n\rangle \|}$$

- In VarQITE, $|\psi(\beta)\rangle$ is replaced with $|\psi(\theta(\beta))\rangle$ with variational parameters θ . Using McLachlan variational principle, the algorithm update variational parameters θ by minimizing

$$\left\| \left(\frac{\partial}{\partial \beta} + H - E_\beta \right) |\psi(\theta(\beta))\rangle \right\|$$

2404.16135, Performant near-term quantum combinatorial optimization

1901.07653, Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution

1812.08767, Theory of variational quantum simulation

Combinatorial problems at the LHC

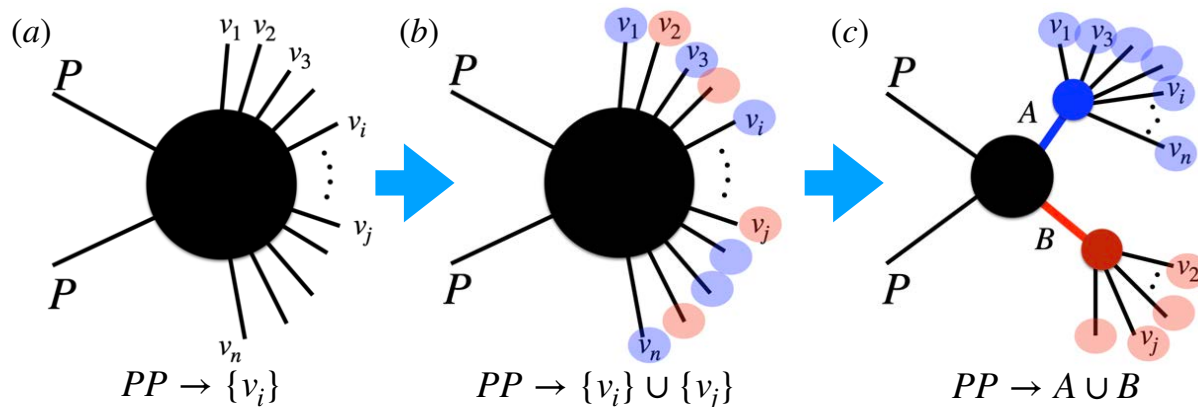


FIG. 1. (a) n -observed particles (b) Dividing n particles into two groups for $2 \rightarrow 2$ process (c) Identified event-topology with A and B .

- Assuming $2 \rightarrow 2$ production with subsequent decays, identification of an event-topology becomes a binary classification, with 2^{n-1} possibilities.

- Combinatorial problem: What would be an efficient way of assigning all observed particles in two decay chains?**

p_i is the momentum of constituent of A if $x_i = 1$

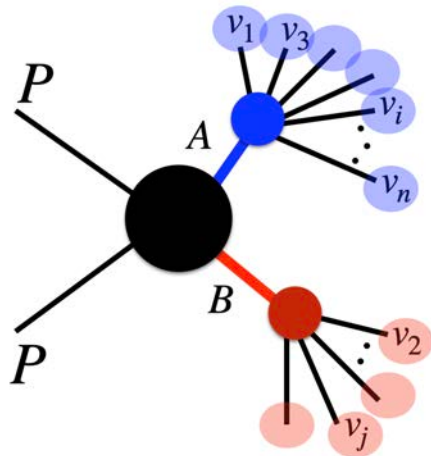
$$P_1 = \sum_i p_i x_i$$

p_i is the momentum of constituent of B if $x_i = 0$

$$P_2 = \sum_i p_i (1 - x_i)$$

Minimize the mass difference: $H = (P_1^2 - P_2^2)^2$ for all possible combinations of x_i

Combinatorial problems in the top quark production



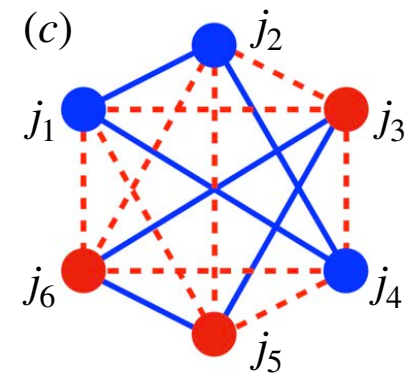
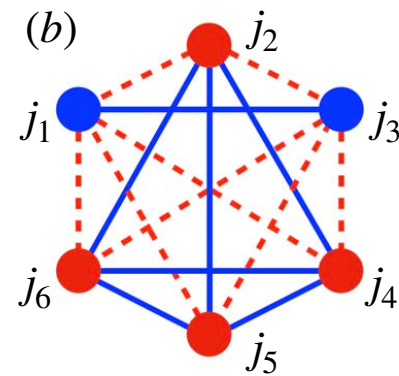
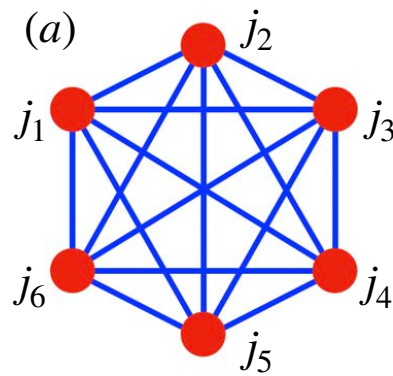
$$PP \rightarrow A \cup B$$

$$H_0 = (P_1^2 - P_2^2)^2$$

$$P_1 = \sum_{i=1}^n p_i x_i,$$

$$P_2 = \sum_{i=1}^n p_i (1 - x_i),$$

$$H_P = H_0 + \lambda H_1$$



$$\begin{aligned} H_0 &= \left(\sum_{ij} P_{ij} x_i x_j - \sum_{ij} P_{ij} (1 - x_i)(1 - x_j) \right)^2 \\ &= \left(\frac{1}{4} \sum_{ij} P_{ij} [(1 + s_i)(1 + s_j) - (1 - s_i)(1 - s_j)] \right)^2 \\ &= \left(\sum_{ij} P_{ij} s_i \right)^2 = \sum_{ij} J_{ij} s_i s_j, \end{aligned}$$

$$\begin{aligned} H_1 &= (P_1^2 + P_2^2) \\ &= \frac{1}{4} \sum_{ij} P_{ij} [(1 + s_i)(1 + s_j) + (1 - s_i)(1 - s_j)] \\ &\rightarrow \frac{1}{2} \sum_{ij} P_{ij} s_i s_j, \end{aligned}$$

$$x_i = \frac{1 + s_i}{2}$$

$$J_{ij} = \sum_{k\ell} P_{ik} P_{j\ell},$$

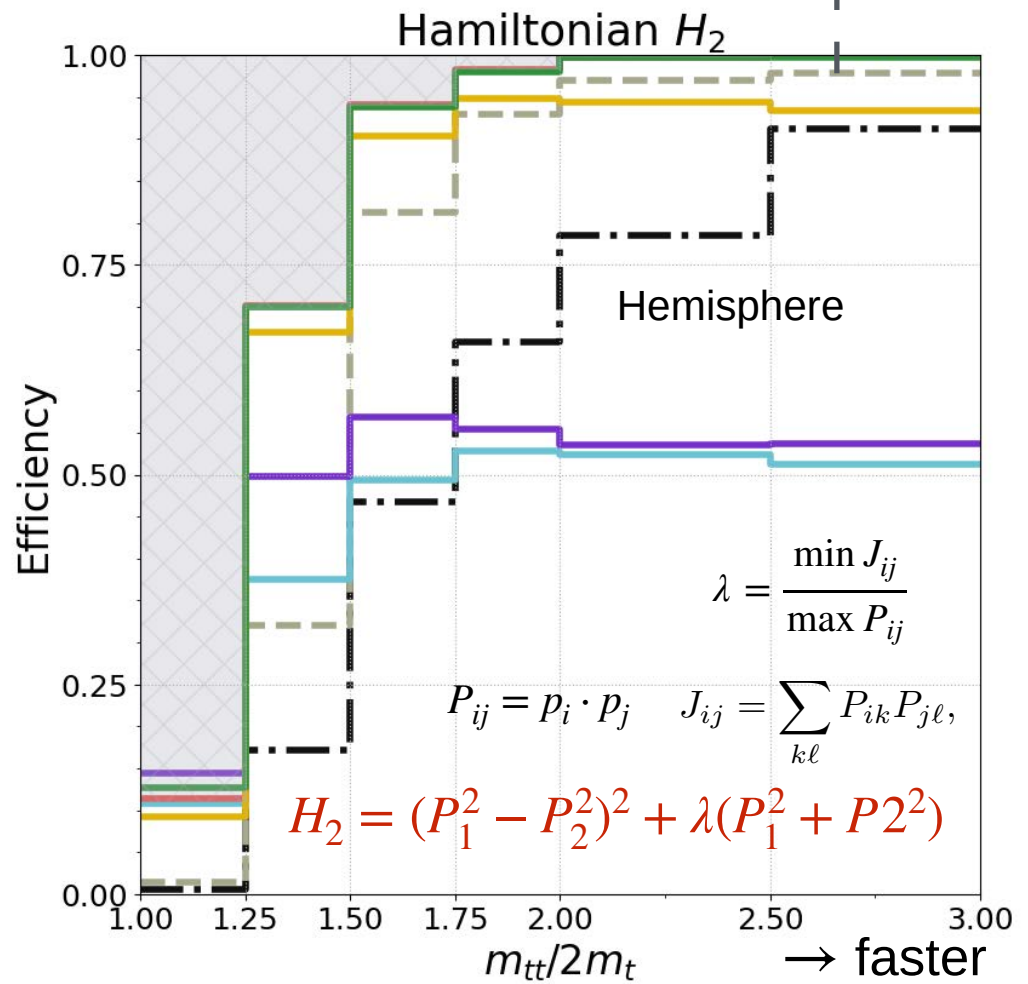
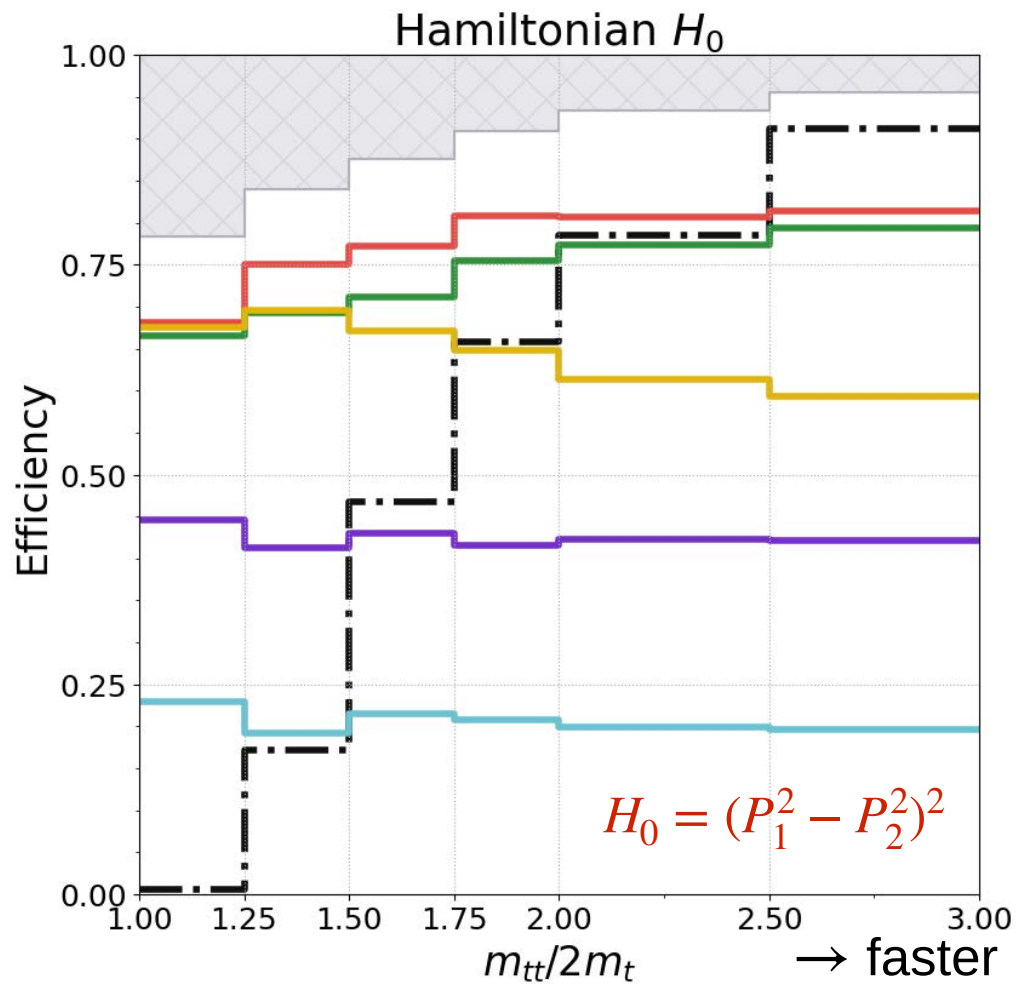
$$P_{ij} = p_i \cdot p_j$$

$$\lambda = \frac{\min J_{ij}}{\max P_{ij}}$$

- Simulated various quantum algorithms using PennyLane to resolve combinatorial ambiguity in the top quark production (6 jets)
 - QAOA, ma-QAOA, XQAOA: Adam optimizer used
 - VarQITE, FALQON: no classical optimization needed
- Compared performance against hemisphere method and classical ML (using SPANet)
- Event generation with MG5 with proper off-shell effects (t and W)
 - Parton-level events and smeared events (for detector effects)

$$\frac{v}{c} = \sqrt{1 - \frac{4m_t^2}{m_{t\bar{t}}^2}}$$

Comparison



QAOA ($p = 8$) $\varepsilon = 42.36\%$	XQAOA ($p = 8$) $\varepsilon = 47.28\%$	ma-QAOA ($p = 8$) $\varepsilon = 78.91\%$	FALQON ($p = 250$) $\varepsilon = 74.83\%$	VarQITE $\varepsilon = 78.98\%$	Hemisphere Method $\varepsilon = 49.98\%$	Quantum Annealer $\varepsilon = 67.10\%$
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$$\frac{m_{t\bar{t}}}{2m_t} = 1$$

$$v = 0$$

$$\frac{m_{t\bar{t}}}{2m_t} = 1.5$$

$$v = 0.75c$$

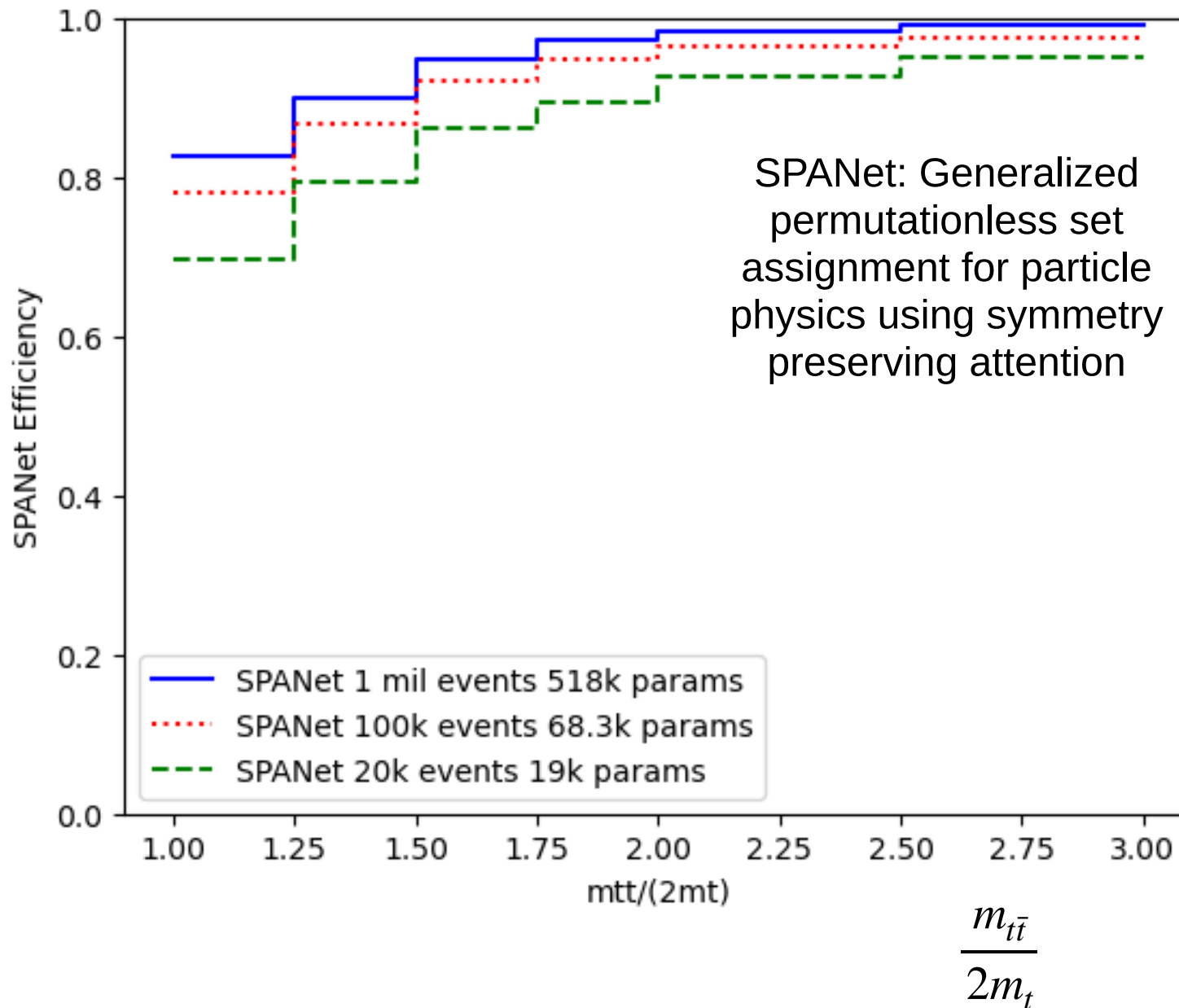
$$\frac{m_{t\bar{t}}}{2m_t} = 2$$

$$v = 0.87c$$

$$\frac{m_{t\bar{t}}}{2m_t} = 3$$

$$v = 0.94c$$

Combinatorial problems in the top quark production



Combinatorial problems in the top quark production

Methods	matching accuracy (efficiency)		n_{train}	$n_{\text{parameters}}$	depth (p)	n_{CNOT}	n_{R_Z}	n_{R_X}	n_{R_Y}
	parton-level	smeared events							
Hemisphere	50%	48%	N/A						
QAOA	43%	41%	N/A	16	8	240	120	48	0
ma-QAOA	79%	77%		168					
XQAOA	47%	46%		216					48
FALQON	78%	76%		2	250	7,500	3,750	1,500	0
VarQITE	79%	77%		16	1	30	15	30	0
SPANet	91%	70%	5×10^5	10^6	N/A				
	81	62%	2×10^4	1.9×10^3					

TABLE I. Summary of the performance of various methods and the corresponding parameters.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

Currently running these algorithms on IonQ

Discussion

- Real hardware run: IonQ with $n=6$
- Scalability: $pp \rightarrow AB \rightarrow j_1 \cdots j_n$, $n > 6$
- Can we do this with hadrons ($n \sim O(100)$)?

High-energy reaction

aa

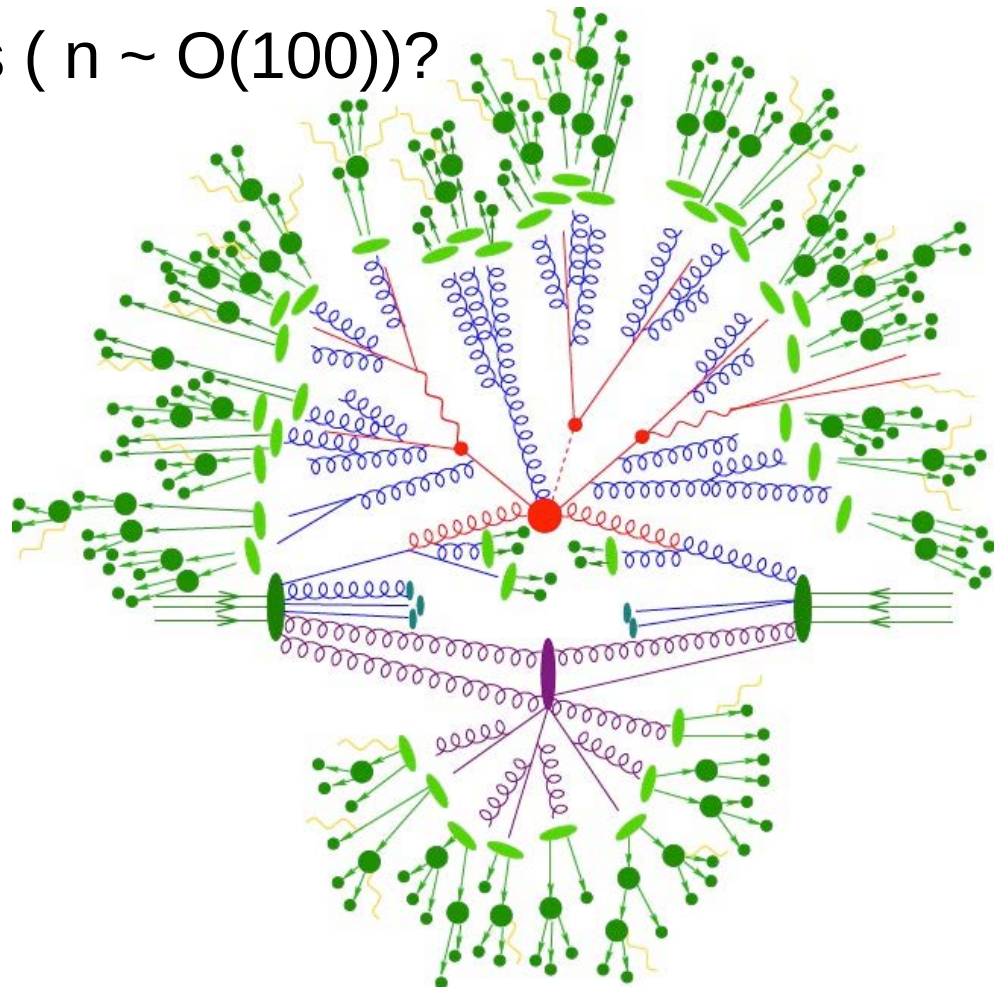
Radiation cascade

aa

Secondary scatterings

aa

Hadronization

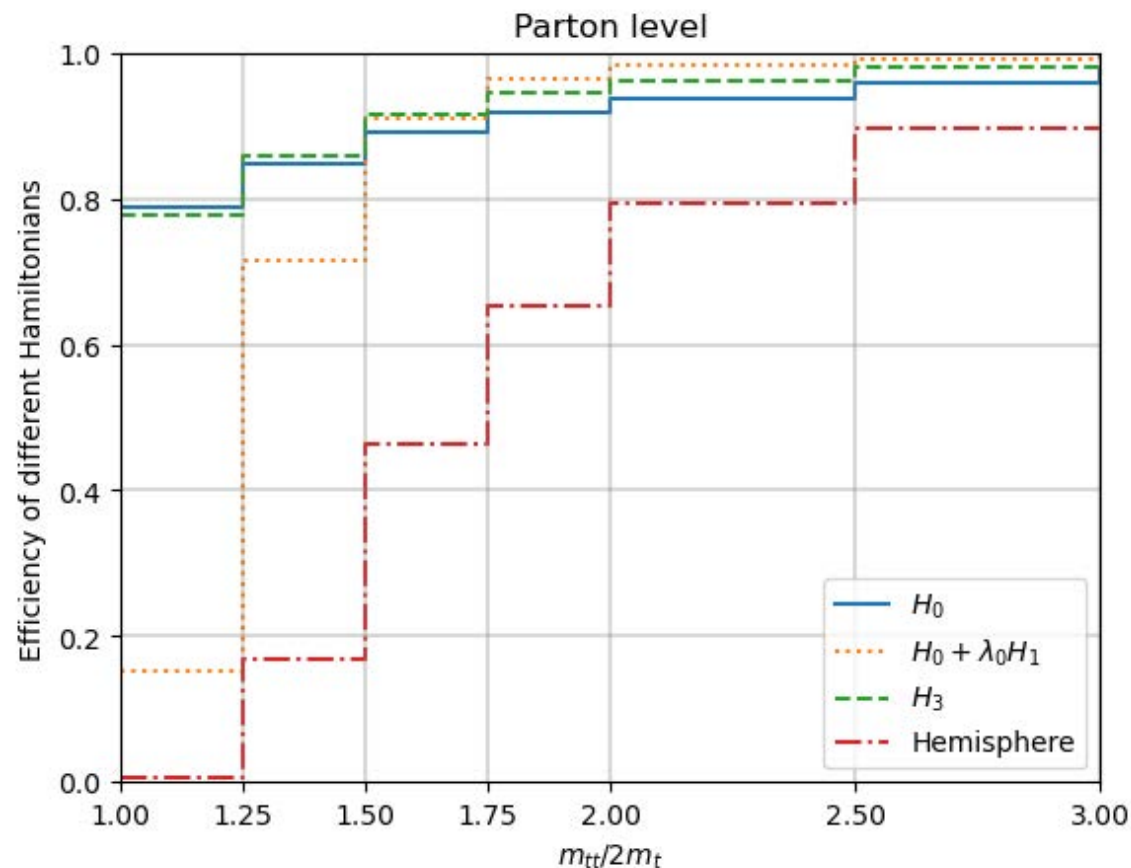


Discussion

- Real hardware run: IonQ with $n=6$
- Scalability: $pp \rightarrow AB \rightarrow j_1 \cdots j_n, n > 6$
- Can we do this with hadrons ($n \sim O(100)$)?
- Higher order terms

$$\begin{aligned}
 H_3 &= H_0 H_1 \\
 &= (P_1^2 - P_2^2)^2 \cdot (P_1^2 + P_2^2) \\
 &= \frac{1}{2} \left[\left(\sum_{k,\ell} P_{k\ell} \right) \sum_{i,j} J_{ij} s_i s_j + \sum_{i,jk\ell} J_{ij} P_{k\ell} s_i s_j s_k s_\ell \right]
 \end{aligned}$$

- QUBO is extended to a problem with higher order (quartic) terms.
- 85% of the time, the ground state of H_3 resolves the combinatorial problem.
- What quantum algorithm can simulate this Hamiltonian?

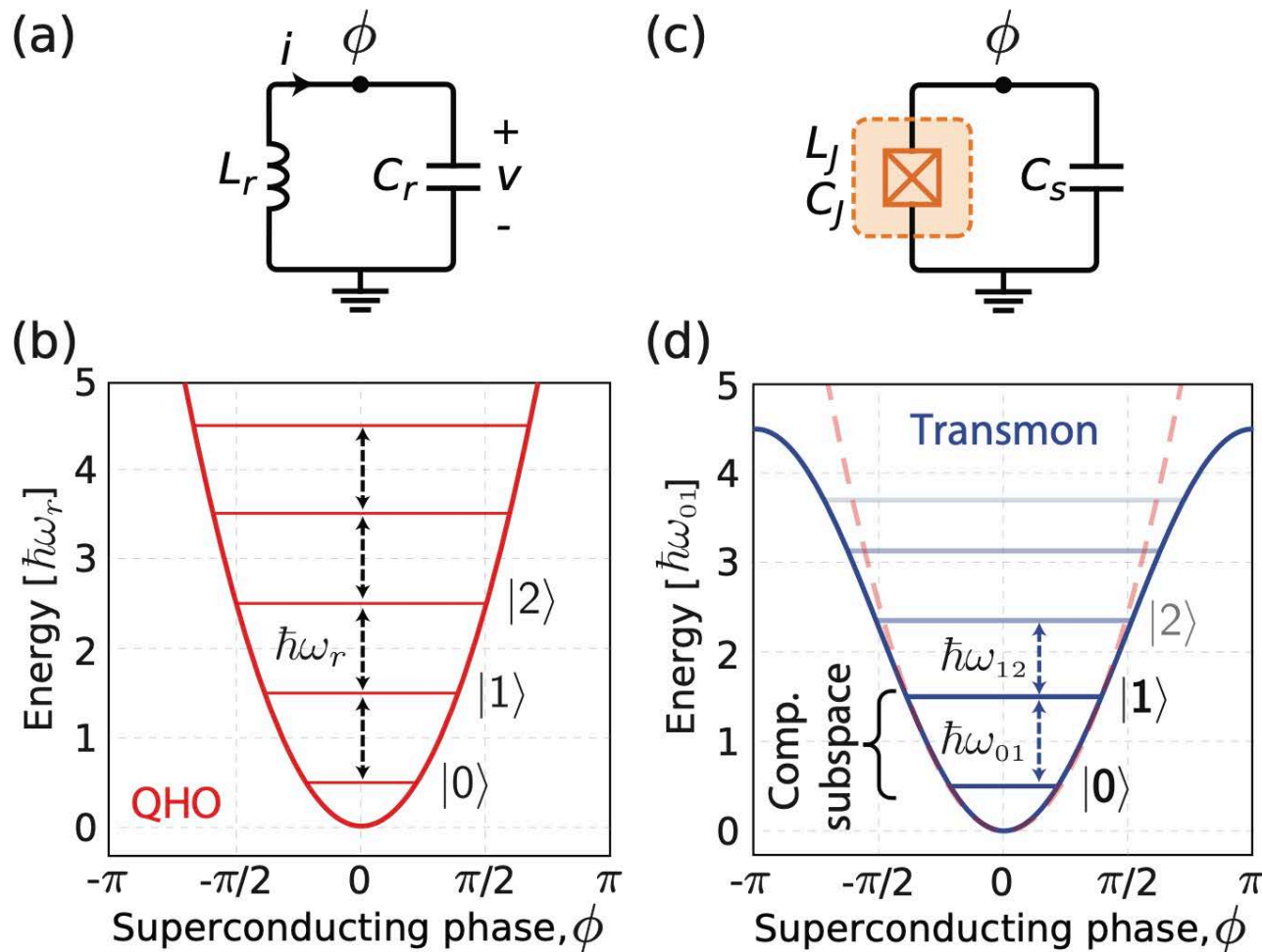


Discussion

- Real hardware run: IonQ with $n=6$
- Scalability: $pp \rightarrow AB \rightarrow j_1 \cdots j_n, n > 6$
- Can we do this with hadrons ($n \sim O(100)$)?
- Higher order terms
- Full error correction
- Combinatorial problems with $t\bar{t} + \text{ISR}$ or $HHH \rightarrow (b\bar{b})(b\bar{b})(b\bar{b})$ or $t\bar{t}t\bar{t} \rightarrow (jjjj)(jjjj)(jjjj)(jjjj)$
 - For $t\bar{t} + \text{ISR}$, naive extension requires twice more qubits for n particles (two binary variables, x_i and y_i for $i = 1, \dots, n$)
 - Qubits with efficient embedding: $2 \otimes 2 = 3 \oplus 1$
 - Qutrits ($x = 0, \pm 1$)
- Combinatorial problems with missing transverse momentum
 - Multi-jets + missing transverse momentum

Qubit sensors for rare decays of
dark matter and cosmic neutrinos

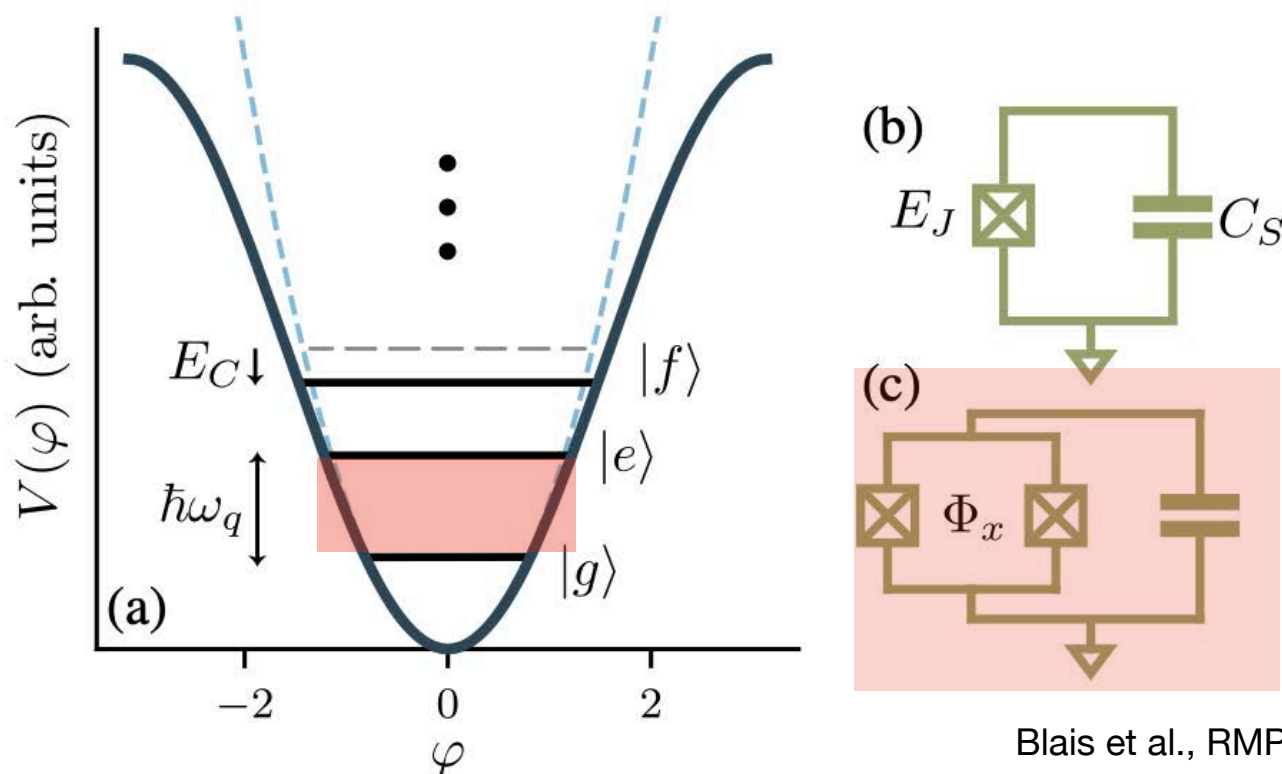
Transmon qubit = Capacitor + Josephson junction



P. Krantz et.al Applied Physics Reviews 6, 021318 (2019)

- Josephson nonlinearity breaks the harmonic spectrum, leading to discrete, unequal energy levels.
- A system of two lowest states can be considered as qubit with ω_{01} .

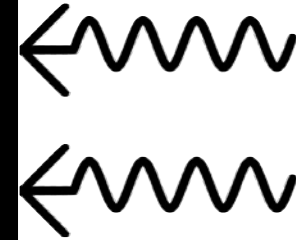
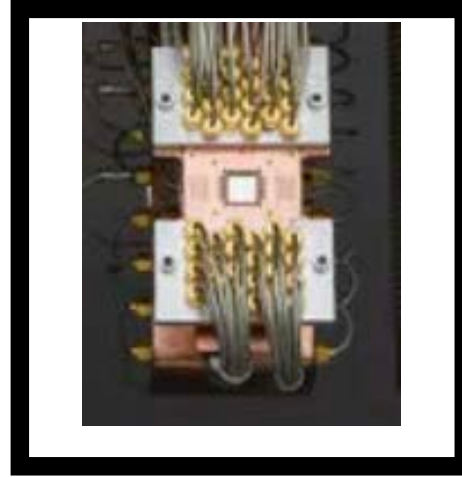
Transmon qubit = Capacitor + Josephson junction



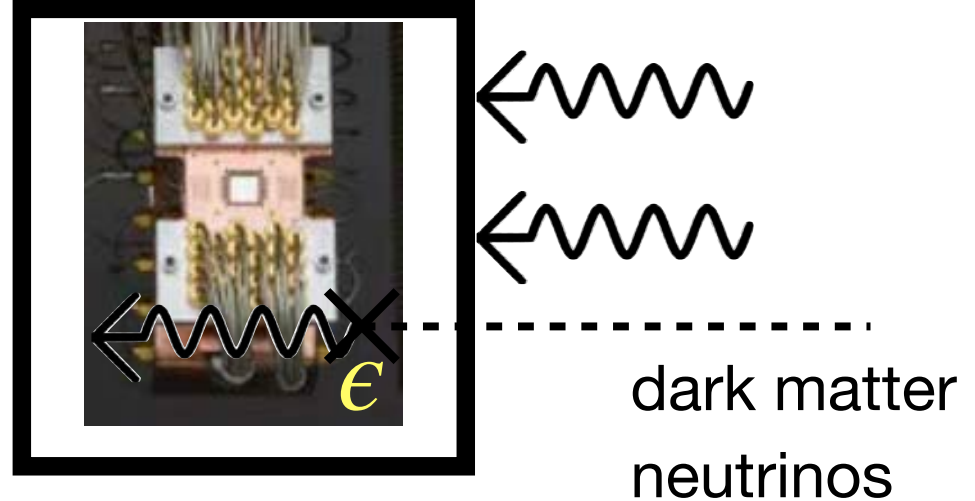
Blais et al., RMP 93 025005 (2021)

- "By using a superconducting quantum interference device (**SQUID**) rather than a single junction, the **frequency of the transmon qubit becomes flux tunable**."
- Transmon qubits have discrete energy levels that can be controlled.
- One can scan the frequency ω_q in the (c) SQUID-based system.

$$\omega \simeq \mathcal{O}(1 - 10) \text{ GHz} \quad \lambda \simeq \mathcal{O}(3 - 30) \text{ cm}$$

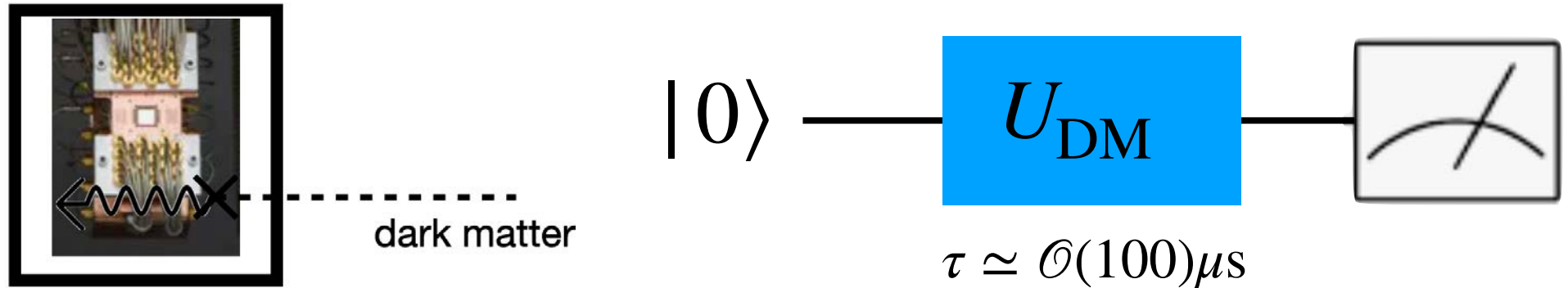


- For typical device parameters, $\omega \simeq \mathcal{O}(1 - 10)\text{GHz}$ with $\lambda \simeq \mathcal{O}(3 - 30)\text{cm}$
- At refrigerator temperatures $\mathcal{O}(1 - 10)\text{mK}$, a few-GHz transition satisfies $k_B T \ll \hbar \omega$.
 - Thus thermal excitation is strongly suppressed, and the qubit stays in the ground state.



- For typical device parameters, $\omega \simeq \mathcal{O}(1 - 10)\text{GHz}$ with $\lambda \simeq \mathcal{O}(3 - 30)\text{cm}$
- At refrigerator temperatures $\mathcal{O}(1 - 10)\text{mK}$, a few-GHz transition satisfies $k_B T \ll \hbar\omega$.
 - Thus thermal excitation is strongly suppressed, and the qubit stays in the ground state.
- “Dark Matter” could penetrate the shield, and induce a weak electric field that couples to qubit, $E^{(\text{ext})} = \epsilon\sqrt{2\rho_{\text{DM}}}\sin m_X t$, here ϵ is a kinetic-mixing parameter, ρ_{DM} is the local DM density.
- **If the DM-induced photons fall within this frequency band, they can resonantly excite the qubit - effectively turning dark-matter interactions into observable qubit errors.** For a de Broglie wavelength in this range, $m_{\text{DM}} \sim \mathcal{O}(4 - 40)\mu\text{eV}$

Dark Matter as an operator U_{DM}



- Signal: a weak classical field: wave-like DM, high-frequency gradational waves
- The interaction between DM and qubits, $H_{\text{DM}} = \epsilon(\cos \alpha \sigma_X + \sin \alpha \sigma_Y)$ will drive transitions from $|0\rangle$ to $|1\rangle$.
- The Goal to estimate the signal strength ϵ .

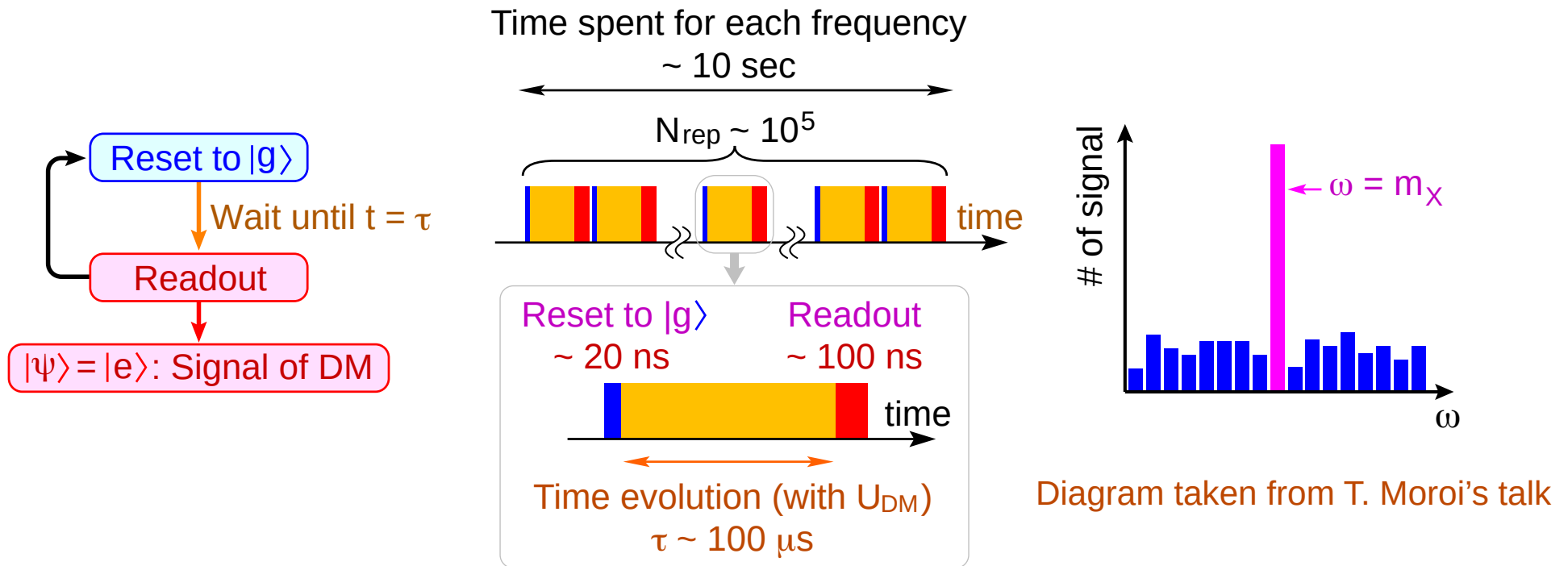
1. Evolve: System evolves from $|0\rangle$ (ground state of free Hamiltonian) under H_{DM} . For a weak signal $\epsilon t \ll 1$,

$$|\psi(0)\rangle = |0\rangle \longrightarrow |\psi(t)\rangle = U(t)|\psi(0)\rangle \approx |0\rangle - ie^{i\alpha} \epsilon t |1\rangle$$

2. Measure: Project $|\psi(t)\rangle$ onto $|0\rangle$ and $|1\rangle$.

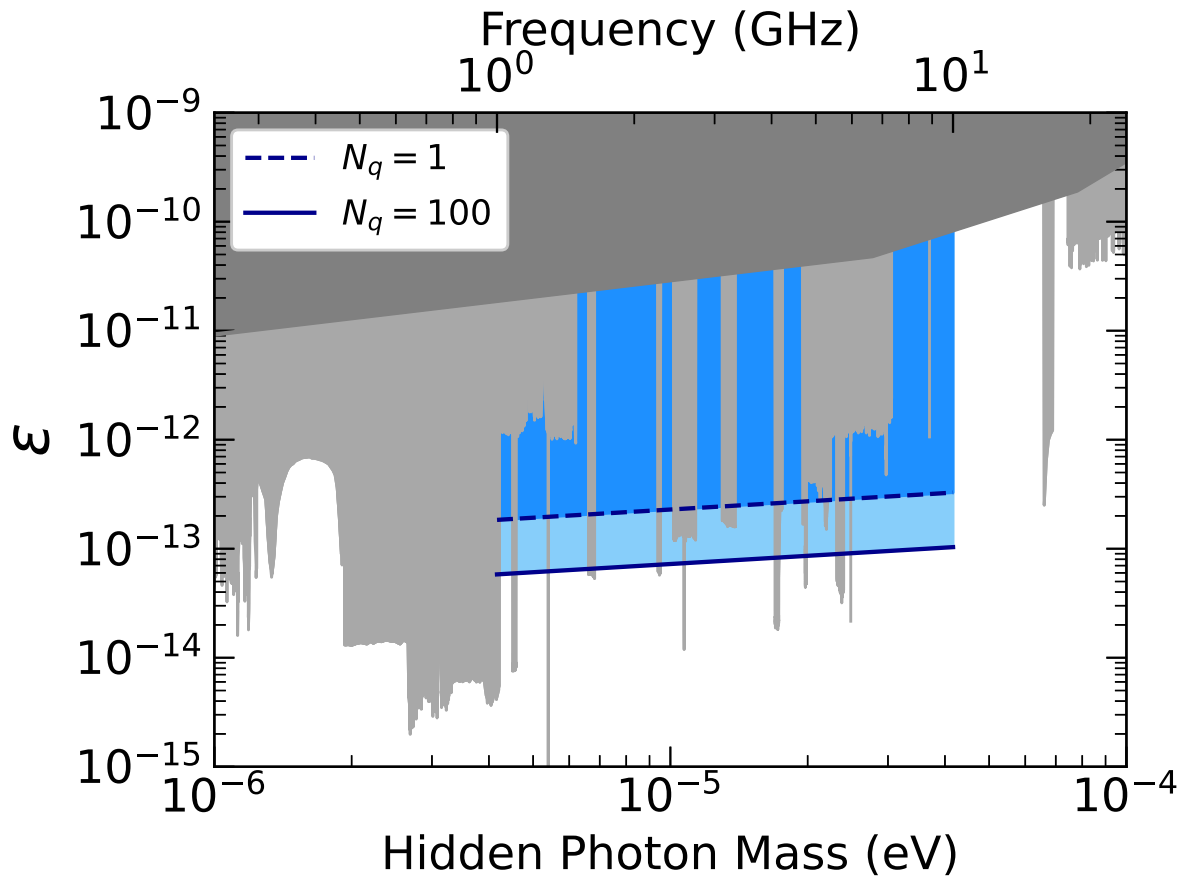
$$\text{Probability of finding } |1\rangle : p_1 = \left| \langle 1 | \psi(t) \rangle \right|^2 = \epsilon^2 t^2 \quad \text{for } t \ll \tau$$

Dark Matter as an operator U_{DM}



- Qubits are exposed to microwaves from dark matters over the coherence time $\mathcal{O}(100)\mu\text{s}$
- One can repeat a cycle of (Reset $\rightarrow U_{\text{DM}} \rightarrow$ Measurement) as many as $\mathcal{O}(10^5)$ during 10 seconds.
 - Time for reset qubits $\sim \mathcal{O}(10)\mu\text{s}$
 - Time for measurement $\simeq \mathcal{O}(100)\text{ns}$

Discovery reach with 1 year frequency scan ($1 \leq f \leq 10$ GHz)



For hidden photon dark matter
2212.03884, PRL

$$d = 100 \text{ } \mu\text{m}$$

$$C = 0.1 \text{ pF}$$

$$Q = 10^6$$

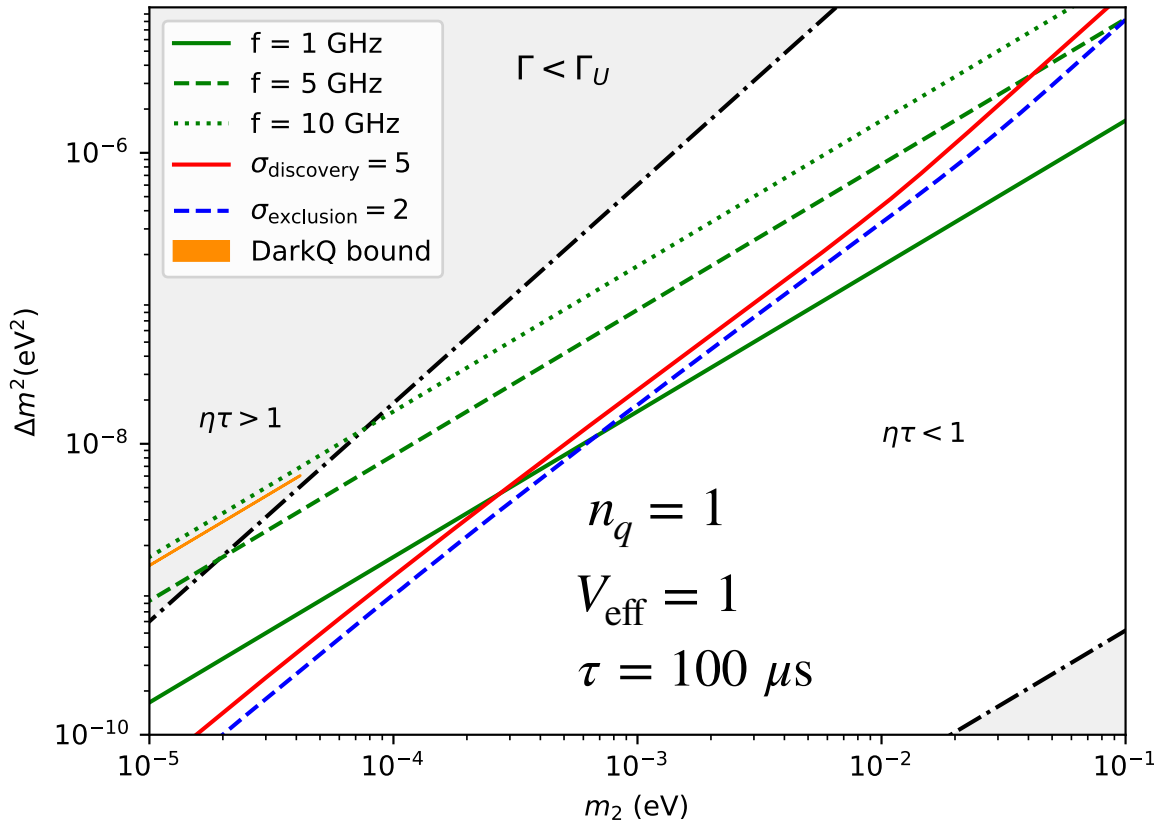
$$\text{Error rate / qubit} = 0.1 \%$$

⇒ Using qubit, we may probe parameter region unexplored

⇒ We hope to use qubit for the detection of other DMs

$$\mathcal{L} \ni -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu + e\bar{\psi}_e\gamma^\mu\psi_e(A_\mu^{(\text{EM})} + \epsilon X_\mu)$$

For decay photon, $X_2 \rightarrow X_1 + \gamma$



- Scanning the frequency range from **1 - 10 GHz** with a quality factor $Q = 10^6$, which sets the frequency resolution to $\delta f = f/Q$.
- This corresponds to about 2.3×10^6 resolvable frequency bins across the **1-10 GHz** range
- One year continuous scan (3.15×10^7 s) yields 14s of integration time per frequency bin, corresponding to $N_{\text{try}} = \mathcal{O}(10^5)$ / bin

$$p_* = 0.069$$

$$p_* \simeq p_*^0 \times \cos^2 \Theta \left(\frac{V_{\text{eff}} \rho_{\text{DM}}}{0.45 \text{ GeV cm}^{-3}} \right) \left(\frac{\Delta m^2}{m_2^2} \right) \left(\frac{d}{100 \text{ } \mu\text{m}} \right)^2 \left(\frac{C}{0.1 \text{ pF}} \right) \left(\frac{f}{1 \text{ GHz}} \right) \left(\frac{\tau}{100 \text{ } \mu\text{s}} \right)^3$$

- Assuming a local dark matter density $\rho_{\text{DM}} = 0.45 \text{ GeV/cm}^3$ and decay rate $\Gamma \lesssim \Gamma_U = 2.299 \times 10^{-18} \text{ s}^{-1}$ ensuring that X_2 remains stable over the age of the universe.

Quantum Enhancement of the Signal Rate

- Suppose we have N identical sensors. What is the maximum sensitivity?

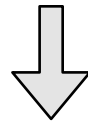
Without entanglement

$$|0\rangle \xrightarrow{\text{DM}} |0\rangle - i\epsilon\tau |1\rangle \Rightarrow p = \epsilon^2\tau^2$$

$$|0\rangle \xrightarrow{\text{DM}} |0\rangle - i\epsilon\tau |1\rangle \Rightarrow p = \epsilon^2\tau^2$$

$\vdots$

$$|0\rangle \xrightarrow{\text{DM}} |0\rangle - i\epsilon\tau |1\rangle \Rightarrow p = \epsilon^2\tau^2$$



$$p_{\text{total}} = 1 - (1 - p)^N \approx Np = N\epsilon^2\tau^2$$

Classical

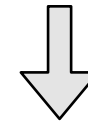
- N independent systems are prepared and separately detected

With entanglement

$|\psi\rangle \equiv$ some entangled state

$$|\psi\rangle \xrightarrow{\text{DM}} |\psi\rangle + O(N\epsilon\tau) |\psi_{\perp}\rangle$$

An example of such a state is the GHZ state.



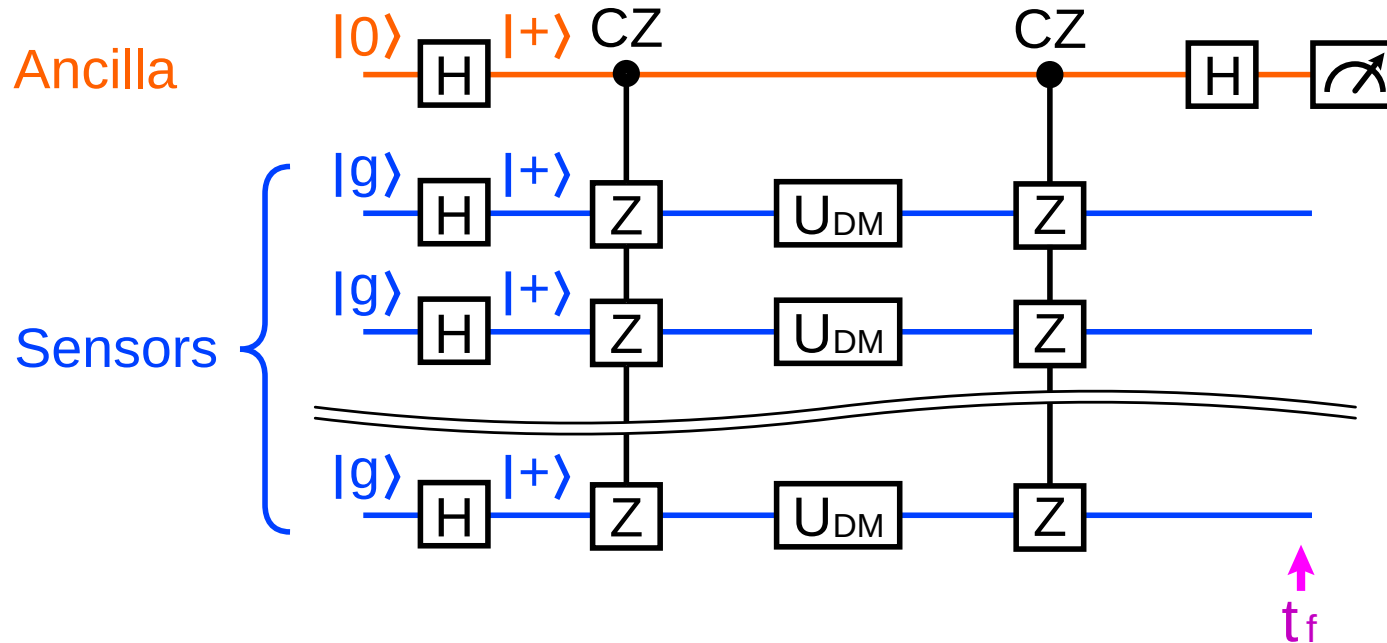
$$p \sim N^2\epsilon^2\tau^2 \quad ?$$

Quantum

- Highly correlated configurations are measured collectively with a single measurement that encompasses all the systems.

Quantum Enhancement of the Signal Rate

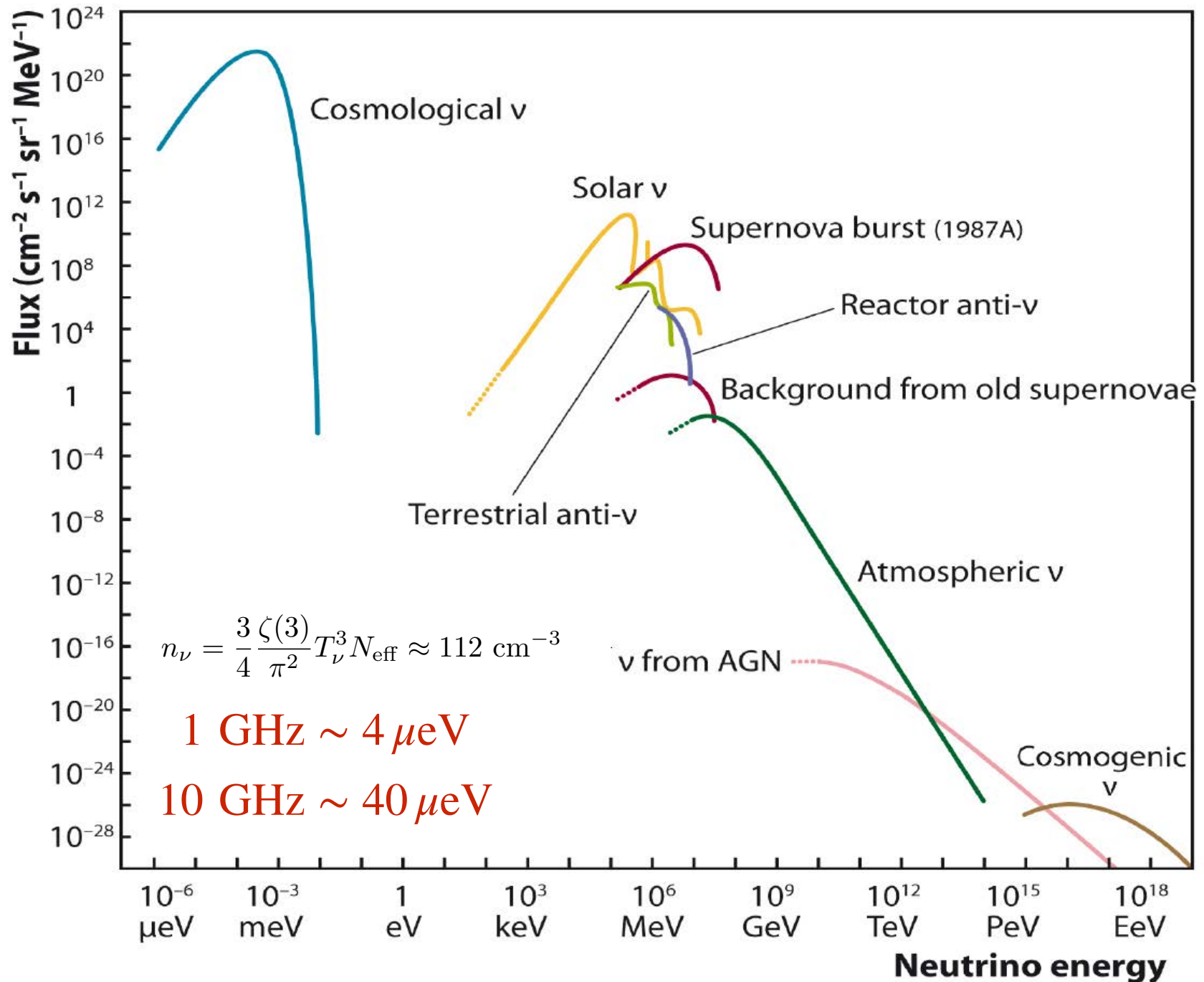
One measurement cycle for the signal enhancement



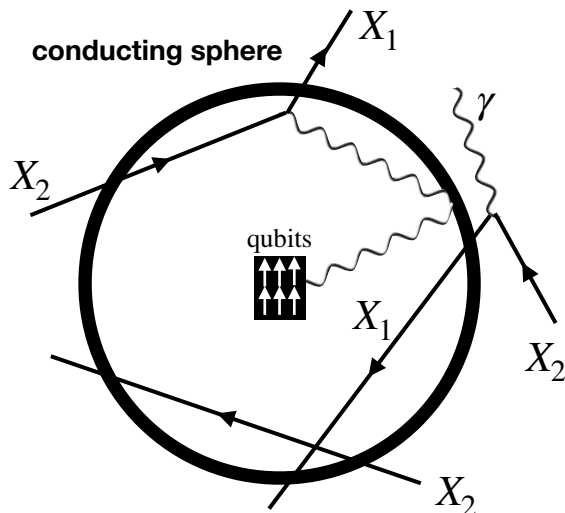
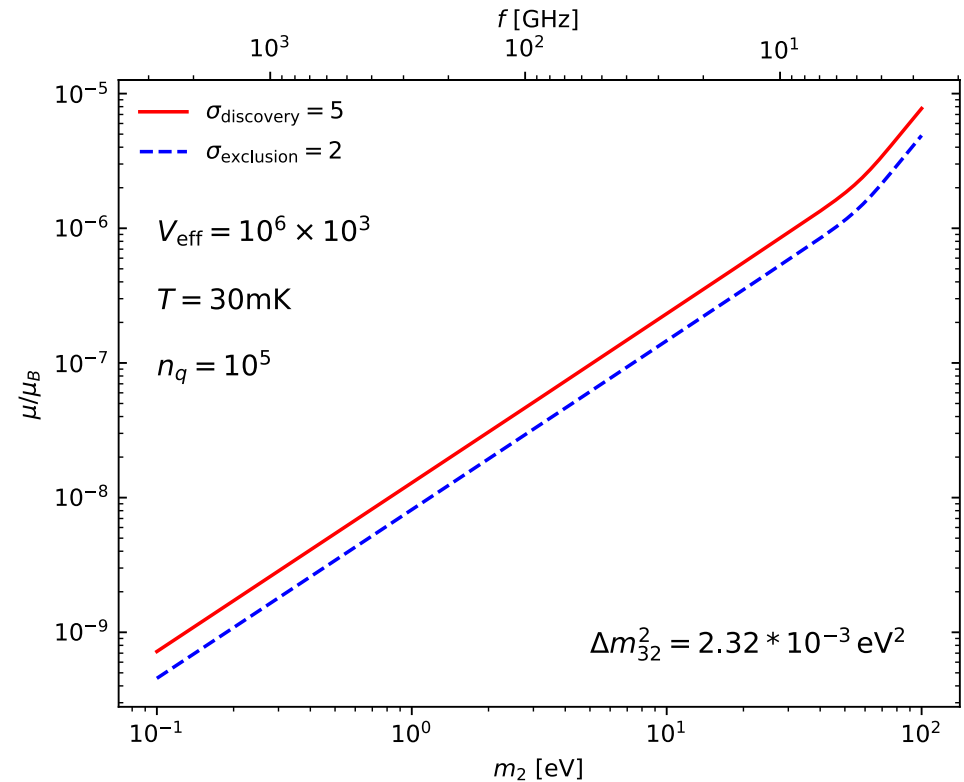
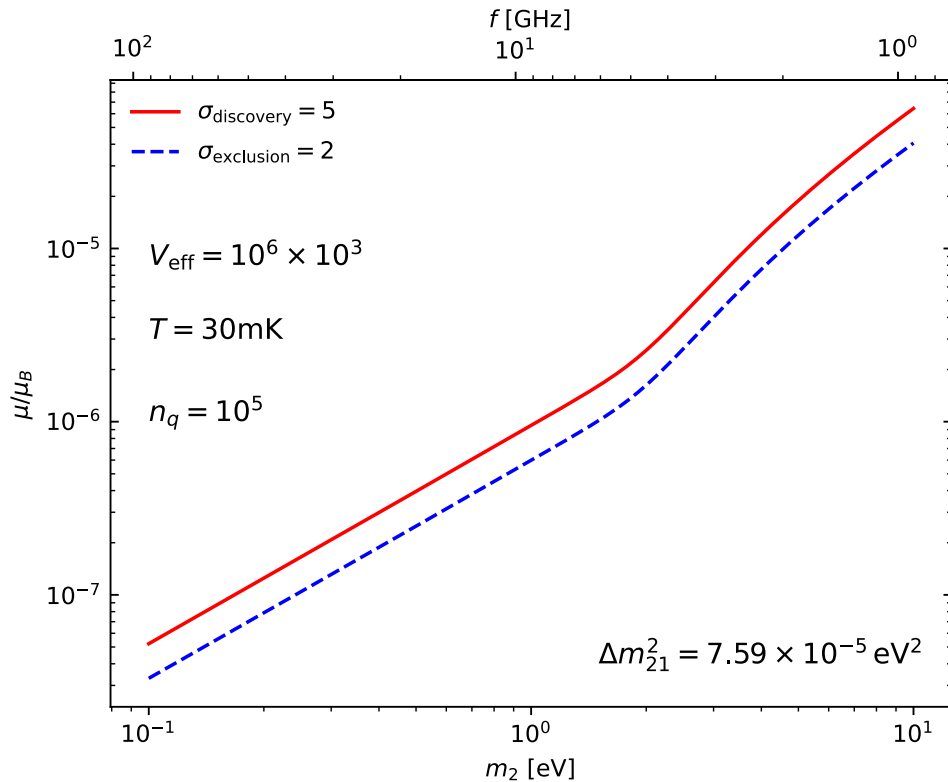
$$|\Psi(t_f)\rangle = \left(\cos N_q \delta |0\rangle + i \sin N_q \delta |1\rangle \right) \otimes |+\rangle^{\otimes N_q}$$

$\Rightarrow$ Ancilla qubit can be excited: $P_{0 \rightarrow 1} \simeq \sin^2 N_q \delta \simeq N_q^2 \delta^2$

Neutrino fluxes from different sources as a function of the neutrino energy



Bounds on neutrino transition magnetic moment



Decay volume

$$n_2 \rightarrow n_2 \left(\frac{V_{\text{eff}}}{\text{cm}^3} \right)$$

$$V_{\text{eff}} = 1 \text{ m}^3 = 10^6 \text{ cm}^3$$

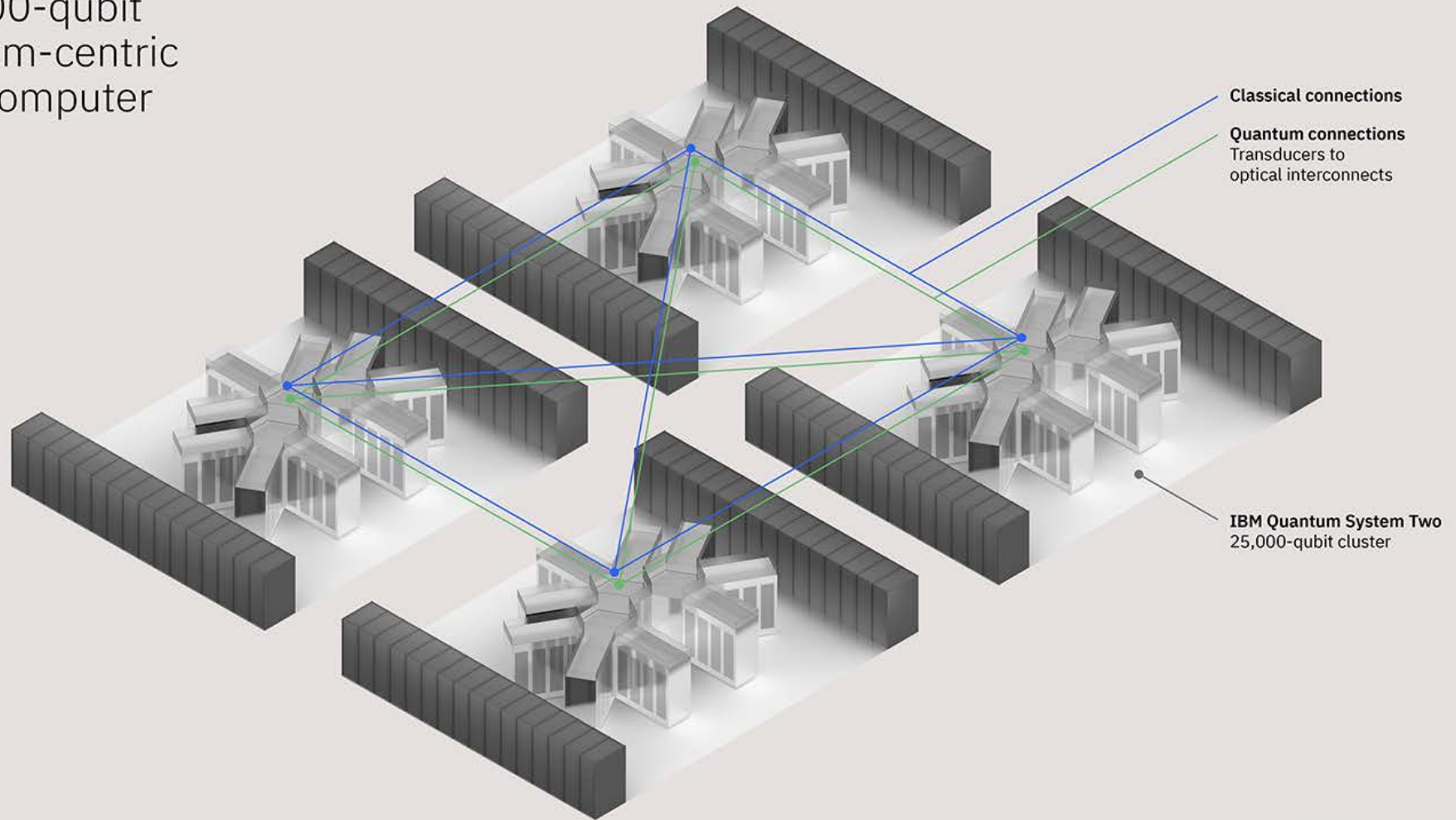
$$P_* \simeq n_q (\eta \tau)^2$$

$$n_q = 10^5$$

Current reactor bounds are $\mu < 10^{-11} \mu_B$ 2508.09139

Charting the course to 100,000 qubits

100,000-qubit
quantum-centric
supercomputer
—
2033

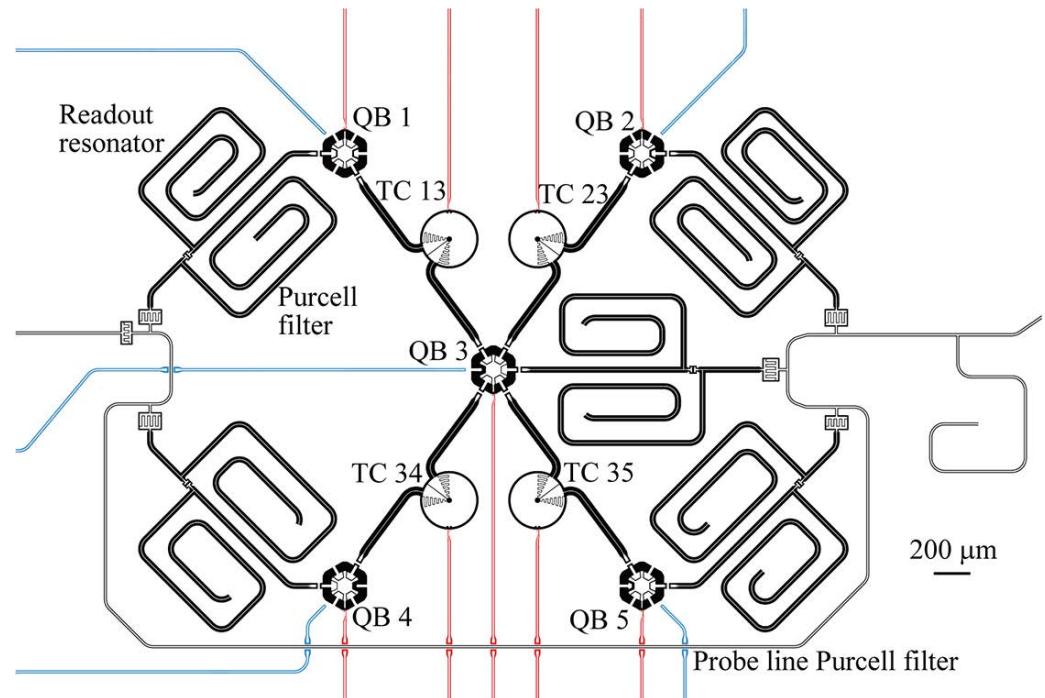


IBM Quantum

- All other quantum computing companies (D-wave, IonQ, Google etc) are also targeting Large-Scale, Fault-Tolerant Quantum Computers for next 10-20 years.
- The future may arrive sooner than we expect.

Discussion

- High frequency gravitational wave detection
- What about other quantum devices
 - Transmon, trapped ion, Rydberg, what else?
 - Ge-based spin qubits
- Quantum algorithms that could enhance beyond n_q^2
- IQM Spark 5 qubits



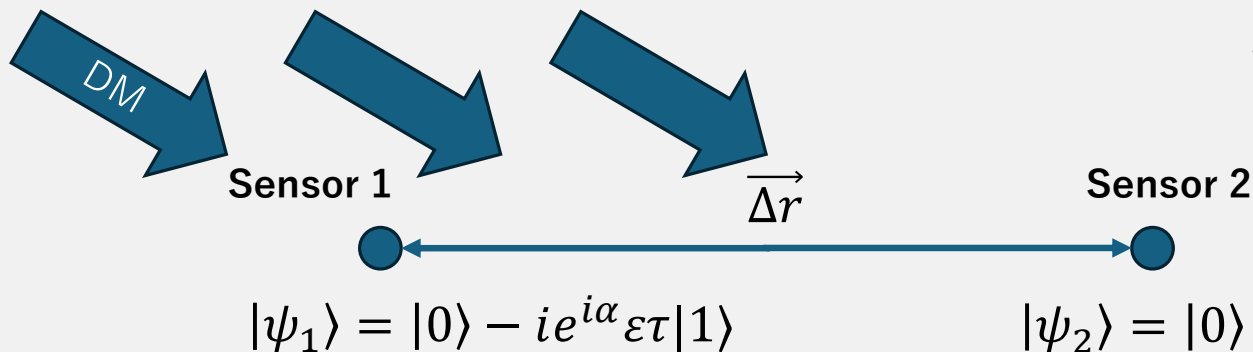
Quantum networks for dark matter flows

Measuring Dark Matter Velocity

Classical Protocol [Derevianko '18]

For simplicity, I consider only two sensors here

1. **Interaction:** The DM signal field interacts with two sensors
2. **Measurement:** Measure an observable σ_x , for each sensor independently:
 $\langle \sigma_x \rangle = 2\varepsilon\tau \sin \alpha$
3. **Correlation:** Look for **coincident events** where both sensors show a response, and correlate the outcomes.



Velocity is encoded in α :

$$\alpha' = \alpha - \vec{k} \cdot \vec{\Delta r}$$

Only the difference is meaningful

$$\alpha' - \alpha = -\vec{k} \cdot \vec{\Delta r}$$

The Problem with the Classical Protocol

- α fluctuates randomly, preventing simple time averaging
- To extract correlation, we need a **coincident detection event** (i.e., both sensors "click").
- The probability of a single click is $p_1 \sim (\varepsilon\tau)^2 \Rightarrow p_1 p_2 \sim (\varepsilon\tau)^4$

Measuring Dark Matter Velocity

Quantum Protocol: Interference with distributed sensors

Step 1: Entangling Measurement (Projection)

- Two-sensor state: $|\psi_1 \otimes \psi_2\rangle = (|0\rangle - ie^{i\alpha}\varepsilon\tau|1\rangle) \otimes [|0\rangle - ie^{i(\alpha - \vec{k} \cdot \vec{\Delta r})}\varepsilon\tau|1\rangle]$
- Measure $P = |10\rangle\langle 10| + |01\rangle\langle 01|$, which succeeds with probability $\langle P \rangle = 2\varepsilon^2\tau^2$
- State after measurement: $P|\psi_1 \otimes \psi_2\rangle \sim |10\rangle + e^{-ik\Delta r}|01\rangle \equiv |\Psi\rangle$

Step 2: Information Readout

- Measure $M = -i|01\rangle\langle 10| + i|10\rangle\langle 01|$
- $\langle M \rangle = -\sin k\Delta r$

Measuring a Projection Operator P

$$|\phi\rangle \rightarrow \begin{cases} P|\phi\rangle & \text{with probability } p = \langle \phi | P | \phi \rangle \\ (1 - P)|\phi\rangle & \text{with probability } 1 - p \end{cases}$$

e.g. Measure $P_1 = |1\rangle\langle 1|$ for one qubit state $|\varphi\rangle$
After measurement: $|1\rangle$ with $p = |\langle \varphi | 1 \rangle|^2$ or $|0\rangle$

State of the art: Scalable Quantum Network

4

Color center-based Quantum repeater network by two leaders, R. Hanson in QuTech, Netherlands and M. Lukin in MIT, USA

nature

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[nature](#) > [articles](#) > [article](#)

Article | [Open access](#) | Published: 15 May 2024

Entanglement of nanophotonic quantum memory nodes in a telecom network

C. M. Knaut, A. Suleymanzade, Y.-C. Wei, D. R. Assumpcao, P.-J. Stas, Y. Q. Huan, B. Machielse, E. N. Knall, M. Sutula, G. Baranes, N. Sinclair, C. De-Eknamkul, D. S. Levonian, M. K. Bhaskar, H. Park, M. Lončar & M. D. Lukin

[Nature](#) **629**, 573–578 (2024) | [Cite this article](#)

Nature **629**, 573 (2024)

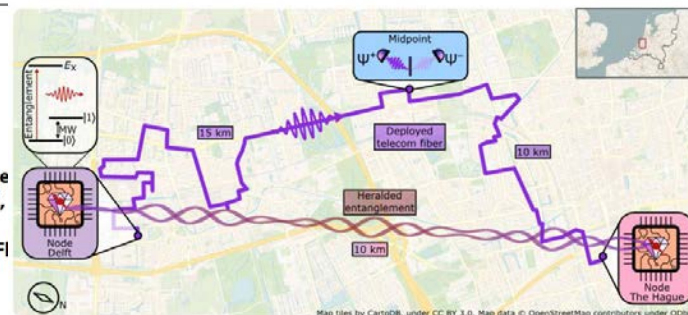
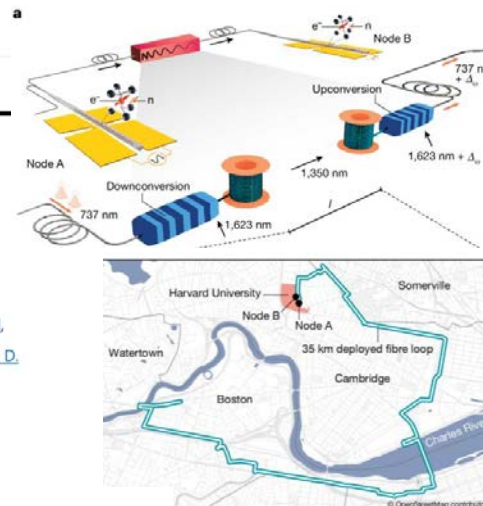
SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Metropolitan-scale heralded entanglement of solid-state qubits

Arian J. Stolk^{1†}, Kian L. van der Enden^{1†}, Marie-Christine Slater^{1‡}, Ingmar te Raa-De Pieter Botma¹, Joris van Rantwijk¹, J. J. Benjamin Biemond^{1,2}, Ronald A. J. Hagen^{1,2}, Rodolf W. Herfst^{1,2}, Wouter D. Koek^{1,2}, Adrianus J. H. Meskers^{1,2}, René Vollmer^{1,2}, Erwin J. van Zwet^{1,2}, Matthew Markham³, Andrew M. Edmonds³, J. Fabian Geus^{4,5}, Fl. Bernd Jungbluth⁴, Constantin Haefner^{4,5}, Christoph Tresp⁶, Jürgen Stuhler⁶, Stephan Ritter⁶, Ronald Hanson^{1*}

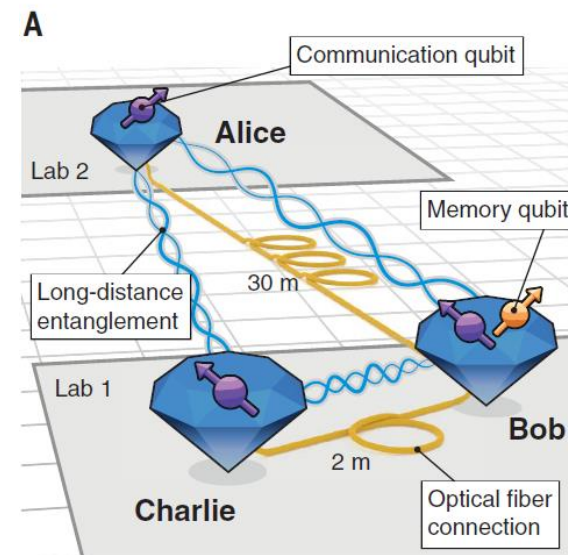
Sci. Adv. **10**, eadp6442(2024).



QUANTUM NETWORKS

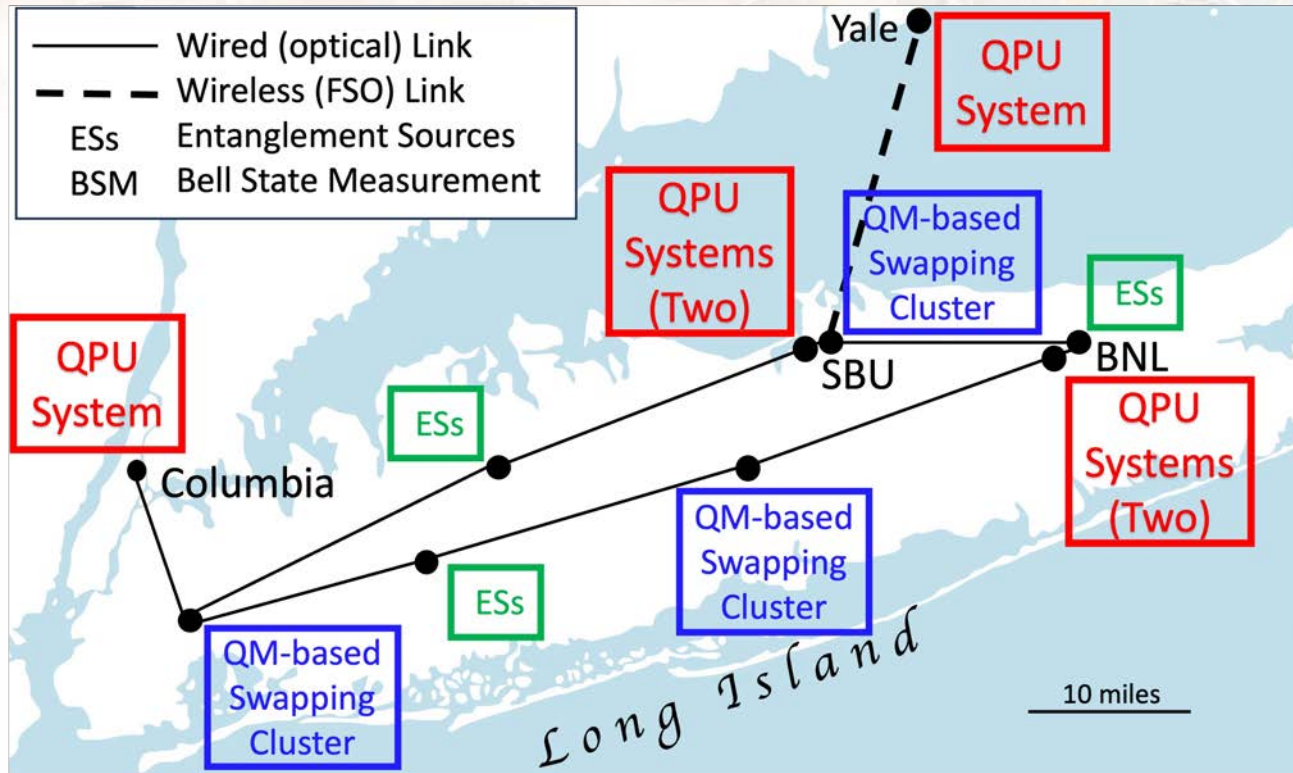
Realization of a multinode quantum network of remote solid-state qubits

M. Pompili^{1,2†}, S. L. N. Hermans^{1,2†}, S. Baier^{1,2†}, H. K. C. Beukers^{1,2}, P. C. Humphreys^{1,2§}, R. N. Schouten^{1,2}, R. F. L. Vermeulen^{1,2}, M. J. Tiggeleman^{1,2¶}, L. dos Santos Martins^{1,2}, B. Dirkse^{1,2}, S. Wehner^{1,2}, R. Hanson^{1,2*}



Science **372**, 259 (2021)

SCY QNet Requirement 3: Interconnecting QPUs using QRs



Technical readiness:

- Building QPUs based of atomic traps for Rb, Sr and Yb.
- Demonstrated QFC between telecom and IR photons.

SCY-QNet goals:

- Create and transmit atom-photon entanglement from the QPUs at O-band.
- Perform entanglement swapping between the QPU qubits and the QM Banks.

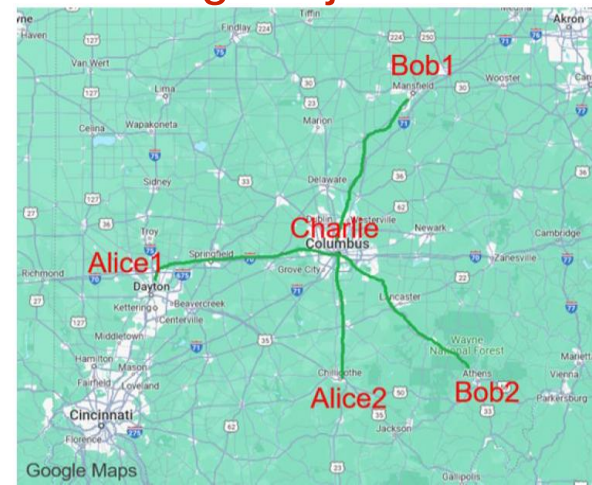
CQISE Quantum Networking Project



P2P QKD



MDI QKD CONCEPT

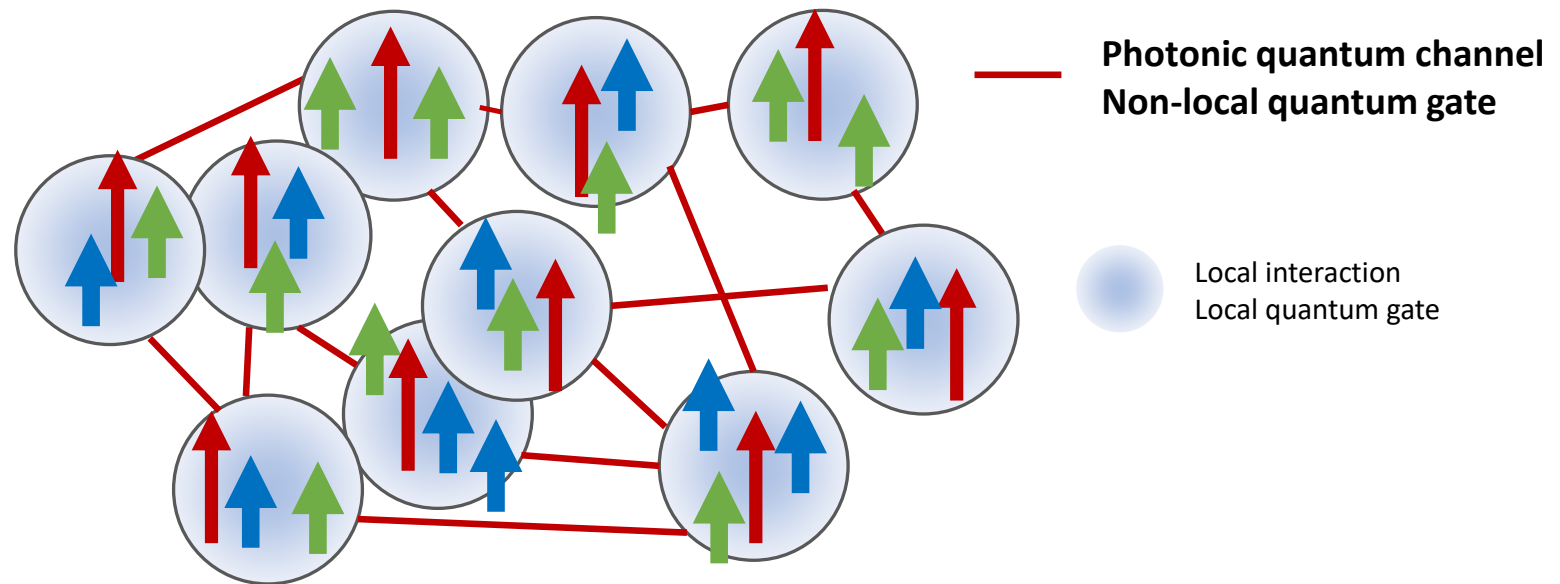


STAR NETWORK MDI QKD CONCEPT

Emerging partnerships

- AFIT
- NGRC
- Cisco
- Honda
- Huntington Bank
- Columbus State
- Starlab

Quantum network: From one big noisy chip to many quiet chips with photonic links.



- ❑ Distributed quantum computing & Quantum computing modular architecture
- ❑ Quantum communication (quantum internet, quantum repeater network)

Colloquium: Quantum Networks with Trapped Ions. Rev. Mod. Phys. (2010)

Scaling the Ion Trap Quantum Processor. Science (2013)

Technical roadmap for fault-tolerant quantum computing, NQIT Technical Roadmap (2016)

Material platforms for spin-based photonic quantum technologies, Nat. Rev. Mater. (2018)

Quantum Internet: A vision for the road ahead, Science (2018)

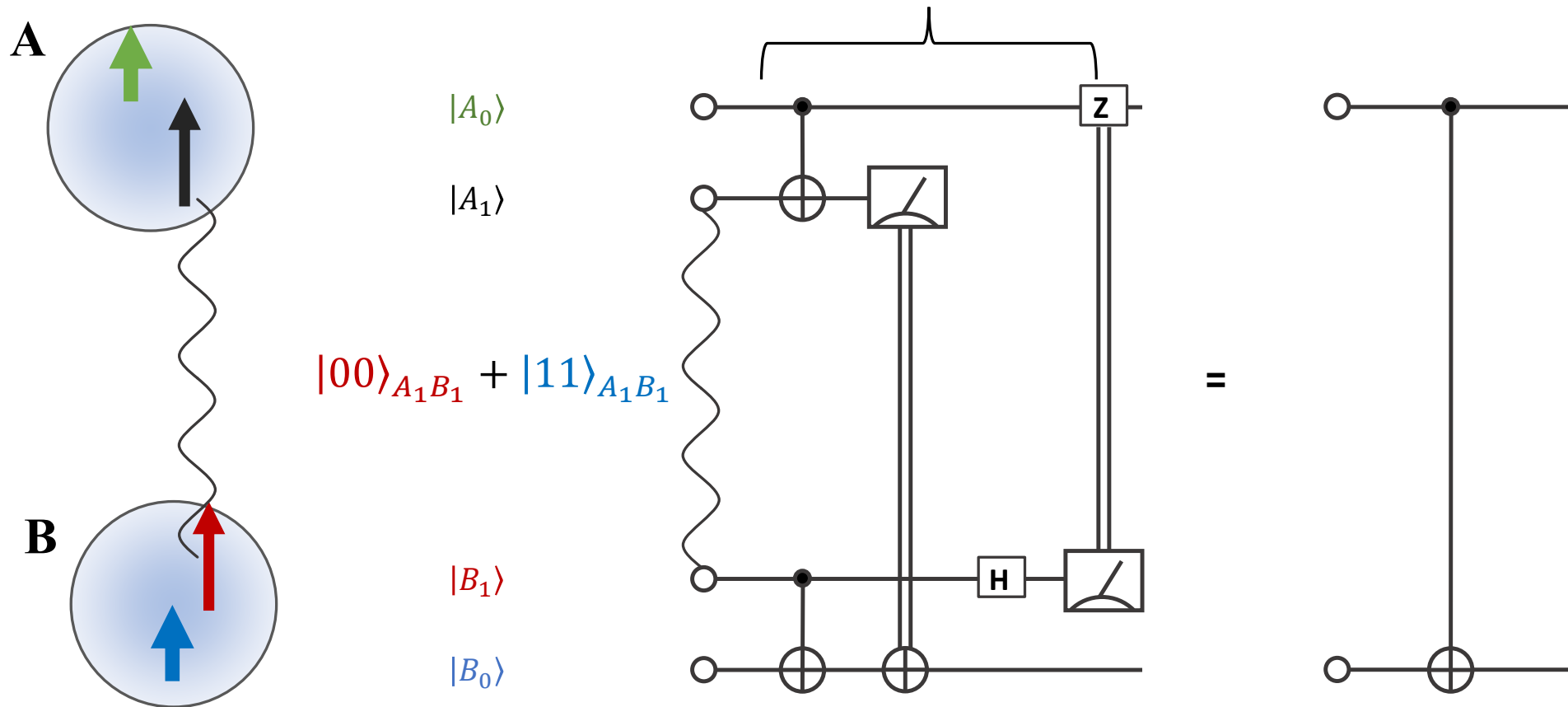
Problems with one big chip

- ❑ As N grows, control lines $\propto N$, noise/crosstalk $> N$, uniformity $\downarrow$
- ❑ I/O fan-out $\uparrow$
- ❑ Crosstalk $\uparrow$
- ❑ Calibration overhead $\uparrow$
- ❑ classical/quantum noise $\uparrow$

Nonlocal two qubit gate

7

Remotely entangled qubit pair + Local operations = Nonlocal CNOT gate



Distributed Quantum Computation Based on Small Quantum Registers. Phys. Rev. A 2007
Optimal Local Implementation of Nonlocal Quantum Gates. Phys. Rev. A 2000

- Involvement with National labs and government agencies
 - NASA, DoD, DOE, NSF, . . .
 - Many private foundations: Templeton, Keck . . .



NASA Quantum Artificial Intelligence Laboratory (QuAIL)

QuAIL is the space agency's hub for assessing and advancing the potential of quantum computers to impact computational challenges faced by the agency in the decades to come.

Overview

Quantum computing is an emerging computational paradigm with the potential to revolutionize diverse areas of future-generation computational systems. QuAIL's mandate is to determine and advance the potential for quantum computation to enable more ambitious, efficient, and safer NASA missions in the future. The team focuses on hard computational problems whose solution would impact NASA's aeronautics, Earth and space sciences, and space exploration missions.

The QuAIL team tracks and contributes to advances in algorithms, both near- and longer-term, for optimization, machine learning, simulation of condensed-matter, high-energy physics, chemistry and materials, and differential equations and computational fluid dynamics. While the group is a theory and numerical team, a key component of this work is close collaboration with quantum hardware groups. Hardware-algorithm co-design is central to the team's work, with algorithmic needs feeding into the hardware design and prioritization of hardware capabilities, hardware limitations feeding into algorithm design, and hardware noise characterization feeding into the design of protocols for error mitigation and correction. Key to this research has been advancing tools, including classical circuit simulation and open quantum system modeling, leading to an improved understanding of co-design principles across multiple architectures, including gate-model, adiabatic, and measurement-based architectures.



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Home : Our Work : Areas of Research : Quantum Research

Quantum Research

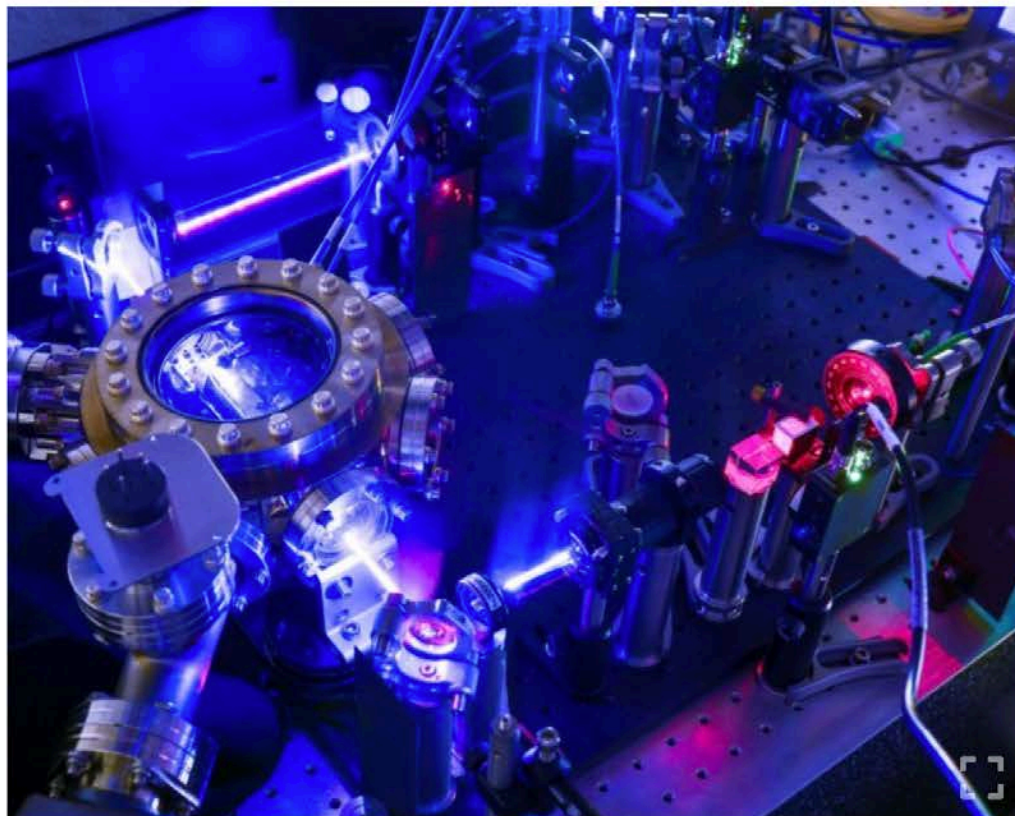
Email: nrlpao@nrl.navy.mil

History

The beginning of research into Quantum Information Science (QIS) can be traced to theoretical physicist, Richard Feynman, Ph.D. In 1982 he proposed that computers that take advantage of quantum mechanical principles may have certain advantages over classical computers. Since then, researchers have proposed or demonstrated various methods of using quantum information to achieve results that cannot be obtained with classical physics. One of the most striking proposals is the quantum computer itself.

Army designates Quantum Information Science Research Center

By U.S. Army DEVCOM Army Research Laboratory Office of Strategic Communications July 5, 2023



TOP STORIES

JANUARY 20, 2025

National Guard Bureau leaders meet with Guardsmen supporting 60th Presidential Inauguration

JANUARY 17, 2025

42nd Infantry Division heads to Warfighter exercise

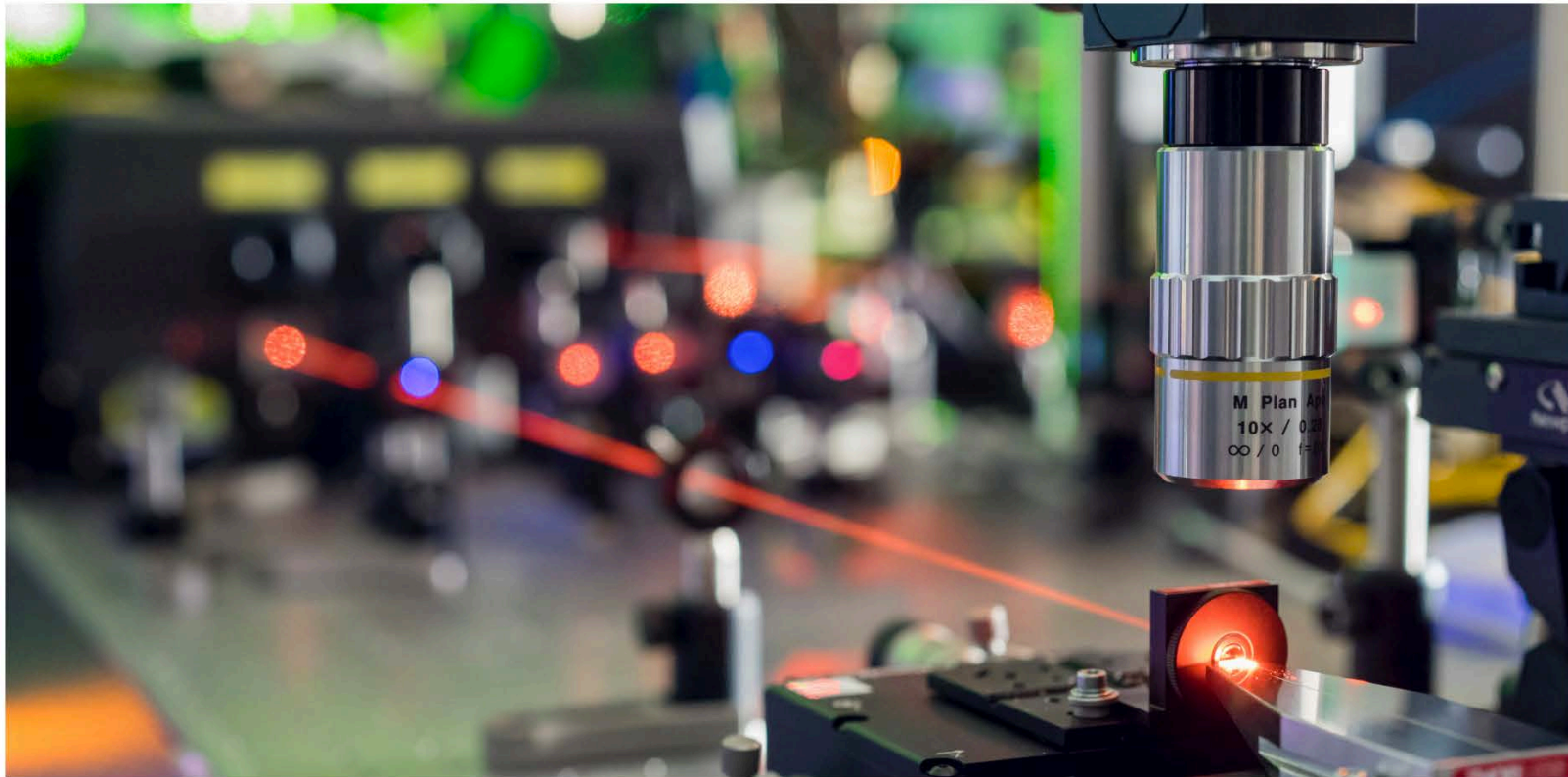
JANUARY 16, 2025

Bradley improvement tested at US Army Yuma Proving Ground

JANUARY 1, 2025

Army provides Soldiers, Families variety

AFRL – Quantum Information Science & Technology



AFRL/RITQ - Quantum Networking and Computing

- Integrated Photonics and Photon Qubits
- Quantum Algorithms
- Superconducting and Hybrid Quantum Systems
- Trapped Ions

Contact Information:

Email: ri.quantum.gov@afrl.af.mil

- Active collaboration between industry, quantum experts and quantum computing companies.



Menu

AIRBUS

Quantum technologies

A potential game-changer in aerospace





JPMorgan Chase

Samsung
Newsroom

Corporate Products Social Impact Press Resources 🔍 🌐

Overview | P

The First Step to a Quantum-Safe Future With Samsung Knox

on February 4, 2025



Revolution

Bo
Se
#O

Our ap

Our tec



- Many schools and internship opportunities for graduate, undergraduate and high school students!
 - REU program
 - Workshops and conferences



Quantum, Computing, Mathematics, & Physics Summer Camp

WHAT IS QCaMP?

- Two summer camps introducing high school teachers and students to quantum concepts and careers



2025 Camps

Teachers: June 16-July 18, 2025

Students: June 30-July 25, 2025

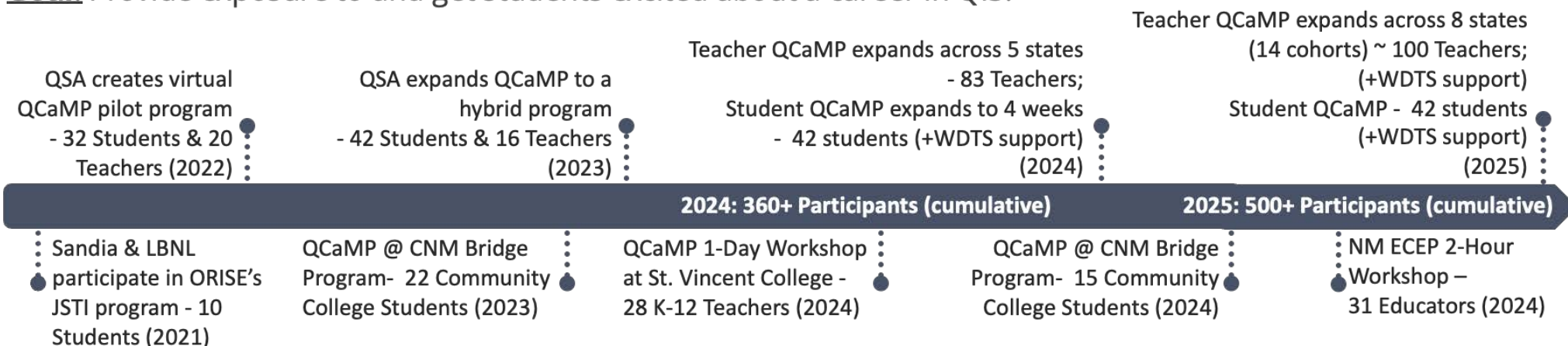
sandia.gov/quantum/qcamp/

QCaMP GOALS AND GUIDING PRINCIPLES

Goal: Reduce barriers. Stipends for all. No prerequisites! Hands-on puzzles/experiments throughout.

Goal: Give teachers tools to introduce quantum topics to their students, allowing us to **reach more students**.

Goal: Provide exposure to and get students excited about a career in QIS.



203 educators and 163 students reached since program inception!

Taken from Megan Ivory's presentation at QIST workshop 2025

Quantum Ideas Summer School

The Quantum Ideas Summer School is open to Undergraduates, Graduate Students, and Industry participants. Support for Undergraduate and Graduate students is funded by the National Science Foundation (PHY-2325080).

[2025 Summer School policy guidelines](#)

2025 Summer School

Application are now closed for 2025 Summer School June 9-13 at Duke University.

Application notifications will be sent no later than Friday 11 April.

Confirmed Instructors:

[Kenneth Brown \(Duke\)](#)

[Emily Edwards \(Duke\)](#)

[Natalie Klco \(Duke\)](#)

[Huangqian Loh \(Duke\)](#)

[Akimasa Miyake \(UNM\)](#)

[Crystal Noel \(Duke\)](#)

[Gokul Ravi \(U Michigan\)](#)

more instructors to be announced

Duke, since 2019

For advanced
undergraduate and
graduate students

2024 Session

Undergraduate and Graduate student support funded by the National Science Foundation award PHY-2325080 and outlined in [Participant Travel Guidelines](#). Summer STAQ 2019-2024 was funded under National Science Foundation award PHY-1818914.

For more details, contact the [STAQ Administrator](#).

2023 Session

Lecture Notes and Session Recordings

[Day 1 | Quantum Information Basics](#)

[Day 2 | Ions and QECC](#)

(Day 3 - Offsite programming. No lectures)

[Day 4 | Superconductors and Algorithms](#)

[Day 5 | Quantum Architecture and Optimization](#)

2022 Session

Lecture Notes

[DAY 1 | Basics with Aram Harrow](#)

[DAY 2 | Basics with Akimasa Miyake](#)

[DAY 3 | Algorithms with Peter Love](#)

[DAY 4 | Grover's Algorithm and Quantum Hype with Shelby Kimmel](#)

[DAY 5 | Quantum Error Correction with Ken Brown](#)



ORNL

The 2025 **U.S. Quantum Information Science Summer School (USQIS)** will be hosted by the Quantum Science Accelerator at Lawrence Berkeley National Laboratory. Dates and application information TBA.



FNAL

For undergraduate and graduate students

Quantum Computing Summer School Fellowship



**Developing new leaders in the theory,
application, and programming of
quantum computers**

For undergraduate and
graduate students

**The school will be held from June 8th to August
14th 2026.**

The Quantum Computing Summer School is an immersive 10-week curriculum that includes tutorials from world-leading experts in quantum computation as well as one-on-one mentoring from LANL staff scientists who are conducting cutting-edge quantum computing research. Summer school fellowship recipients will be exposed to the theoretical foundations of quantum computation and will become skilled at programming commercial quantum computers, such as those developed by IBM, Quera, IonQ, Quantinuum, DWave. Roughly twenty students (with the precise number determined based on the applicant pool) will be awarded a fellowship from LANL for the summer school. The fellowship amount ranges from \$10,000 to \$20,000, based on academic rank. All students (undergraduate and graduate) are encouraged to apply.

Student Opportunities

Including summer schools, career fairs, Quantum Thursdays, and Department of Energy internships

QIS 101: Foundations of Quantum Information Science

Virtual Summer School

Application period open through April 3, 2026

A six-week virtual program to introduce undergraduate and graduate students, recent college graduates, and academic educators to the exciting world of quantum information science (QIS). Students will learn how quantum algorithms can outperform classical computers. Through a series of demonstrations and hands-on programming labs, students will learn how to use IBM Qiskit — a world-class software package for working with quantum computers. Students will hear from guest scientists, including Brookhaven scientific researchers and C2QA leadership, and virtually tour some of Brookhaven Lab's facilities.

[Apply for QIS 101](#)

Application due April 3, 2026

\$500 weekly stipend

Undergraduate and graduate

QIS 300-Level Course Series

Virtual Self-Study Tutorials

C2QA researchers design virtual, 300-level quantum information science (QIS) courses to address critical gaps in the QIS ecosystem. Taught by leading quantum experts, each course offers insights into QIS challenges and includes workshops for participants to develop the technical skills needed to address these challenges. The QIS 300-level courses are intended for advanced undergraduate and graduate students, postdoctoral researchers, and early career scientists. Though the courses are typically offered in the summer, the lectures are posted online at the end of the course. Previously offered courses include:

- [QIS 301](#): Quantum Characterization, Verification, and Validation
- [QIS 302](#): A Practical Guide to Superconducting Qubit Experiments
- [QIS 303](#): Quantum Error Mitigation

[Explore courses](#)

Quantum Thursdays

Virtual Lecture Series

Hosted by the U.S. Department of Energy, Office of Science, National Quantum Information Science Research Centers ([NQISRC](#)), Quantum Thursdays features leading quantum information scientists sharing recent research accomplishments and that underway at the five NQISRCs and within the sector. In addition, lectures discuss the various career paths and opportunities available at the NQISRCs and within this field. New lectures will be announced as they become available.

[See past lectures](#)

Quantum Computing Internship for Physics Undergraduates Program (QCIPU)

Fermi National Accelerator Laboratory

Program Description

Three- week undergraduate physics students interested in quantum computing for the simulation of physics participate in this three-week program to gain hands-on Python and Qiskit programming experience. Daily lectures on the fundamentals of quantum mechanics and quantum computing are additional program components. The program culminates in participants running their own code on an IBM quantum computer.

QCIPU is a training platform that supports the next generation of scientific research and engineering professionals.

Eligibility

- Enrollment in a U.S. college or university majoring in physics, math, computer science, or engineering
- Completed year of introductory level physics course
- Eligible to work in the United States

Application Procedure

Before applying have all requirements ready. Once submitted, access to the application will no longer be available to upload any missing requirements.

- Online Application – [Application Walkthrough Video Guide](#)
- Resume
- Unofficial transcript
- A short statement of interest in the topic (Two paragraphs is sufficient, no more than a page)

Key Dates

Application Dates

November 3, 2025 – January 17, 2026

Offer Date

February 19, 2026

Acceptance Date

February 28, 2026

Program Dates

June 8 – June 26, 2026

The link is live during the application period.

APPLY

Questions?

[Email](#)



Google Summer of Code 2025

Introduction

In 2025 ML4SCI plans to participate in the program as a GSoC umbrella organization. The ML4SCI organization plans to partner with the [Google Summer of Code](#) in 2025 to broaden student participation in machine learning projects over a wide variety of scientific fields. ML4SCI participants will be mentored by scientists at top research universities and laboratories on research projects at the cutting edge of science. Projects span a wide range of scientific domains, including physics, astronomy, planetary science, quantum information science and others.

For Students

In 2025 GSoC students work with their mentors for 175 hrs to produce open-source codes that apply machine learning solutions to solve science problems. Projects span three evaluation periods that allow for students and mentors to collaborate on their project and evaluate student progress. Detailed rules for the GSoC program can be found [here](#). Interested students should look at the ideas page and contact the mentors. Candidates will be asked to complete an evaluation test for each project they apply to demonstrate the skills needed for the respective projects. In the next step, students will produce a proposal which will be evaluated for final student selection.

Please see the [official GSoC Timeline](#)

Projects in 2025



[Google Summer of Code](#) program is intended for advanced students who are already familiar with AI or quantum computing. [CMS](#) is a high-energy physics experiment at the [Large Hadron Collider](#) (LHC) at [CERN](#). It is a general-purpose detector that is designed to observe any new physics phenomena that the LHC might reveal. CMS acts as a giant, high-speed camera, taking 3D “photographs” of particle collisions from all directions up to 40 million times each second. The CMS collects few tens of Peta-Bytes of data each year and processes them through Worldwide LHC Computing Grid infrastructure around the globe.

[List of proposals](#)



[DeepLense](#) is a deep learning pipeline for particle dark matter searches with strong gravitational lensing.

[List of proposals](#)

The Adjoint Is All You Need: Characterizing Barren Plateaus in Quantum Ansätze

Enrico Fontana,^{1,2} Dylan Herman,^{1,*} Shouvanik Chakrabarti,¹ Niraj Kumar,¹
Romina Yalovetzky,¹ Jamie Heredge,^{1,3} Shree Hari Sureshbabu,¹ and Marco Pistoia¹

¹*Global Technology Applied Research, JPMorgan Chase*

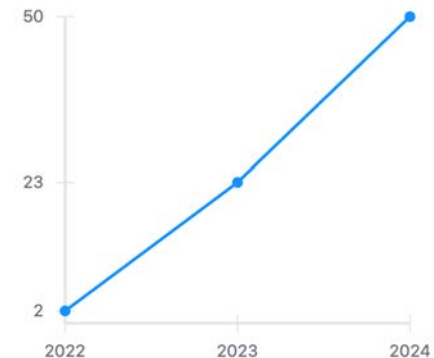
²*Computer and Information Sciences, University of Strathclyde*

³*School of Physics, The University of Melbourne*

Using tools from the representation theory of compact Lie groups, we formulate a theory of Barren Plateaus (BPs) for parameterized quantum circuits whose observables lie in their dynamical Lie algebra (DLA), a setting that we term Lie algebra Supported Ansatz (LASA). A large variety of commonly used ansätze such as the Hamiltonian Variational Ansatz, Quantum Alternating Operator Ansatz, and many equivariant quantum neural networks are LASAs. In particular, our theory provides, for the first time, the ability to compute the variance of the gradient of the cost function of the quantum compound ansatz. We rigorously prove that, for LASA, the variance of the gradient of the cost function, for a 2-design of the dynamical Lie group, scales inversely with the dimension of the DLA, which agrees with existing numerical observations. In addition, to motivate the applicability of our results for 2-designs to practical settings, we show that rapid mixing occurs for LASAs with polynomial DLA. Lastly, we include potential extensions for handling cases when the observable lies outside of the DLA and the implications of our results.

Nature Commun. 15 (2024) 1, 7171
2309.07902

Citations per year



INTERNSHIP

Research Technologies Summer Associate Program

(Applications currently closed)

Program information

Learn more about our Research Technologies internship program

What You'll Do

Who We're Looking For

What We Offer

Valued qualities

We are seeking colleagues with excellent analytical, quantitative and problem solving skills and demonstrated research ability. We value strong communication skills and the ability to present findings to a non-technical audience.

Key skills

- Enrolled in a master's or Ph.D. degree program in computer science, math, engineering, sciences or other quantitative fields
- Experience with quantum computing algorithms and applications
- Excellent knowledge of Python programming
- Working knowledge of common research evaluation frameworks and techniques
- No prior experience with financial markets required

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What we're looking for:

- A strong background in data security. **This could include experience such as:**
 - *Strong understanding of data security principles in modern application contexts*
 - Implementing modern encryptions algorithms
 - *Delivering key management systems, key distribution protocols, or working with hardware security modules*
 - *Identifying and mitigating real-world security vulnerabilities and attack vectors*
- You've managed or technically led a team of developers and owned application development from planning to delivery.
- You bring hands-on engineering experience and have tackled complex, ambiguous problems through code.
- You use your technical background to coach developers and elevate team performance

Bonus points for:

- Experience with post-quantum cryptography (e.g., lattice-based algorithms) in theory or practice
- Familiarity with evolving cryptographic standards and bodies (e.g., NIST), and how they shape implementation decisions

• You drive delivery and long-term planning, shaping roadmaps that balance

Summary

- Quantum Information Science (QIS) and Quantum Computing (QC) open new opportunities by connecting diverse fields: physics, mathematics, computer science, and engineering.
- Although it may take time to mature, QIS/QC will eventually become an integral part of our daily lives, much like AI/ML have become today.
- Challenges remain: scalability, noise mitigation, algorithmic development, and workforce training.
 - But progress is accelerating rapidly.
- Quantum computing today is where classical computing was in the 1950s: slow, expensive, and limited, yet full of promise to redefine science and technology.
- It is crucial to explore where quantum computers can make a real impact from fundamental science to applications in our society.
- We need young, creative, and imaginative minds (YOU!) to shape this emerging quantum future.

Ad

- A workshop on “Quantum Science and Technology for Fundamental Physics with a Global Perspective”
 - to be held at the University of Kansas in Lawrence, Kansas, from May 27–30, 2026
 - May 27-28: Lectures
 - Introduction to quantum algorithms by Thomas Wong (Creighton University)
 - Quantum simulation by Meurice, Yannick (U of Iowa)
 - Qiskit/PennyLane by Nathan Thomson (Wichita State University)
 - May 28-29: Invited and contributed talks
 - May 30: Open discussion
- Undergraduate and graduate students are welcome.

THE MAP OF QUANTUM COMPUTING

CLASSICAL COMPUTERS

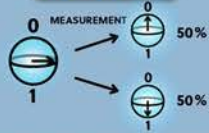


CLASSICAL VS. QUANTUM

QUANTUM COMPUTERS



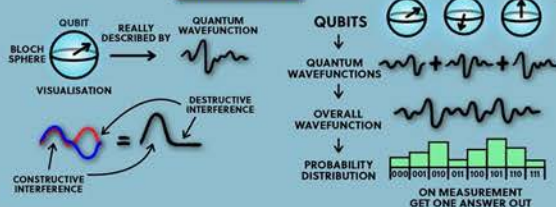
SUPERPOSITION



ENTANGLEMENT



INTERFERENCE

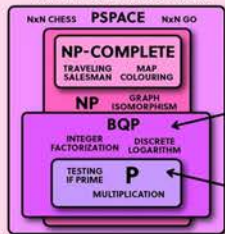


QUANTUM ALGORITHMS

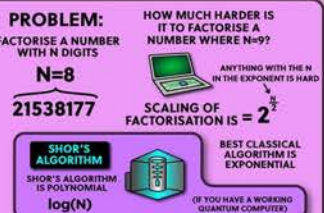


COMPLEXITY THEORY

HOW MUCH HARDER IS IT TO SOLVE THE PROBLEM AS THE PROBLEM GETS LARGER?



QUANTUM COMPLEXITY THEORY



CLASSICAL COMPUTERS



NON-COMPUTABILITY

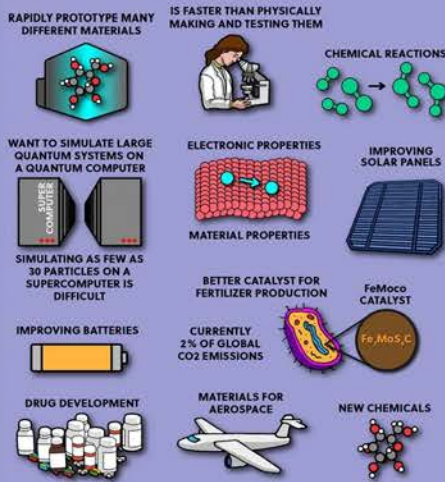


GROVER'S ALGORITHM

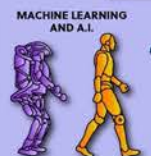


POTENTIAL APPLICATIONS OF QUANTUM COMPUTERS

QUANTUM SIMULATION



OPTIMIZATION PROBLEMS

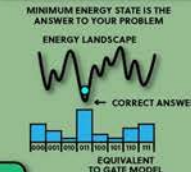


FINANCIAL MODELING

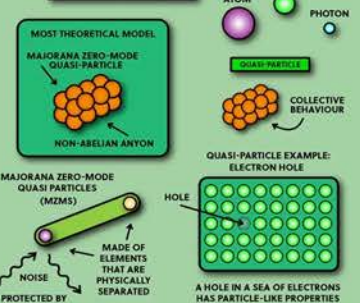


MODELS OF QUANTUM COMPUTING

ADIABATIC QUANTUM COMPUTING



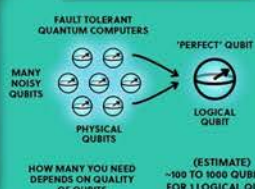
TOPOLOGICAL QUANTUM COMPUTING



QUANTUM ANNEALING



QUANTUM ERROR CORRECTION

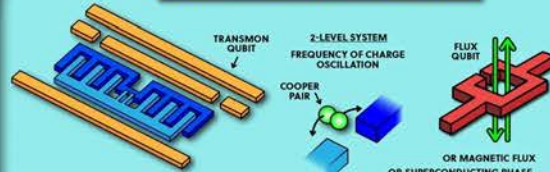


OBSTACLES

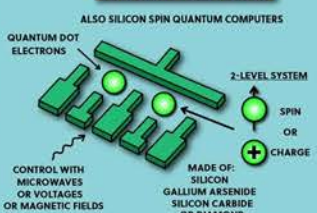


PHYSICAL REALISATIONS

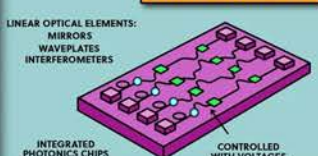
SUPERCONDUCTING QUANTUM COMPUTERS



QUANTUM DOT QUANTUM COMPUTERS



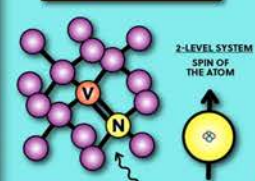
LINEAR OPTICAL QUANTUM COMPUTERS



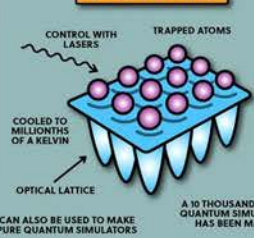
TRAPPED ION QUANTUM COMPUTERS



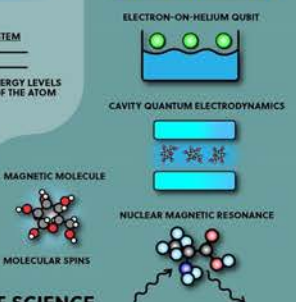
COLOUR CENTRE QUANTUM COMPUTERS



NEUTRAL ATOMS IN OPTICAL LATTICES



OTHER APPROACHES



Extra slides

\$ vs Tiger

- Teaching quantum information science to high-school and early undergraduate students by Sophia Economou, Terry Rudolph, Edwin Barnes, 2005.07874
- You encounter two doors:
Money behind at least one door

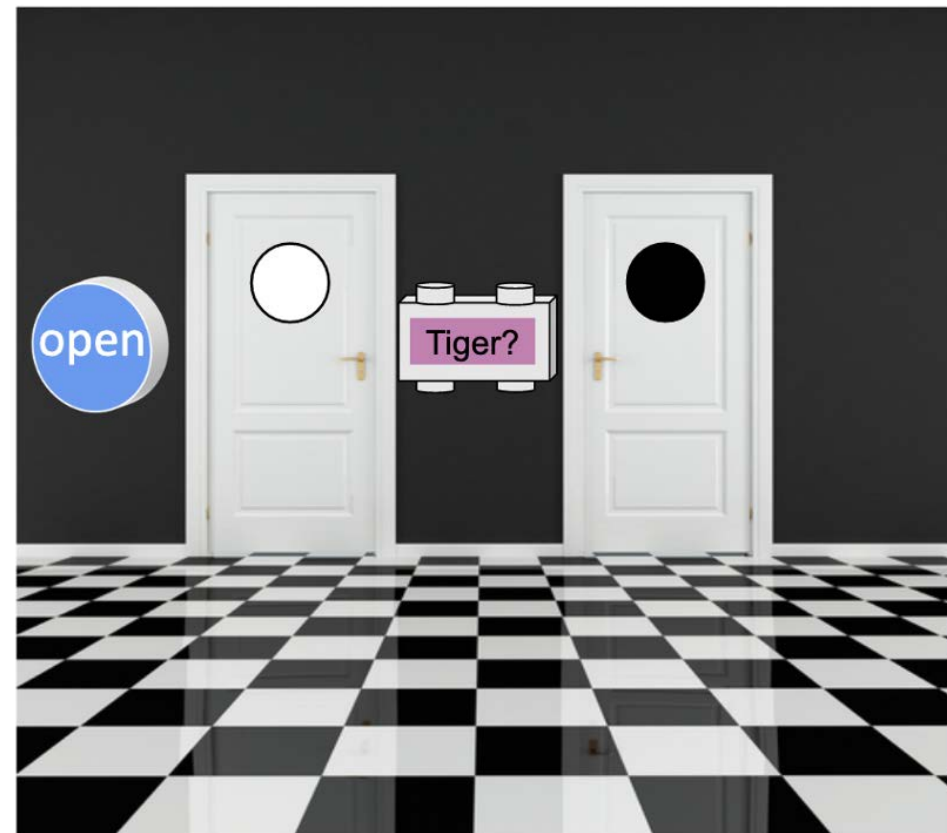
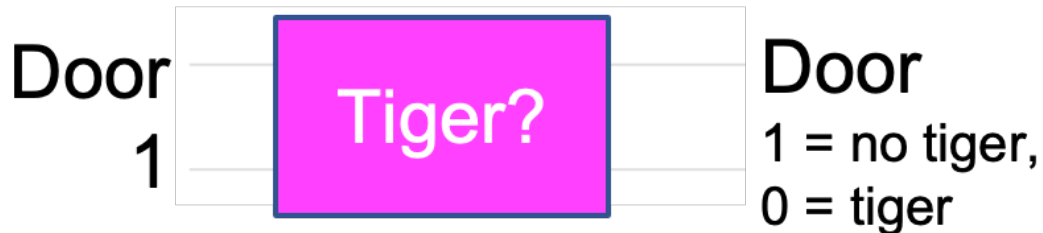


- Tiger might be lurking behind one door



\$ vs Tiger

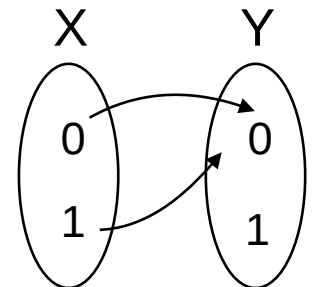
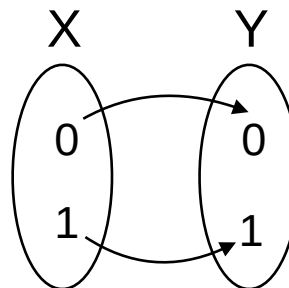
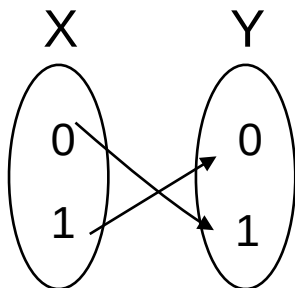
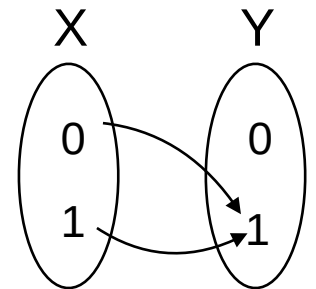
- The button on the left opens both doors
- YOU WANT TO BE SURE THERE'S NO TIGER BEHIND EITHER DOOR BEFORE YOU PUSH THE "OPEN" BUTTON
- The device in the middle will tell you **if there is a tiger behind the door that you ask about** – but **you only get to use it once**



Deutsch Algorithm

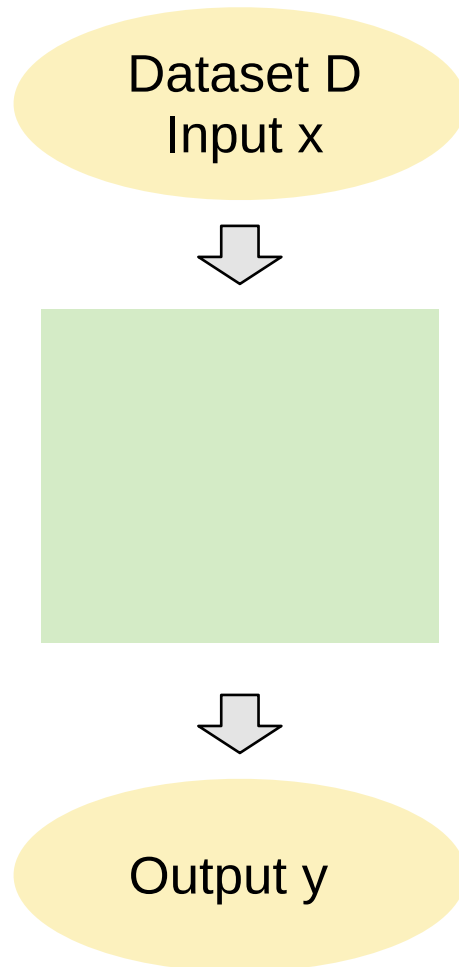
1985

- We want to find out whether a particular function, with one input bit and one output bit is constant or balanced. **Classically, we need to evaluate the function twice** (i.e., for input = 0 and input = 1), but remarkably, **we only need to evaluate the function once using quantum algorithm**, by using Deutsch's algorithm.
- Consider a simple function, $f(x) : \{0,1\} \longrightarrow \{0,1\}$
- For possible functions
 - Identity: $f(0) = 0$ and $f(1) = 1$
 - Bit-flip function: $f(0) = 1$ and $f(1) = 0$
 - Constant function: $f(x) = 0$ or $f(x) = 1$

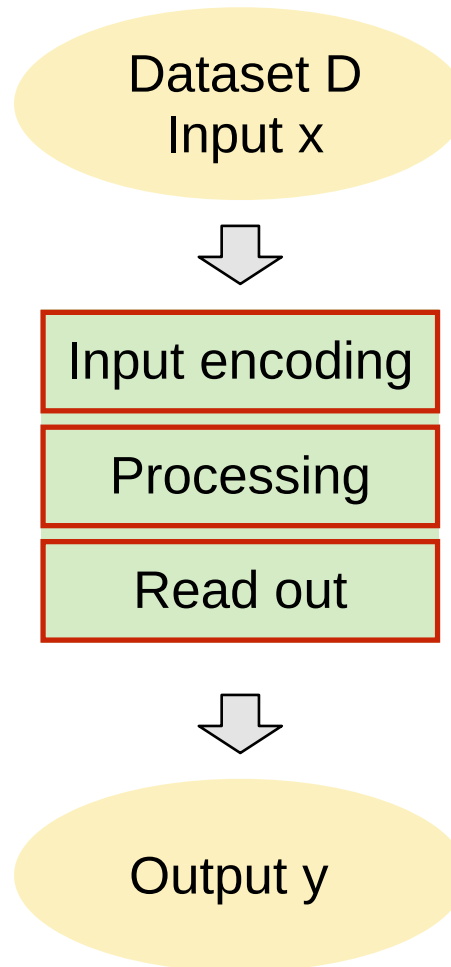


Quantum Algorithms and Data Embedding

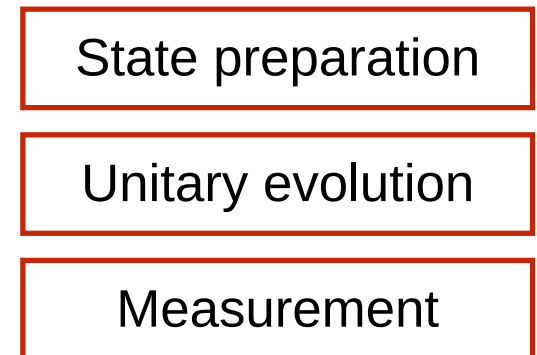
Classical Algorithm



Quantum Algorithm



Quantum System



Quantum Physics: states,
amplitudes,
Hamiltonian,
time.

Encoding

Data



State of a
quantum system

basis encoding of binary string (1, 0),
i.e. representing integer 2

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

amplitude encoding of unit-length
complex vector $(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$

Hamiltonian encoding of a matrix A

$$U = e^{-iH_A t}$$

time-evolution encoding of a scalar t

data encoding in different parts of the state and
operator description

Basis encoding:

maps a collection of items into the states forming an orthonormal basis of the Hilbert space of the considered quantum system.

The orthonormal basis $\{|x\rangle\}_{x \in X}$, called **computational basis**, is made by the eigenstates of a reference observable measured on the considered quantum system. For instance, a bit can be encoded into a qubit by the mapping $0 \rightarrow |0\rangle$, $1 \rightarrow |1\rangle$. Then the n -bit strings $(x_1 \cdots x_n)$ can be encoded into the states of n qubits forming an orthonormal basis of a 2^n -dimensional Hilbert space H_n :

$$\mathbb{B}^n \ni (x_1 \cdots x_n) \mapsto |x_1 \cdots x_n\rangle \in H_n$$

→ can prepare superposition of data that can be processed in parallel, e.g.

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle$$

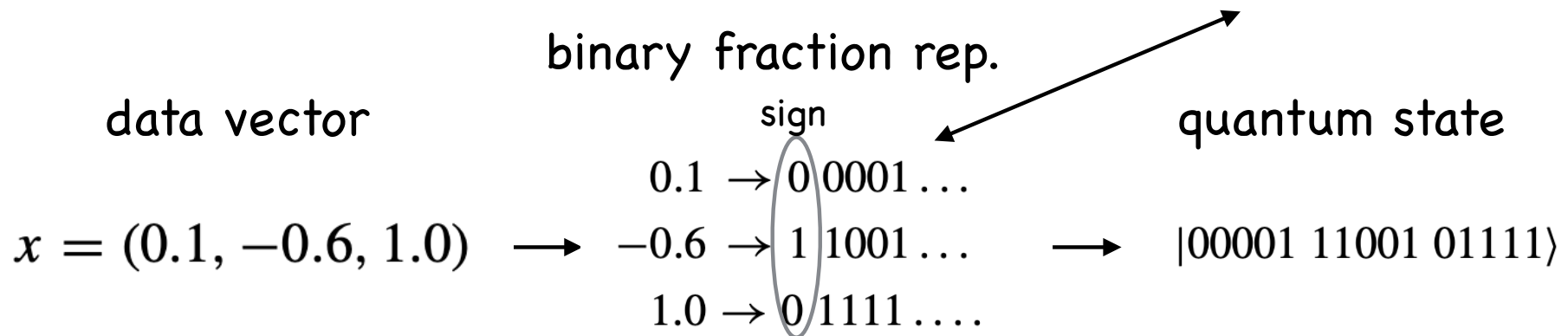
(ex) max-cut problem

Binary encoding into basis states

Represent numbers as binaries, each binary digit requires a qubit

basis vector coefficient $\{0,1\}$

$$x = \sum_{k=1}^{\tau-1} b_k \frac{1}{2^k}$$



The number of qubits = $N\tau$

N = # of features

τ = # of bits in binary rep

Time-Evolution Encoding

Associates the input data value x with the time evolution parameter t

$$U(x) = e^{-ixH}$$

In quantum machine learning, this kind of encoding is particularly popular when encoding classical trainable parameters into a quantum circuit. The most common choice are the Pauli rotation gates, in which $H = \frac{1}{2}\sigma_a$ and $a \in \{x, y, z\}$. Successive gates or evolutions of the form $U(x)$ can be used to encode a real-valued vector $\mathbf{x} \in \mathbb{R}^N$

data	Encoded with RY gate applied to initial state $ 0\rangle$
-0.438	$ \psi(-0.438)\rangle = \cos(-0.438/2) 0\rangle + \sin(-0.438/2) 1\rangle$ $\approx 0.976 0\rangle - 0.217 1\rangle$

sine/cosine structure typical for Time-Evolution Encoding → leads to Fourier-type dependence of amplitudes on the inputs

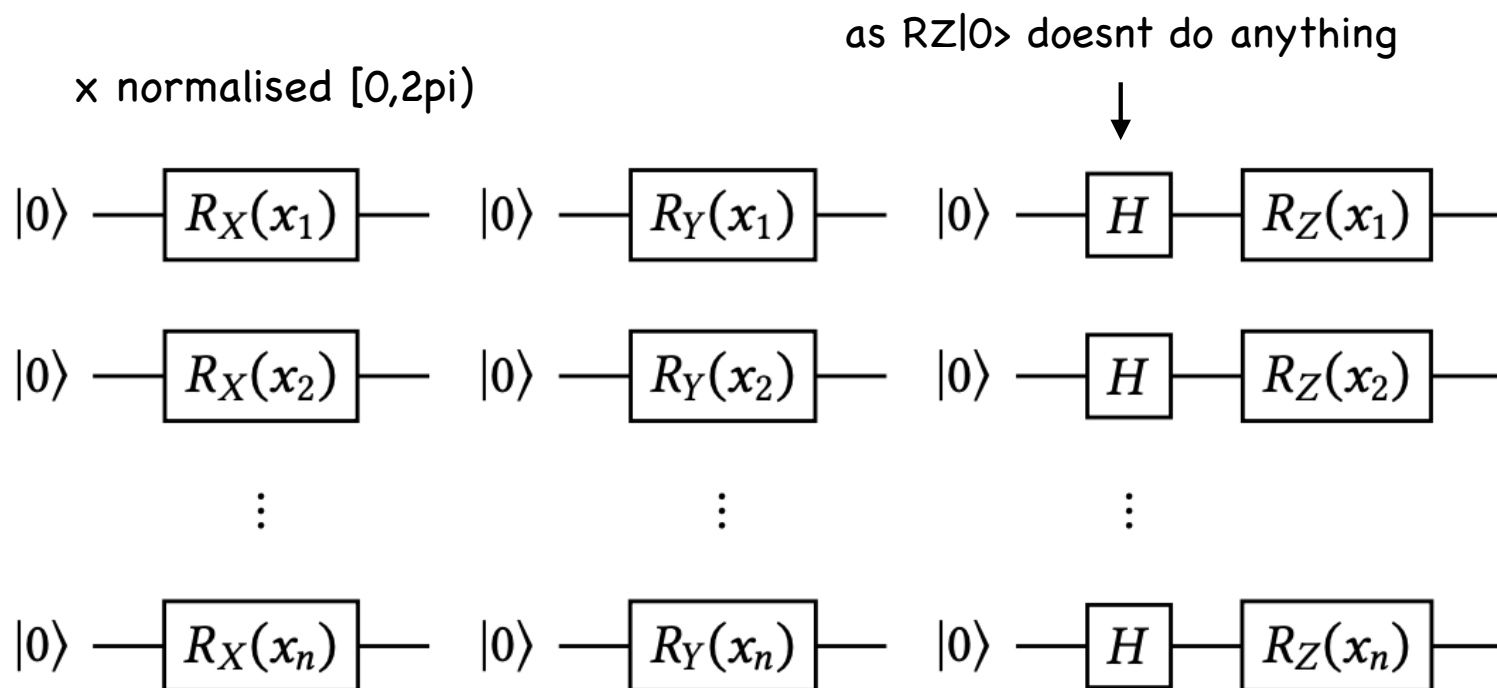
The number of qubits = N

N = # of features

Angle/Rotation encoding

When used on an n -qubit circuit, this feature map of angle encoding can take up to n numerical inputs $x_1, \dots, x_n$. The action of its circuit consists in the application of a rotation gate on each qubit j parametrised by the value x_j . In this feature map, we are using the x_j values as angles in the rotations, hence the name of the encoding.

Example



Hamiltonian Encoding:

For some applications, it can be useful to encode matrices into the Hamiltonian of a time evolution. The basic idea is to associate a Hamiltonian H with a square matrix $\mathbf{A}$. In case $\mathbf{A}$ is not Hermitian, one can sometimes use the trick of encoding

$$H_{\mathbf{A}} = \begin{pmatrix} 0 & \mathbf{A} \\ \mathbf{A}^\dagger & 0 \end{pmatrix}$$

instead, and to perform the computations in two subspaces of the Hilbert space. Hamiltonian encoding allows us to extract and process the eigenvalues of $\mathbf{A}$, for example, to multiply $\mathbf{A}$ or $\mathbf{A}^{-1}$ with an amplitude-encoded vector.

The number of qubits = $\log N$

N = # of features

Hamiltonian Encoding

$$\mathbf{A} = \begin{pmatrix} 0.073 & -0.438 \\ 0.730 & 0.000 \end{pmatrix} \quad H_{\mathbf{A}} = \begin{pmatrix} 0 & \mathbf{A} \\ \mathbf{A}^\dagger & 0 \end{pmatrix} \quad H_{\mathbf{A}} = \begin{pmatrix} 0 & 0 & 0.073 & -0.438 \\ 0 & 0 & 0.730 & 0.000 \\ 0.073 & 0.730 & 0 & 0 \\ -0.438 & 0.000 & 0 & 0 \end{pmatrix}$$

- Eigenvectors and eigenvalues of H

$$\mathbf{v}_1 = (-0.108 \ -0.699 \ 0.704 \ -0.065), \quad \lambda_1 = -0.736$$

$$\mathbf{v}_2 = (0.699 \ -0.108 \ 0.065 \ 0.704), \quad \lambda_2 = -0.435$$

$$\mathbf{v}_3 = (0.699 \ -0.108 \ -0.065 \ -0.704), \quad \lambda_3 = 0.435$$

$$\mathbf{v}_4 = (0.108 \ 0.699 \ 0.704 \ -0.064), \quad \lambda_4 = 0.736.$$

- These eigenvectors form a basis and the most general state α can be decomposed as

$$\alpha = \gamma_1 \mathbf{v}_1 + \gamma_2 \mathbf{v}_2 + \gamma_3 \mathbf{v}_3 + \gamma_4 \mathbf{v}_4$$

- Applying $U(t) = \exp(-iHt)$ $\alpha' = e^{-iH_{\mathbf{A}}t} \alpha = e^{-i\lambda_1 t} \gamma_1 \mathbf{v}_1 + e^{-i\lambda_2 t} \gamma_2 \mathbf{v}_2 + e^{-i\lambda_3 t} \gamma_3 \mathbf{v}_3 + e^{-i\lambda_4 t} \gamma_4 \mathbf{v}_4$.

$$|00\rangle = (1,0,0,0) \quad |01\rangle = (0,1,0,0) \quad |10\rangle = (0,0,1,0) \quad |11\rangle = (0,0,0,1)$$

$$\begin{aligned} |\psi\rangle = & (-0.108e^{-i\lambda_1 t} \gamma_1 + 0.699e^{-i\lambda_2 t} \gamma_2 + 0.699e^{-i\lambda_3 t} \gamma_3 + 0.108e^{-i\lambda_4 t} \gamma_4) |00\rangle \\ & + (-0.699e^{-i\lambda_1 t} \gamma_1 - 0.108e^{-i\lambda_2 t} \gamma_2 - 0.108e^{-i\lambda_3 t} \gamma_3 + 0.699e^{-i\lambda_4 t} \gamma_4) |01\rangle \\ & + (0.704e^{-i\lambda_1 t} \gamma_1 + 0.065e^{-i\lambda_2 t} \gamma_2 - 0.065e^{-i\lambda_3 t} \gamma_3 + 0.704e^{-i\lambda_4 t} \gamma_4) |10\rangle \\ & + (-0.065e^{-i\lambda_1 t} \gamma_1 + 0.704e^{-i\lambda_2 t} \gamma_2 - 0.704e^{-i\lambda_3 t} \gamma_3 - 0.064e^{-i\lambda_4 t} \gamma_4) |11\rangle \end{aligned}$$

- In Dirac notation:

Amplitude encoding:

The number of qubits = $\log N$

N = # of features

Represent classical data as amplitudes of a quantum state

$$2^n = N$$

$$|\psi_{\mathbf{x}}\rangle = \sum_{i=1}^d x_i |\phi_i\rangle \in H$$

$$\sum_i^d |x_i|^2 = 1$$

or for composite systems with $\sum_{ij} |a_{ij}|^2 = 1$

$$\langle \phi_i | \phi_j \rangle = \delta_{ij}$$

$$|\psi_A\rangle = \sum_{i,j=1}^d a_{ij} |\phi_i\rangle \otimes |\phi_j\rangle \in H \otimes H$$

Example:

data vector

normalised and padded data vector

$$\mathbf{x} = (0.1, -0.6, 1.0) \longrightarrow \mathbf{x} = (0.073, -0.438, 0.730, 0.000)$$



quantum state

$$|\psi_{\mathbf{x}}\rangle = 0.073|00\rangle - 0.438|01\rangle + 0.730|10\rangle + 0|11\rangle$$

This could also be encoded as a matrix A

$$A = \begin{pmatrix} 0.073 & -0.438 \\ 0.730 & 0.000 \end{pmatrix}$$

Amplitude encoding uses much less qubits than basis encoding, however, routines to prepare amplitude vectors can be costly

Qsample encoding:

Given a probability distribution p on the finite set X , it can be encoded in the state:

$$|\psi_p\rangle = \sum_{x \in X} \sqrt{p(x)} |x\rangle \in H$$

Repeated measurements on the state $|\psi_p\rangle$ with respect to the computational basis allow to sample the distribution p .

In a sense a hybrid case of basis and amplitude encoding since the information is represented by amplitudes, but the features are encoded in the qubits.

Quantum Algorithms and Data Embedding

	Classical data	Requirement	Quantum state
Basis Encoding	$\vec{x} \in \{0,1\}^{\otimes n}$ $\vec{x} = (x_1, x_2, \dots, x_n) \in \{0,1\}$		$ \psi\rangle = x_1 x_2 \dots x_n\rangle$ $= x_1\rangle \otimes x_2\rangle \otimes \dots \otimes x_n\rangle$
	$\vec{x} \in \mathbb{R}^{2^n}$ $x_i \in \mathbb{R}$	$\sum_{i=1}^{2^n} x_i ^2 = 1$	$ \psi_x\rangle = \sum_{i=1}^{2^n} x_i i\rangle$
Amplitude Encoding	$A \in \mathbb{R}^{2^n \times 2^m} \quad i = 1, \dots, 2^n$ $A_{ij} \in \mathbb{R} \quad j = 1, \dots, 2^m$	$\sum_{i,j} A_{ij} ^2 = 1$	$ \psi_A\rangle = \sum_{i,j} A_{ij} i\rangle \otimes j\rangle$
	$A \in \mathbb{R}^{2^n \times 2^n}$	$\sum_i A_{ii} = 1 \quad A^\dagger = A$ $A_{ij}^* = A_{ji}$	$\rho_A = \sum_{i,j} A_{ij} i\rangle \langle j $
Time-evolution Encoding	$x \in \mathbb{R}$	$x \in [0, 2\pi)$	$U(x) = e^{-ixH}$
Hamiltonian Encoding	$A \in \mathbb{R}^{2^n \times 2^n}$	$A^\dagger = A$	$H_A = A$
	$A \in \mathbb{R}^{2^n \times 2^n}$	$A^\dagger \neq A$ (in general)	$H_A = \begin{pmatrix} 0 & A \\ A^\dagger & 0 \end{pmatrix}$

Quantum Data Embedding

- Quantum Data Embedding Patterns for Quantum AI/ML algorithms
- Data Encoding Patterns for Quantum Computing
- Expanding Data Encoding Patterns For Quantum Algorithms

Data re-uploading for a universal quantum classifier

1907.02085

Kolmogorov–Arnold representation theorem

In [real analysis](#) and [approximation theory](#), the **Kolmogorov–Arnold representation theorem** (or **superposition theorem**) states that every [multivariate continuous function](#) $f: [0, 1]^n \rightarrow \mathbb{R}$ can be represented as a [superposition](#) of continuous single-variable functions.

The works of [Vladimir Arnold](#) and [Andrey Kolmogorov](#) established that if f is a multivariate continuous function, then f can be written as a finite [composition](#) of continuous functions of a single variable and the [binary operation](#) of [addition](#).^[1] More specifically,

$$f(\mathbf{x}) = f(x_1, \dots, x_n) = \sum_{q=0}^{2n} \Phi_q \left(\sum_{p=1}^n \phi_{q,p}(x_p) \right),$$

where $\phi_{q,p}: [0, 1] \rightarrow \mathbb{R}$ and $\Phi_q: \mathbb{R} \rightarrow \mathbb{R}$.

- It solved a more constrained form of Hilbert's thirteenth problem, so the original Hilbert's thirteenth problem is a corollary. In a sense, they showed that the only true continuous multivariate function is the sum, since every other continuous function can be written using univariate continuous functions and summing.

Universal Approximation Theorem

- A feed-forward network with a single hidden layer containing a finite number of neurons can approximate continuous functions on compact subsets of $\mathbb{R}^n$, under mild assumptions on the activation function.

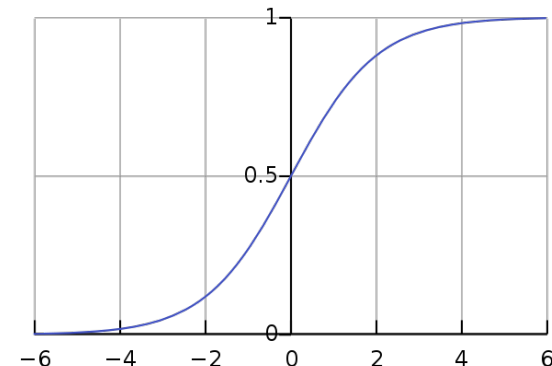
Let $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ be a nonconstant, bounded, and continuous function (called the *activation function*). Let I_m denote the m -dimensional unit hypercube $[0, 1]^m$. The space of real-valued continuous functions on I_m is denoted by $C(I_m)$. Then, given any $\varepsilon > 0$ and any function $f \in C(I_m)$, there exist an integer N , real constants $v_i, b_i \in \mathbb{R}$ and real vectors $w_i \in \mathbb{R}^m$ for $i = 1, \dots, N$, such that we may define:

$$F(x) = \sum_{i=1}^N v_i \varphi(w_i^T x + b_i) \quad \text{as an approximate realization of the function } f; \text{ that is,}$$

$|F(x) - f(x)| < \varepsilon$ for all $x \in I_m$. In other words, functions of the form $F(x)$ are dense in $C(I_m)$.

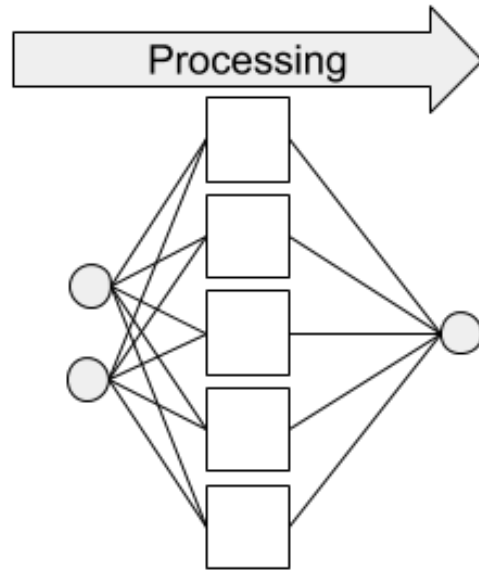
- A. N. Kolmogorov, 1957
- G. Cybenko, 1989 with sigmoid activation
- K. Hornik, 1991, importance of the multilayer architecture
- Z. Lu et al, 2017, with deep neural network and ReLU activation

$$S(x) = \frac{1}{1 + e^{-x}} = \frac{e^x}{e^x + 1}$$

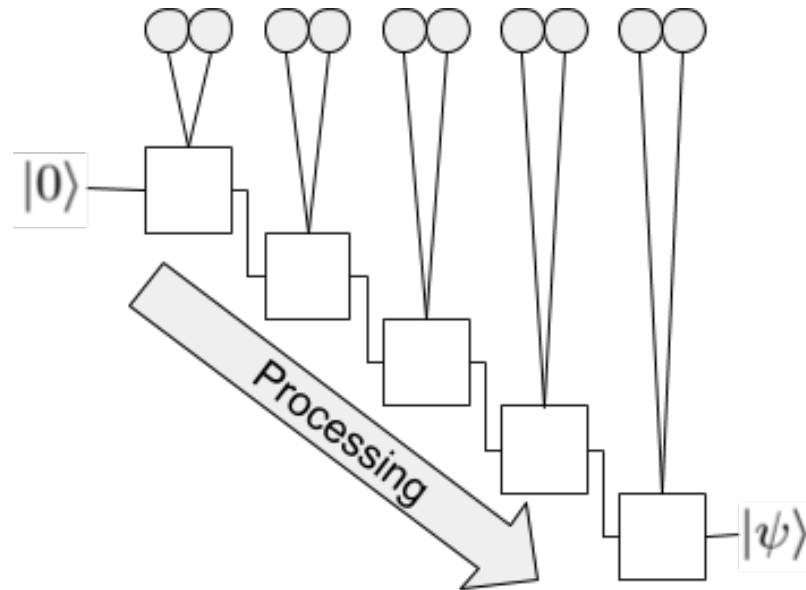


Data re-uploading for a universal quantum classifier

1907.02085



(a) Neural network



(b) Quantum classifier

- Universal approximation theorem
- We can approximate a function $F(\vec{x})$ with $f(\vec{x}, \vec{\theta})$, where $\vec{x}$ is an input feature and $\vec{\theta}$ is a learnable parameter.
- The cost function (ex. MSE) to be minimized is
$$\sum_{i=1}^n |F(\vec{x}_i) - f(\vec{x}_i, \vec{\theta})|^2$$

Single qubit classifier using data re-uploading

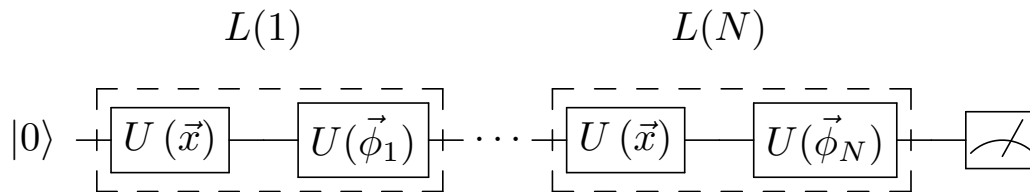
- Consider the three dimensional data, $\vec{x}$. (can be generalized.) 1907.02085
- Data can be re-uploaded using unitary transformation $U(\vec{x})$ rotating the qubit.
- The single-qubit classifier has the following structure: $|\psi\rangle = \mathcal{U}(\vec{\phi}, \vec{x})|0\rangle$

$$\mathcal{U}(\vec{\phi}, \vec{x}) = L(N) \dots L(1)$$

$$L(i) \equiv U(\vec{\phi}_i)U(\vec{x}) \quad \vec{\phi} = (\phi_1, \phi_2, \phi_3)$$

$$\mathcal{U}(\vec{\phi}, \vec{x}) \equiv U(\vec{\phi}_N)U(\vec{x}) \dots U(\vec{\phi}_1)U(\vec{x})$$

$$U(\phi_1, \phi_2, \phi_3) \in SU(2)$$

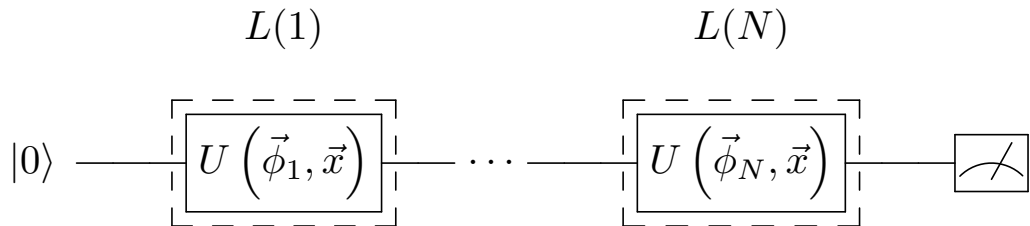


(a) Original scheme

$$L(i) = U(\vec{\theta}_i + \vec{w}_i \circ \vec{x})$$

Hadamard product of $\vec{w}_i$ and $\vec{x}$:

$$\vec{w}_i \circ \vec{x} = (w_i^1 x^1, w_i^2 x^2, w_i^3 x^3)$$



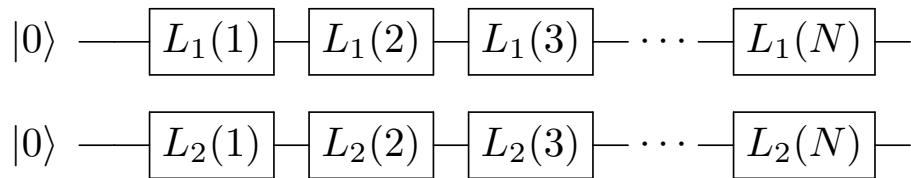
(b) Compressed scheme

$$L(i) = U(\vec{\theta}_i^{(k)} + \vec{w}_i^{(k)} \circ \vec{x}^{(k)}) \dots U(\vec{\theta}_i^{(1)} + \vec{w}_i^{(1)} \circ \vec{x}^{(1)})$$

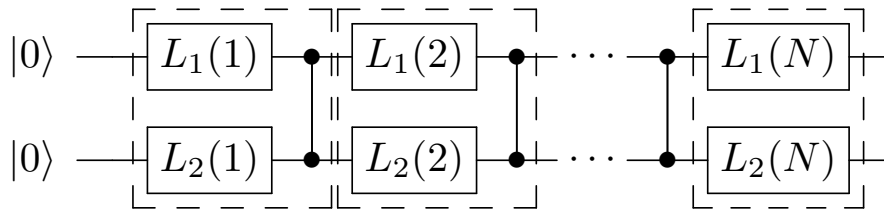
The number of parameters = 3 N

$$U(\vec{\phi}) = U(\phi_1, \phi_2, \phi_3) = e^{i\phi_2\sigma_z} e^{i\phi_1\sigma_y} e^{i\phi_3\sigma_z} \quad \text{or} \quad U(\vec{\phi}) = e^{i\vec{\omega}(\vec{\phi}) \cdot \vec{\sigma}}$$

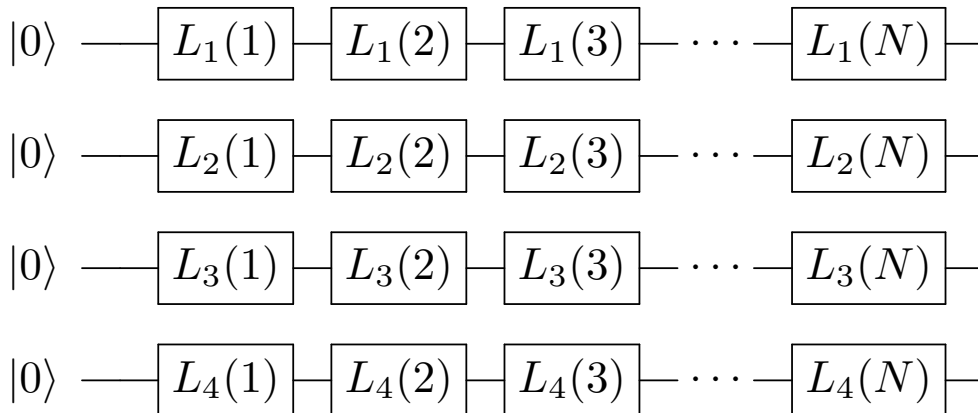
Multi-qubit using data re-uploading



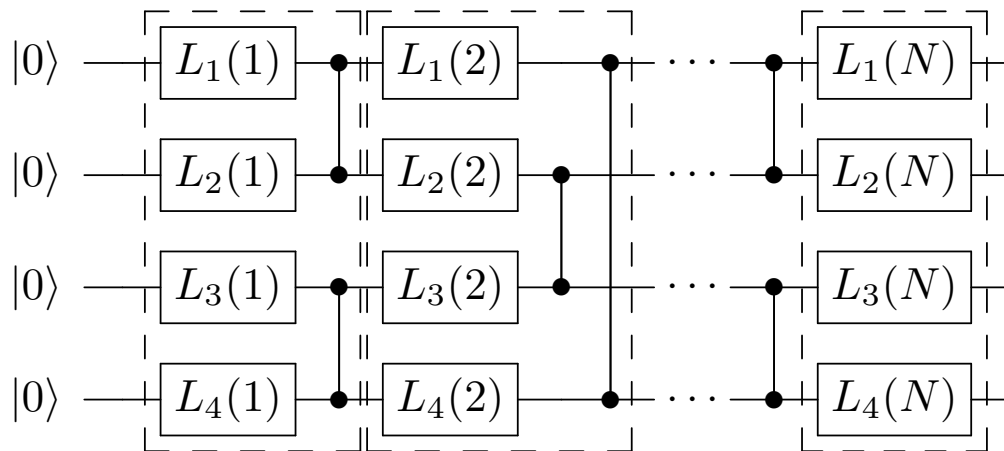
(a) Ansatz with no entanglement



(b) Ansatz with entanglement



(a) Ansatz with no entanglement



(b) Ansatz with entanglement

Single qubit classifier: measurements

- The quantum circuit characterized by a series of processing angle $\{\theta_i\}$ and weights $\{w_i\}$ delivers a final state $|\psi\rangle$. $|\psi\rangle = \mathcal{U}(\vec{\phi}, \vec{x})|0\rangle$
- The critical point in the quantum measurement is to find an optimal way to associate outputs from the observations to target classes.
- This is easily established for a dichotomic classification, where one of two classes A and B have to be assigned to the final measurement of the single qubit.
- In such a case it is possible to measure the output probabilities $P(0)$ for $|0\rangle$ and $P(1)$ for $|1\rangle$. A given pattern could be classified into the A class if $P(0) > P(1)$ and into B otherwise.
- We may refine this criterium by introducing a bias. That is, the pattern is classified as A if $P(0) > \lambda$, and as B otherwise. The λ is chosen to optimize the success of classification on a training set.
- The assignment of classes to the output reading of a single qubit becomes an involved issue when many classes are present.
 - For example, one possible strategy consists on comparing the probability $P(0)$ to four sectors with three thresholds: $0 \leq \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq 1$. Then, the value of $P(0)$ will fall into one of them, and classification is issued.

Single qubit classifier: cost function

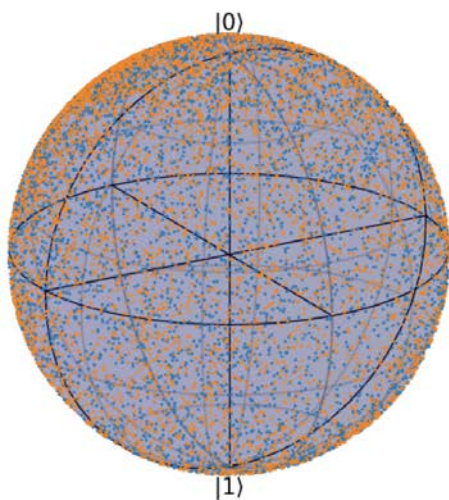
- A fidelity cost function: (fidelity is a measure of similarity of two things.)
 - We want to force the quantum state (data state) $|\psi(\vec{\theta}, \vec{w}, \vec{x})\rangle$ to be as near as possible to one particular state (label state) on the Bloch sphere.
 - The angular distance between the label state and the data state can be measured with the relative fidelity between the two states.
 - Goal is to maximize the average fidelity.

Easy choice: $|0\rangle$ and $|1\rangle$

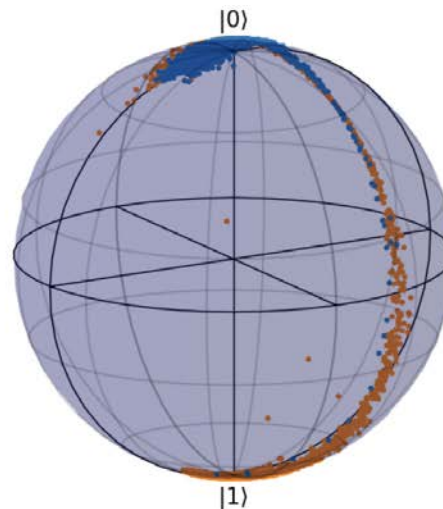
$$\chi_f^2(\vec{\theta}, \vec{w}) = \sum_{\mu=1}^M \left(1 - \left| \langle \tilde{\psi}_s | \psi(\vec{\theta}, \vec{w}, \vec{x}_\mu) \right|^2 \right) \text{ where } |\tilde{\psi}_s\rangle \text{ is the correct label state}$$

of the data points. (M = total number of training data)

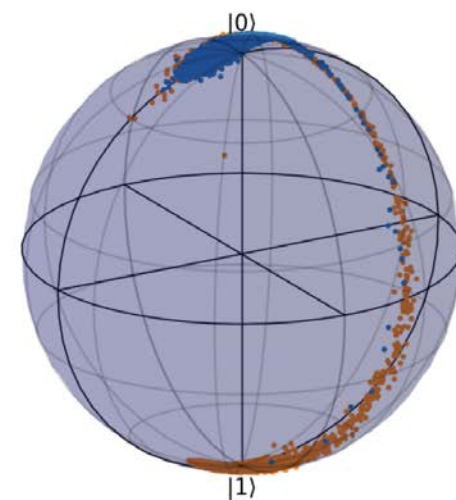
1907.02085



Before training

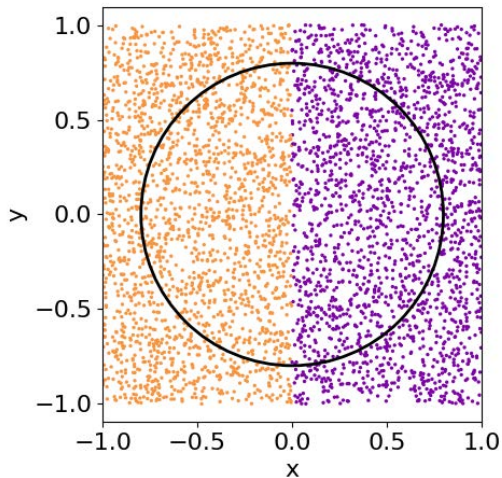


After training

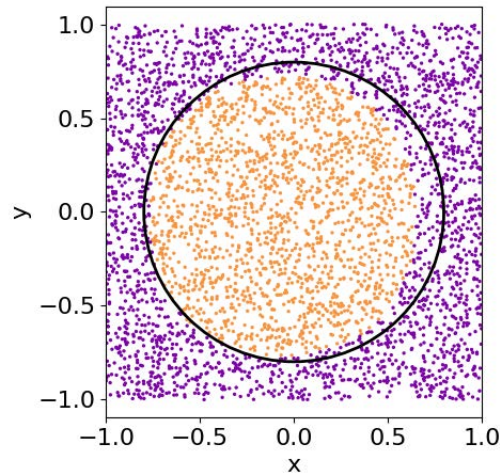


With testing data

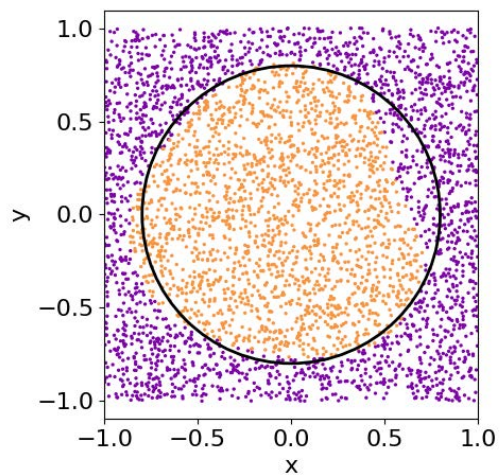
Example: binary classification



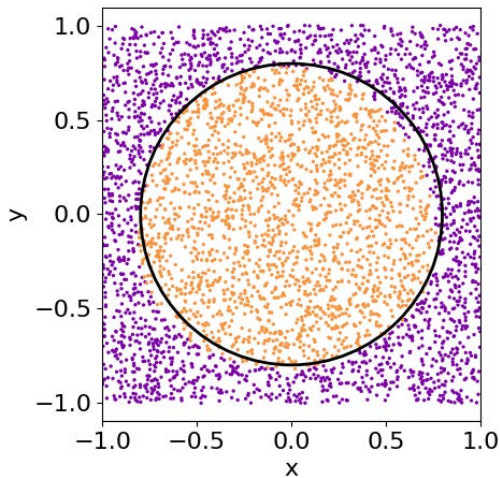
(a) 1 layer



(b) 2 layers



(c) 4 layers

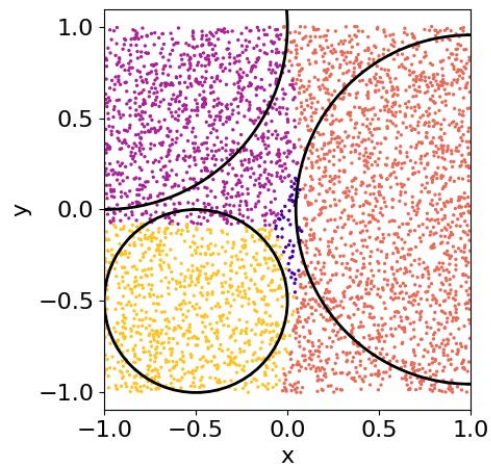


(d) 8 layers

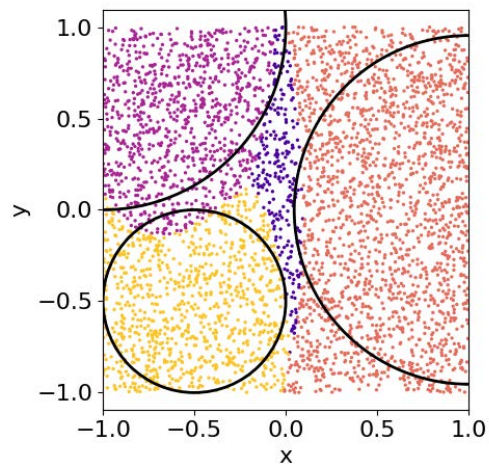
Qubits Layers	χ_f^2	
	1	2
		No Ent. Ent.
1	0.50	0.75 —
2	0.85	0.80 0.73
3	0.85	0.81 0.93
4	0.90	0.87 0.87
5	0.89	0.90 0.93
6	0.92	0.92 0.90
8	0.93	0.93 0.96
10	0.95	0.94 0.96

The number of parameters = 3 N

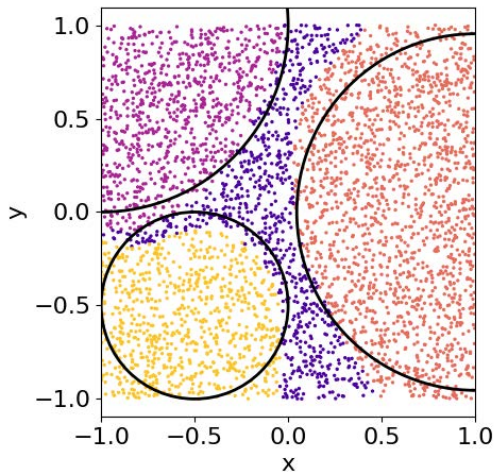
Example: 4 classes



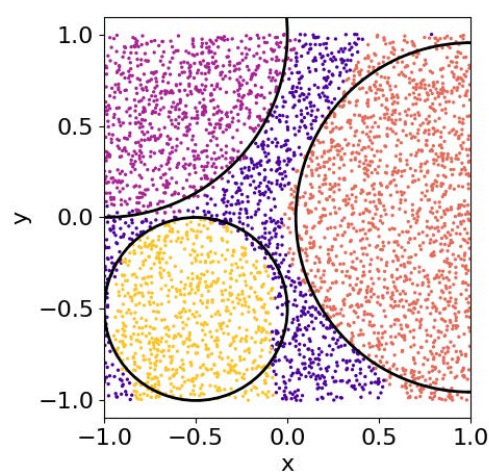
(a) 1 layer



(b) 3 layers



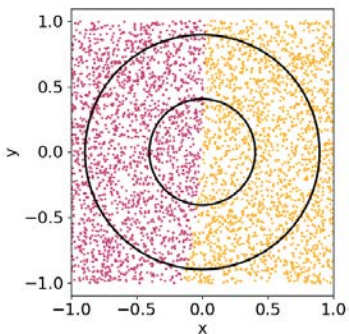
(c) 4 layers



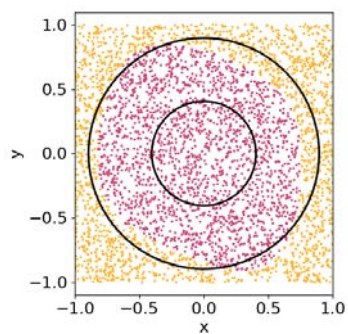
(d) 10 layers

Qubits Layers	χ_f^2	
	1	2
		No Ent. Ent.
1	0.73	0.56 —
2	0.79	0.77 0.78
3	0.79	0.76 0.75
4	0.84	0.80 0.80
5	0.87	0.84 0.81
6	0.90	0.88 0.86
8	0.89	0.85 0.89
10	0.91	0.86 0.90

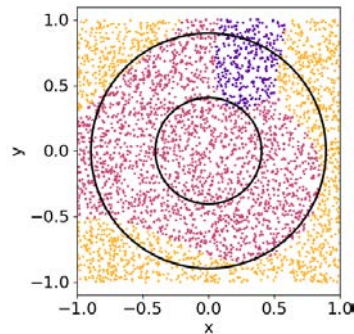
Example: 3 classes



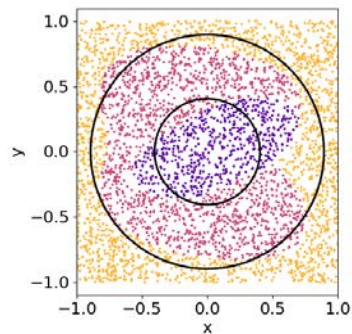
(a) 1 layer



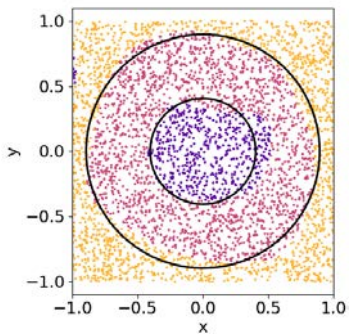
(b) 2 layers



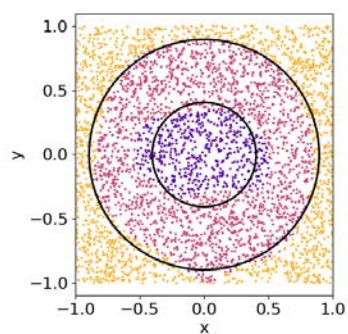
(c) 3 layers



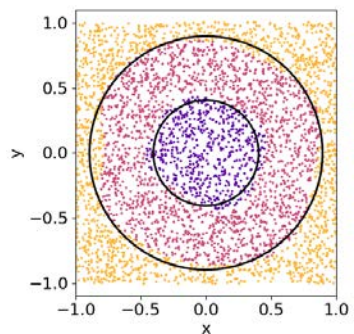
(d) 4 layers



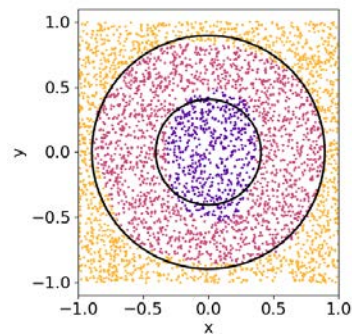
(e) 5 layers



(f) 6 layers



(g) 8 layers



(h) 10 layers

Qubits Layers	χ_f^2		
	1	2	
		No Ent.	Ent.
1	0.34	0.51	—
2	0.57	0.63	0.59
3	0.80	0.68	0.65
4	0.84	0.78	0.89
5	0.92	0.86	0.82
6	0.93	0.91	0.93
8	0.90	0.89	0.90
10	0.90	0.91	0.92

Single qubit classifier: example

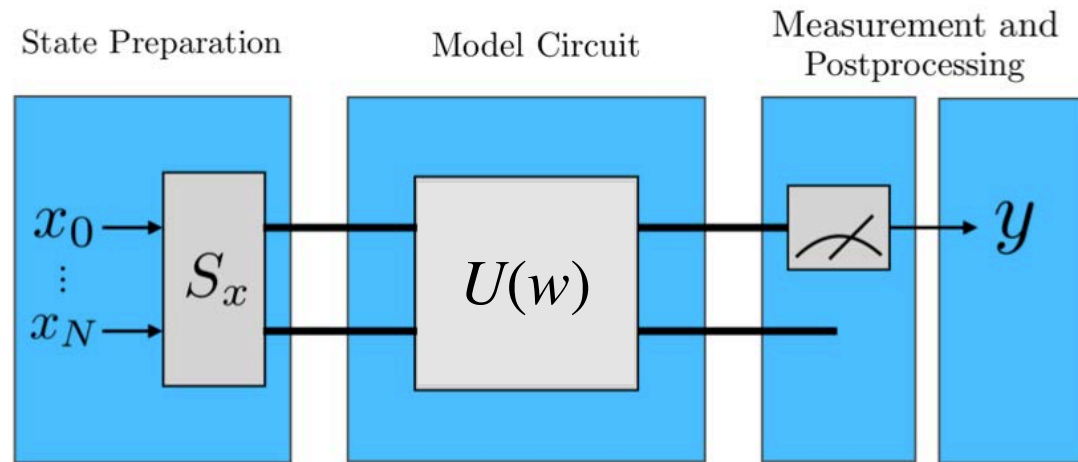
Problem	Classical classifiers		Quantum classifier	
	NN	SVC	χ_f^2	χ_{wf}^2
Circle	0.96	0.97	0.96	0.97
3 circles	0.88	0.66	0.91	0.91
Hypersphere	0.98	0.95	0.91	0.98
Annulus	0.96	0.77	0.93	0.97
Non-Convex	0.99	0.77	0.96	0.98
Binary annulus	0.94	0.79	0.95	0.97
Sphere	0.97	0.95	0.93	0.96
Squares	0.98	0.96	0.99	0.95
Wavy Lines	0.95	0.82	0.93	0.94

Comparison between single-qubit quantum classifier and two well-known classical classification techniques: a neural network (NN) with a single hidden layer composed of 100 neurons and a support vector classifier (SVC), both with the default parameters as defined in scikit-learn python package. This table shows the best success rate, being 1 the perfect classification, obtained after running ten times the NN and SVC algorithms and the best results obtained with single-qubit classifiers up to 10 layers.

4/9/2025 Wednesday

- Final term project due April 30 Wednesday
 - Late submission will not be accepted.
- HW9 due 3pm 4/11 Friday
- This week we will discuss
 - VQC as Fourier series
 - HHL (Quantum Algorithm for Linear Systems of Equations)
- Tutorial on Kernel method (4/11 Friday)
- In-person poster presentations, April 25, 2025, 2-4:15 pm, Gray-Little Hall.
 - Many of undergraduate students will be presenting 3:15pm - 4:15pm.
 - No lecture on 4/25 but we will be meeting in Gray-Little Hall.

Quantum Machine Learning with a Variational Quantum Circuit

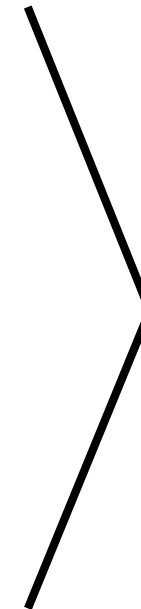
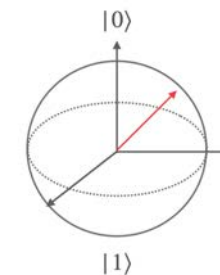
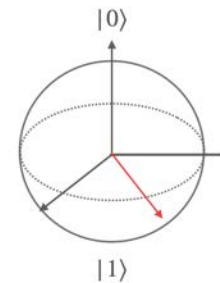


state preparation $\swarrow$ n corresponds to # features

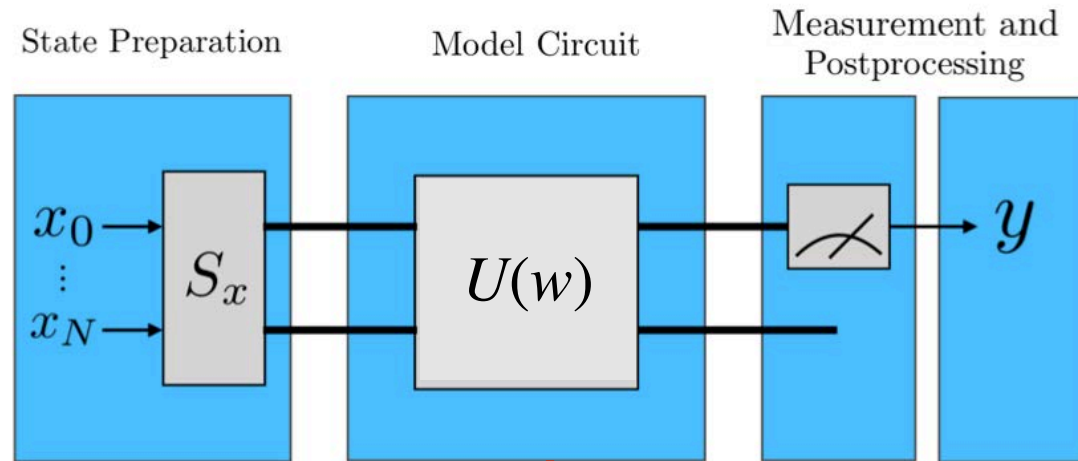
$$x \mapsto S_x |\phi\rangle = S_x |0\rangle^{\otimes n} = |x\rangle$$

e.g. angle encoding

$$|x\rangle = \bigotimes_{i=1}^n \cos(x_i) |0\rangle + \sin(x_i) |1\rangle$$



Quantum Machine Learning with a Variational Quantum Circuit

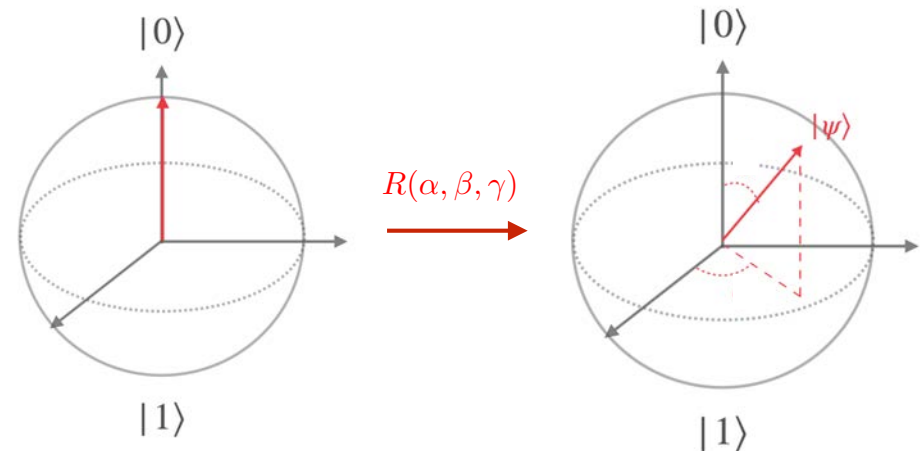
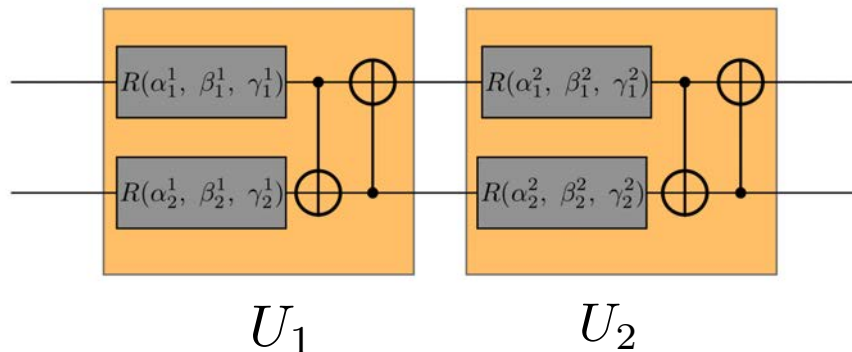


$$|\psi\rangle = U(w)|x\rangle \quad \text{with} \quad U(w) = U_{l_{\max}}(w_{l_{\max}}) \dots U_l(w_l) \dots U_1(w_1)$$

Annotations for the equation above:

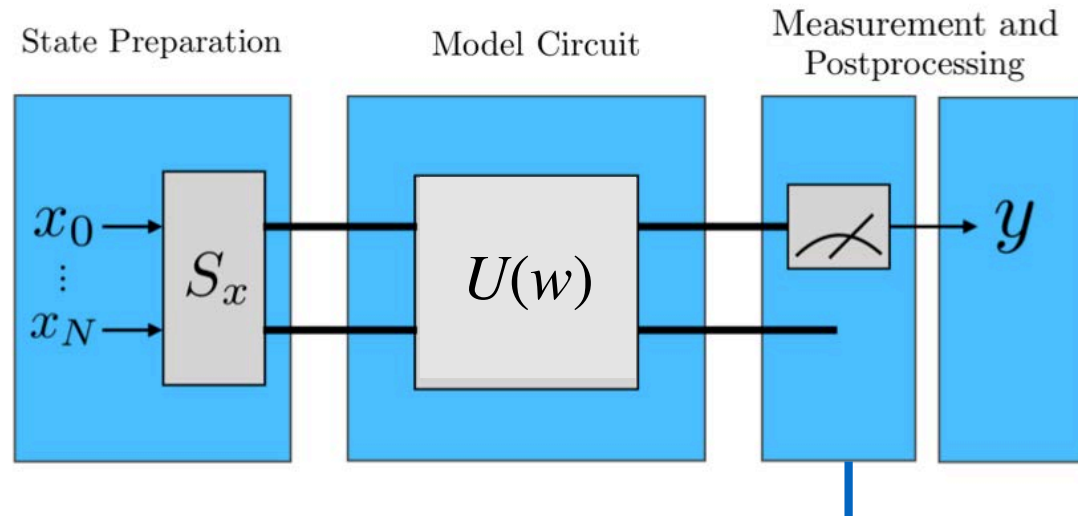
- $U(w)$ is labeled as the **model circuit**.
- w is labeled as **trainable parameters**.
- $|x\rangle$ is labeled as the **prepared state**.

2-layer Variational Quantum Circuit



➡ Rotation + CNOT -> Entanglement

Quantum Machine Learning with a Variational Quantum Circuit



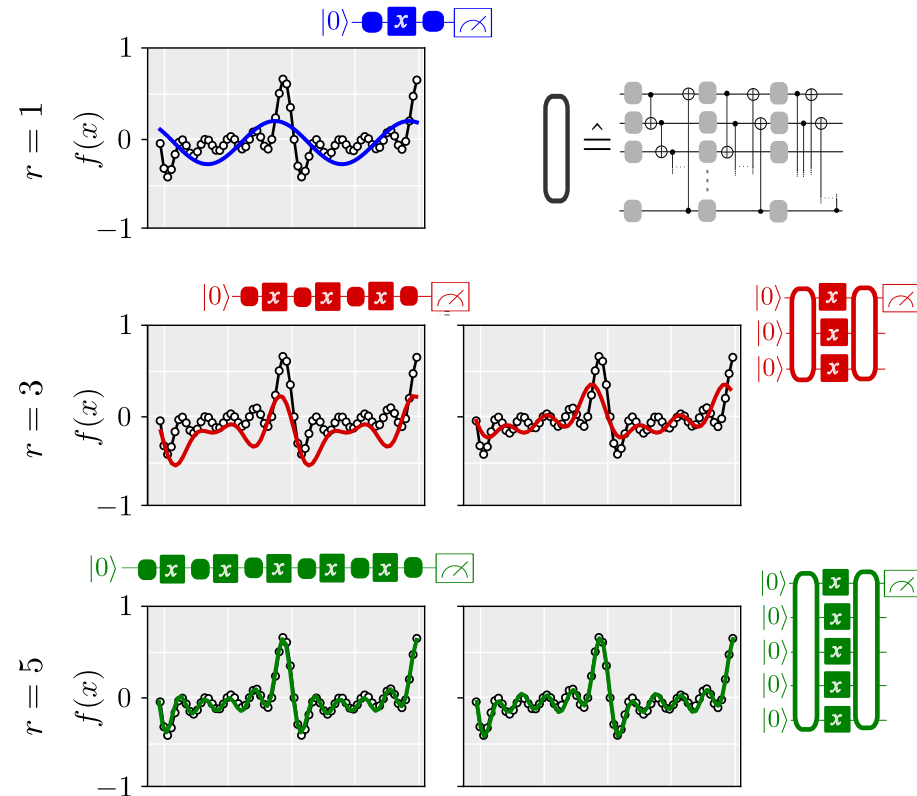
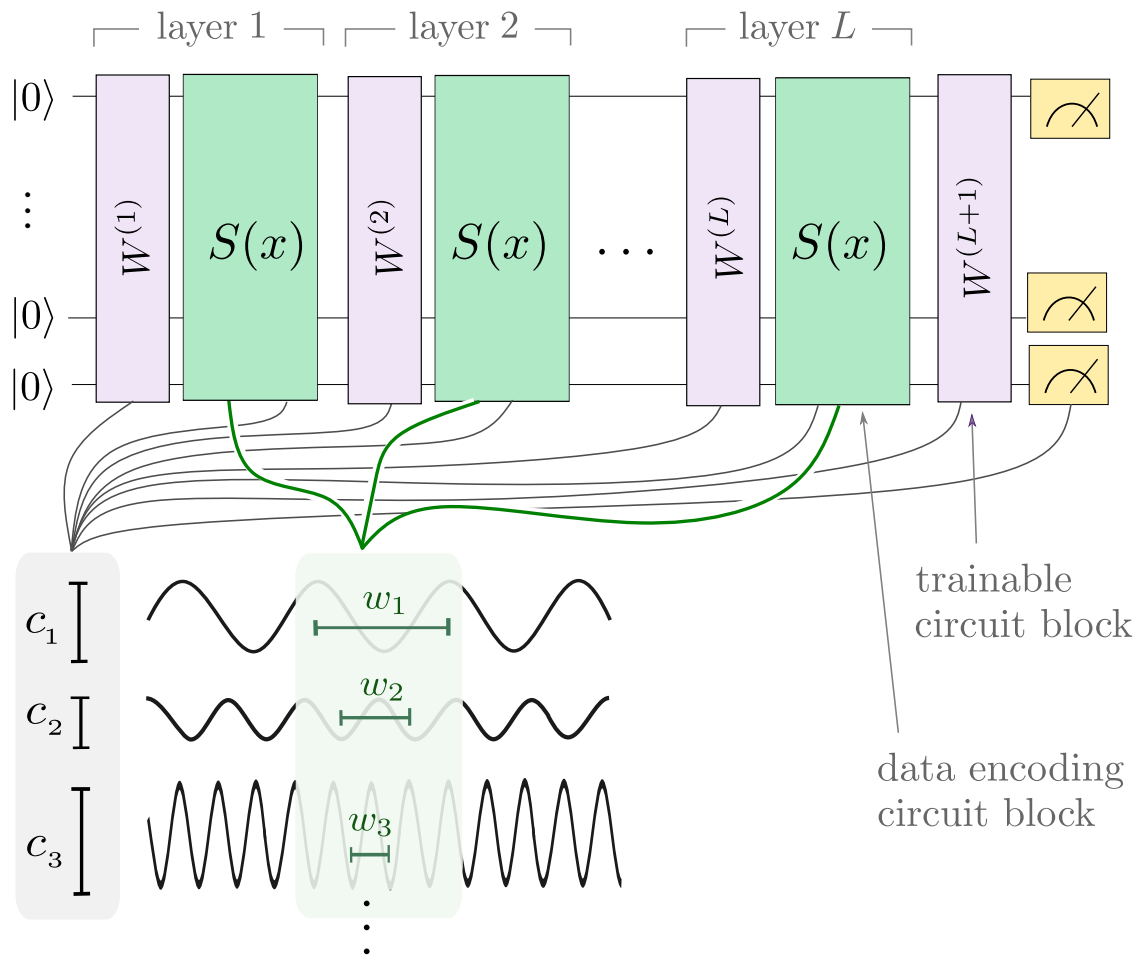
- Entangled state shares information across qubits
- Evaluate expectation value of qubits to construct loss
for supervised S vs B classification one qubit sufficient

$$\mathbb{E}(\sigma_z) = \langle 0 | S_x(x)^\dagger U(w)^\dagger \hat{O} U(w) S_x(x) | 0 \rangle = \pi(w, x) \quad \text{for} \quad \hat{O} = \sigma_z \otimes \mathbb{I}^{\otimes(n-1)}$$

- Quantum network output: $f(w, b, x) = \pi(w, x) + b$
- Changing operator and loss => VQE, VQT, ... (simulate QFT)

The effect of data encoding on the expressive power of variational quantum machine learning models

- VQA as a Fourier series

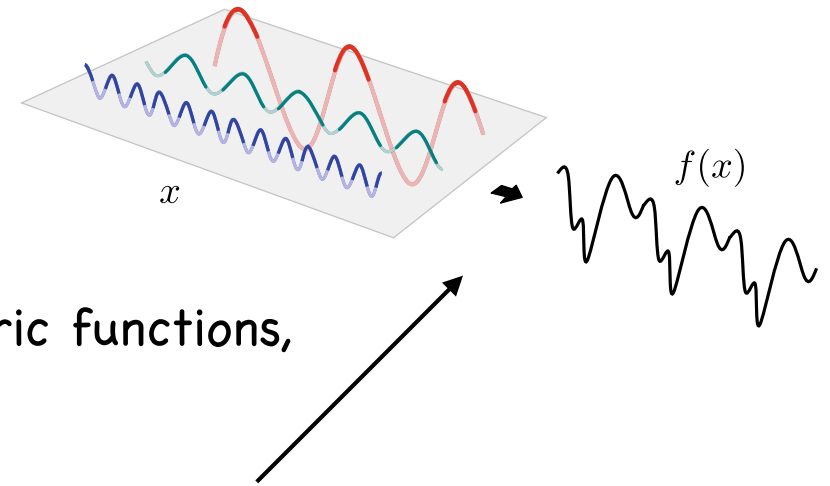


$$f_{\theta}(x) = \sum_{n \in \Omega} c_n(\theta) e^{inx}$$

$$f_{\theta}(x) = \langle 0 | U^{\dagger}(x, \theta) M U(x, \theta) | 0 \rangle$$

$$U(x) = W^{(L+1)} S(x) W^{(L)} \dots W^{(2)} S(x) W^{(1)}$$

Expressibility of model and encoding



- Most encodings result in sum of trigonometric functions, e.g. angle encoding, time evolution encoding
- Fourier series is universal approximator, but problem is that for many encoding strategies quantum models are linear combinations of functions composed of very few frequencies
- If encoding gates are not rich enough $\rightarrow$ model limited irrespective of width or depth in variational circuit
 - $\Rightarrow$ Encoding controls expressivity of model (circuit expressivity $\neq$ model expressivity)
 - $\Rightarrow$ Fourier analysis for VQC (mindful about pre-scaling)
 - $\Rightarrow$ Can hint on where VQC models are particularly useful

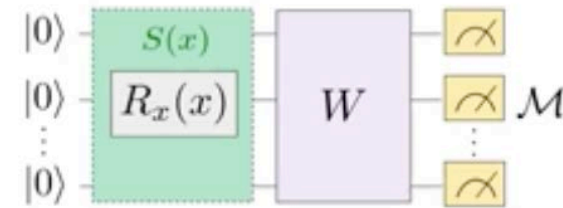
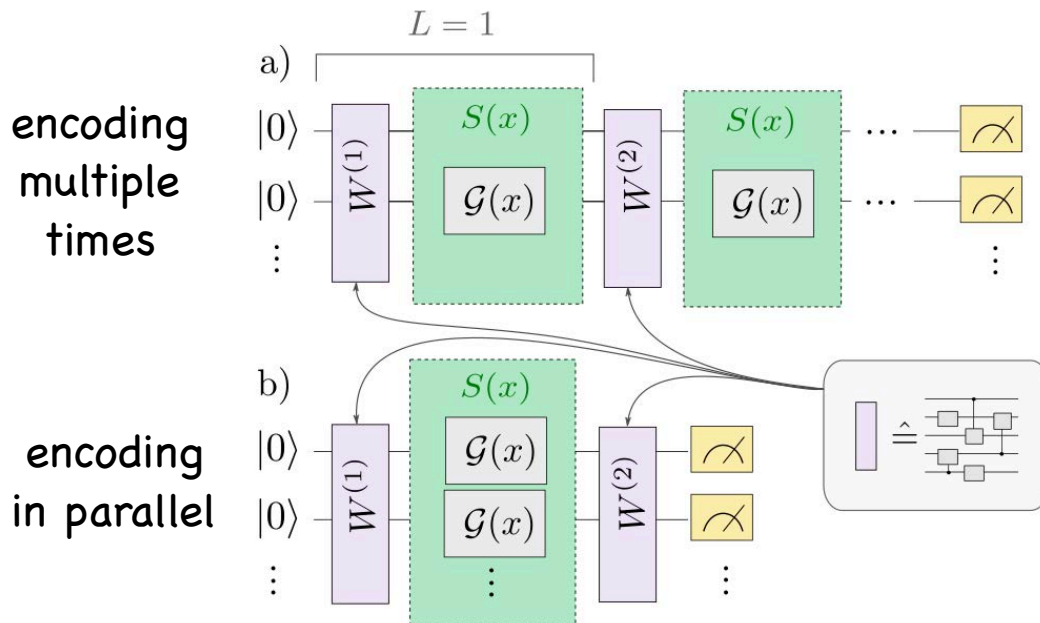
Specifically, for many (angle, time-evolution,...) encoding gates are of form

$$S(x) = e^{-ixG}$$



For Pauli operators output can be expressed as Fourier series

(universal approximator, dependent on number of frequencies)



$$f_{\theta}(x) = \langle \mathcal{M} \rangle_{x,\theta} = A + B \cos(x) - C \sin(x)$$

A, B, C coefficients from
parametrised circuit W

trigonometric structure from
data encoding

See 2008.08605
1907.02085

Simple example for 1-qubit system and 1 encoding operation:

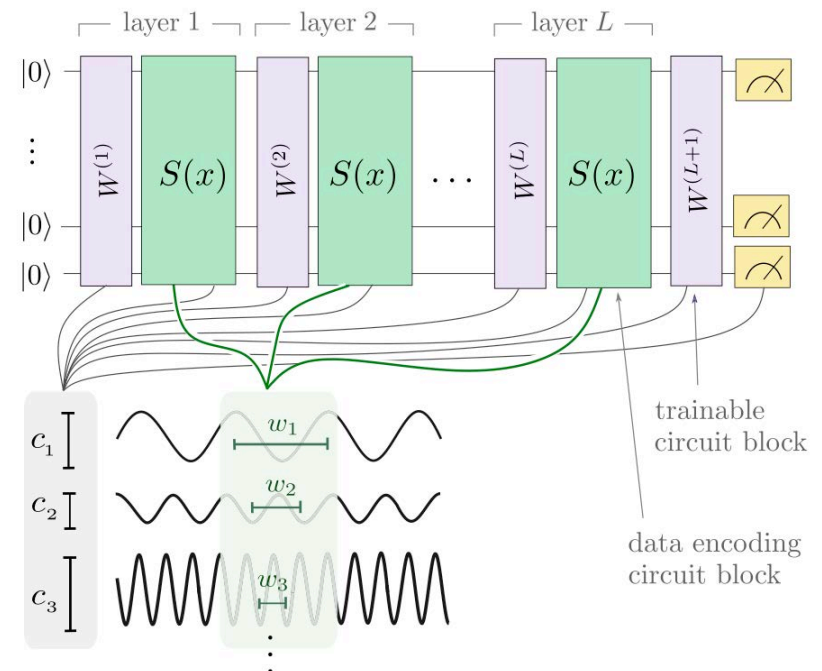
$$f_{\theta}(x) = \langle 0|U^{\dagger}(x, \theta)\sigma_z U(x, \theta)|0\rangle \quad x \in \mathcal{X} = \mathbb{R}$$

take $U(x, \theta) = W(\theta)R_x(x) \quad W(\theta) = \text{Rot}(\theta_1, \theta_2, \theta_3) \quad R_x(x) = e^{-i\frac{x}{2}\sigma_x}$

Model output

$$f_{\theta}(x) = \sum_{n=-1}^1 c_n e^{inx} = c_{-1}e^{-ix} + c_0 + c_1e^{ix} = c_0 + 2\text{Re}(c_1)\cos(x) + 2\text{Im}(c_1)\sin(x).$$

- The encoding of data, using e.g. angle or time-evolution encoding, results in a trigonometric structure of the model's output.
- Complex enough (potentially repeated) encoding results in modes with different frequencies → **universal approximator**



Simple example:

$$|0\rangle \rightarrow \boxed{R_x(x)} \rightarrow \boxed{\text{Rot}(\theta_1, \theta_2, \theta_3)} \rightarrow \boxed{\text{Measurement}} \quad \sigma_z$$

gives the model output $f_\theta(x) = \langle 0 | R_x(x)^\dagger \text{Rot}(\theta_1, \theta_2, \theta_3)^\dagger \sigma_z \text{Rot}(\theta_1, \theta_2, \theta_3) R_x(x) | 0 \rangle$

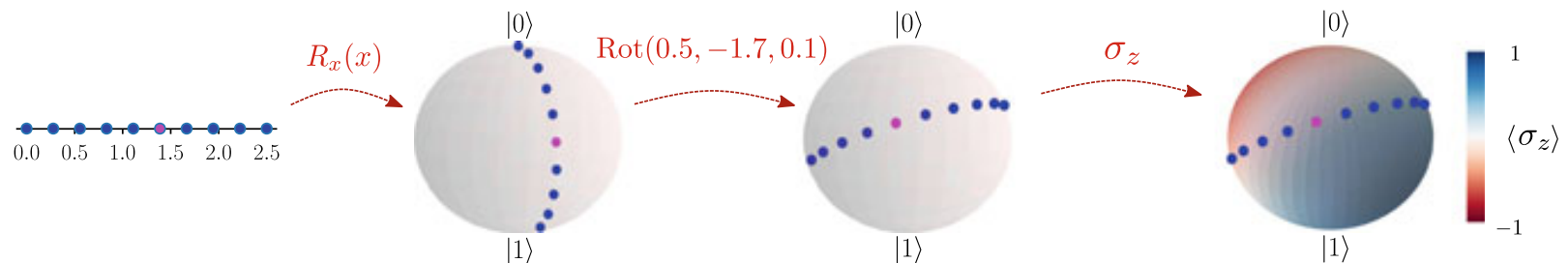
data encoding $|\phi(x)\rangle = R_x(x)|0\rangle = \begin{pmatrix} \cos(\frac{x}{2}) & -i \sin(\frac{x}{2}) \\ -i \sin(\frac{x}{2}) & \cos(\frac{x}{2}) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(\frac{x}{2}) \\ -i \sin(\frac{x}{2}) \end{pmatrix}$

parametrised rotation $|\psi(x, \theta)\rangle = \text{Rot}(\theta_1, \theta_2, \theta_3)|\phi(x)\rangle$

$$= \begin{pmatrix} e^{i(-\frac{\theta_1}{2} - \frac{\theta_3}{2})} \cos(\frac{\theta_2}{2}) \cos(\frac{x}{2}) + i e^{i(-\frac{\theta_1}{2} + \frac{\theta_3}{2})} \sin(\frac{\theta_2}{2}) \sin(\frac{x}{2}) \\ e^{i(\frac{\theta_1}{2} - \frac{\theta_3}{2})} \sin(\frac{\theta_2}{2}) \cos(\frac{x}{2}) - i e^{i(\frac{\theta_1}{2} + \frac{\theta_3}{2})} \cos(\frac{\theta_2}{2}) \sin(\frac{x}{2}) \end{pmatrix}$$

model output $f_\theta(x) = \langle \psi(x, \theta) | \sigma_z | \psi(x, \theta) \rangle = \cos(\theta_2) \cos(x) - \sin(\theta_1) \sin(\theta_2) \sin(x)$

What happens on the Bloch-Sphere



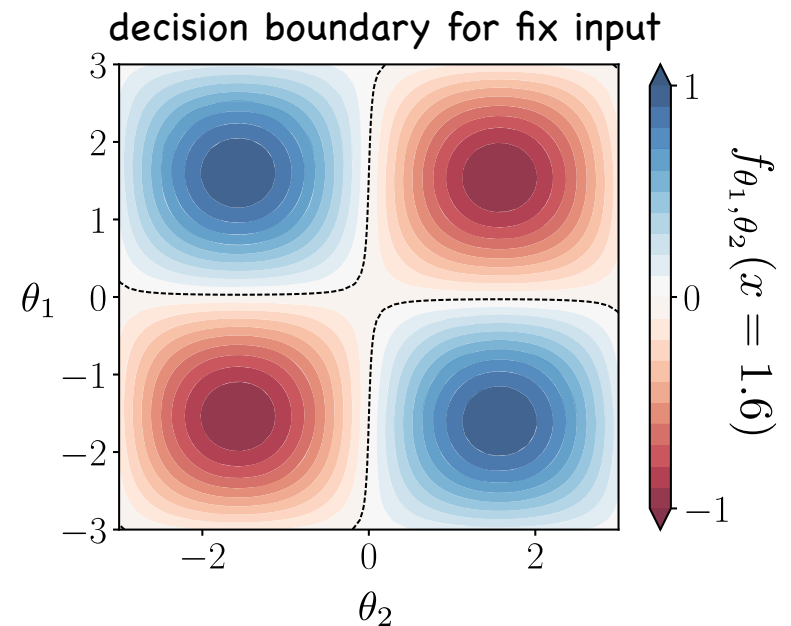
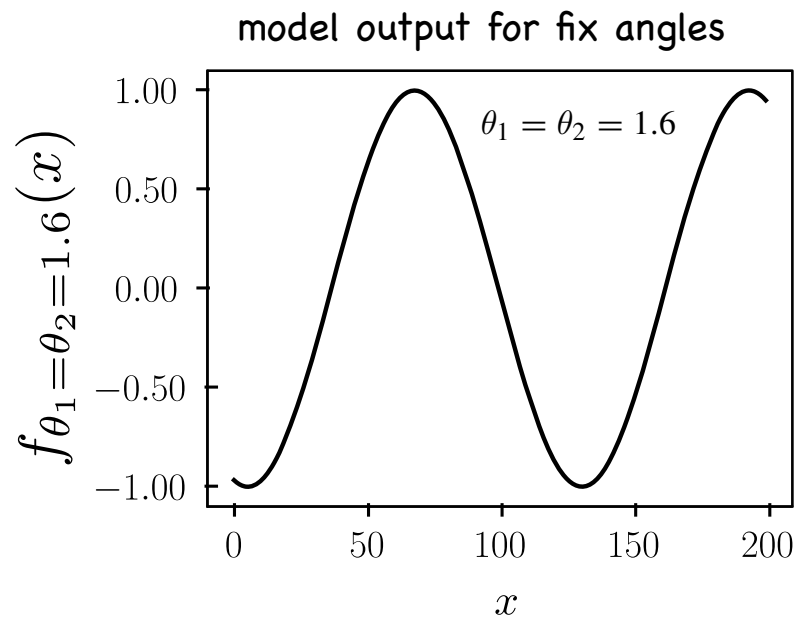
Classifier from simple example:

for binary classifier define e.g.

$$y = \begin{cases} 1 & \text{if } f_{\theta}(x) > 0 \\ -1 & \text{else} \end{cases}$$

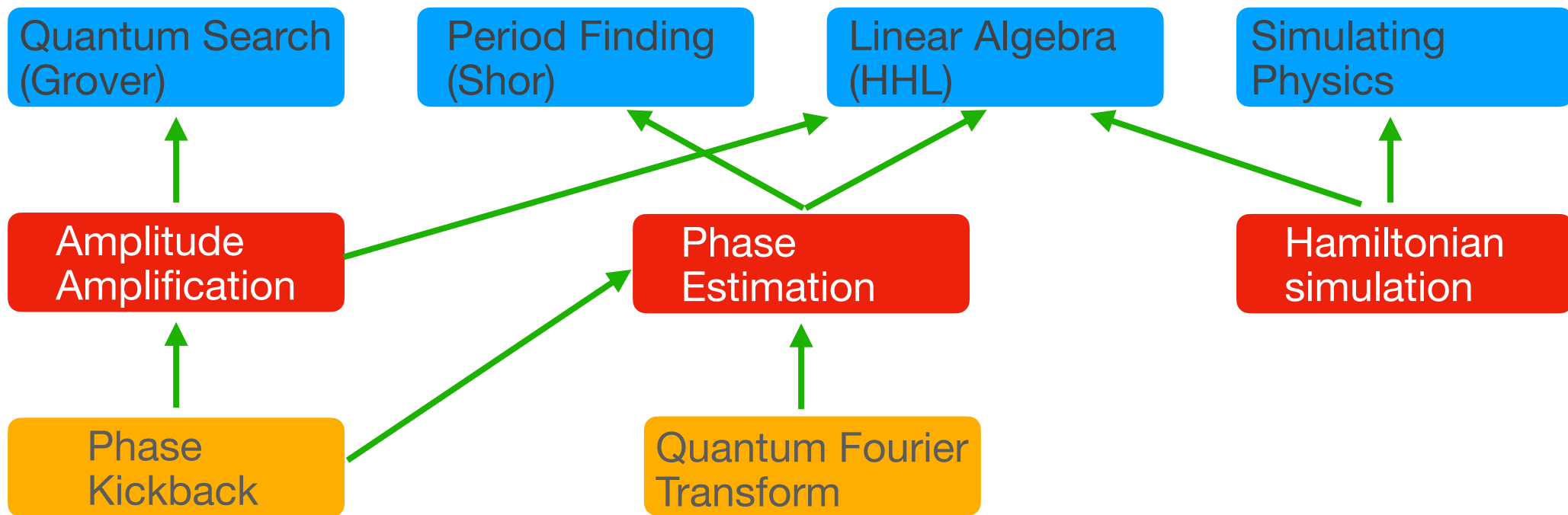
for probabilistic classifier
(density estimator)

$$p(1) = \frac{f_{\theta}(x) + 1}{2}, \quad p_0 = 1 - p_1$$



Quantum Algorithms

Algorithms, Tools, Tricks



The First Wave of Quantum Machine Learning?

PRL 103, 150502 (2009)

PHYSICAL REVIEW LETTERS

week ending
9 OCTOBER 2009



Quantum Algorithm for Linear Systems of Equations

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Solving linear systems of equations is a common problem that arises both on its own and as a subroutine in more complex problems: given a matrix A and a vector $\vec{b}$, find a vector $\vec{x}$ such that $A\vec{x} = \vec{b}$. We consider the case where one does not need to know the solution $\vec{x}$ itself, but rather an approximation of the expectation value of some operator associated with $\vec{x}$, e.g., $\vec{x}^\dagger M \vec{x}$ for some matrix M . In this case, when A is sparse, $N \times N$ and has condition number κ , the fastest known classical algorithms can find $\vec{x}$ and estimate $\vec{x}^\dagger M \vec{x}$ in time scaling roughly as $N\sqrt{\kappa}$. Here, we exhibit a quantum algorithm for estimating $\vec{x}^\dagger M \vec{x}$ whose runtime is a polynomial of $\log(N)$ and κ . Indeed, for small values of κ [i.e., $\text{poly} \log(N)$], we prove (using some common complexity-theoretic assumptions) that any classical algorithm for this problem generically requires exponentially more time than our quantum algorithm.

Complexity of inversion of a regular matrix = $O(N^3)$ by Gaussian elimination method

Complexity of inversion of a sparse matrix = $O(N)$ by conjugate gradient method

$$Ax = b$$

The First Wave of Quantum Machine Learning?

PRL **103**, 150502 (2009)

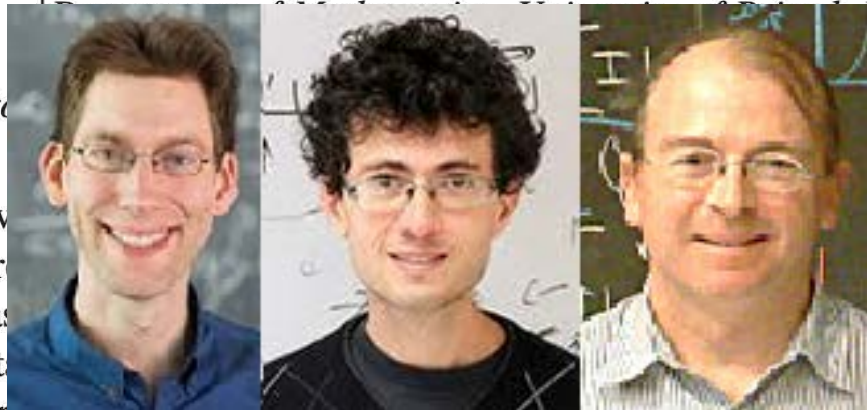
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Quantum Algorithm for Linear Systems of Equations

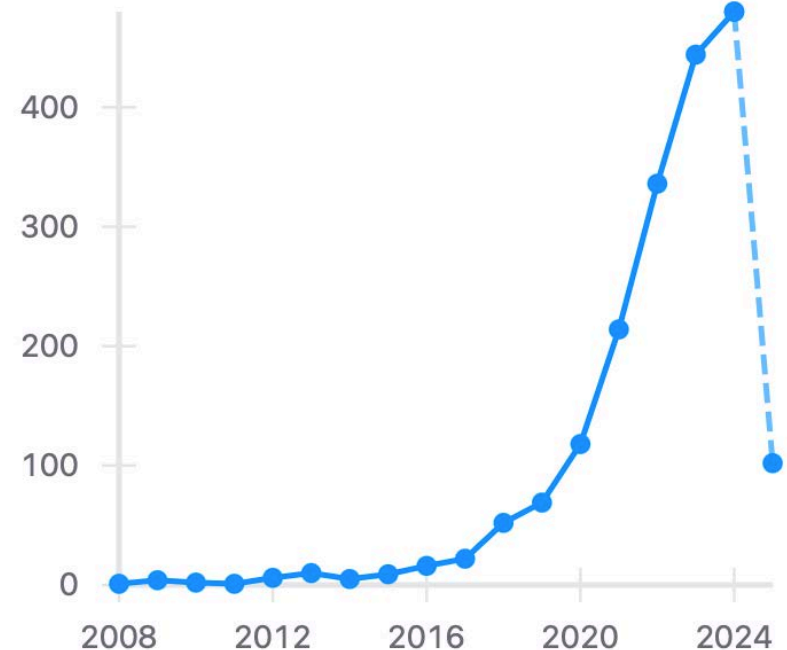
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Solving a linear system of equations $Ax = b$ in more than one dimension is the case of interest. If A is sparse, $N \times N$ and has condition number κ , the fastest known estimate $\tilde{x}^\dagger M \tilde{x}$ in time scaling roughly as $N\sqrt{\kappa}$. Here, we estimate $\tilde{x}^\dagger M \tilde{x}$ whose runtime is a polynomial of $\log(N)$ and κ . Indeed, we prove (using some common complexity-theoretic assumptions) that this problem generically requires exponentially more time than our algorithm.

Citations per year



$$Ax = b$$

Complexity of inverse
Complexity of inverse

Quantum Computing for High-Energy Physics

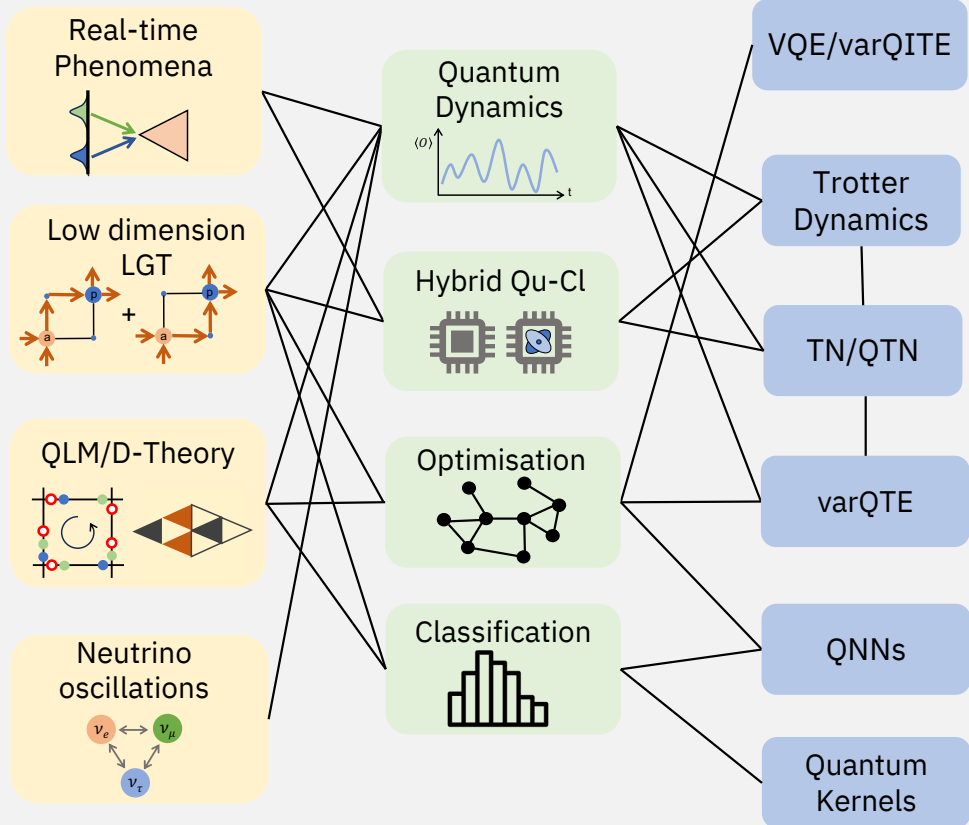
State of the Art and Challenges

Summary of the QC4HEP Working Group

2307.03236

Theoretical
physical
model
systems

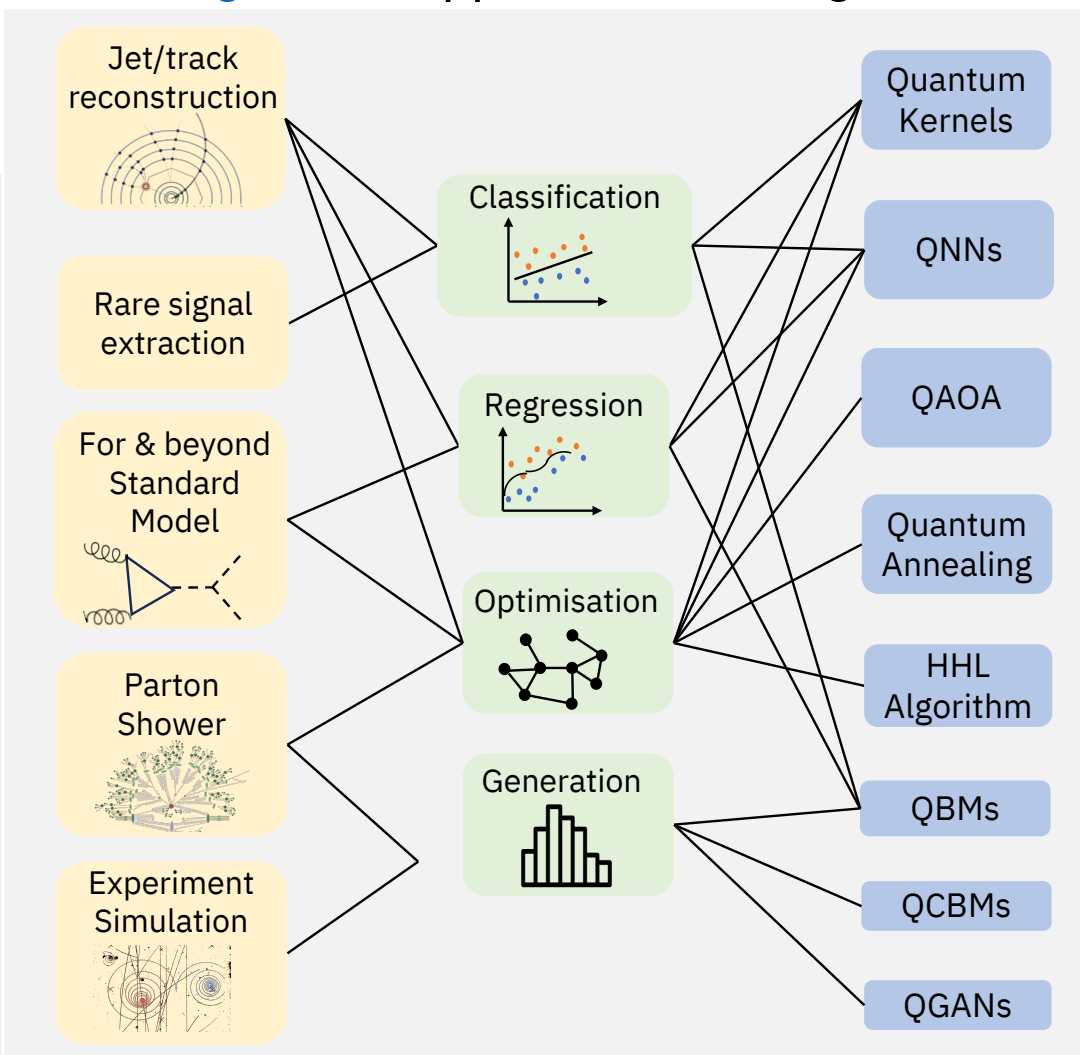
corresponding
approaches



Experimental
challenges

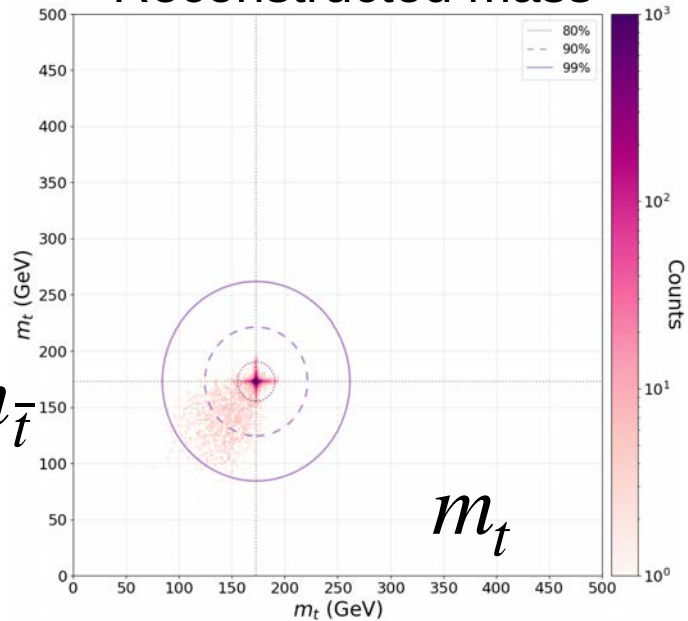
corresponding
approaches

quantum
algorithms

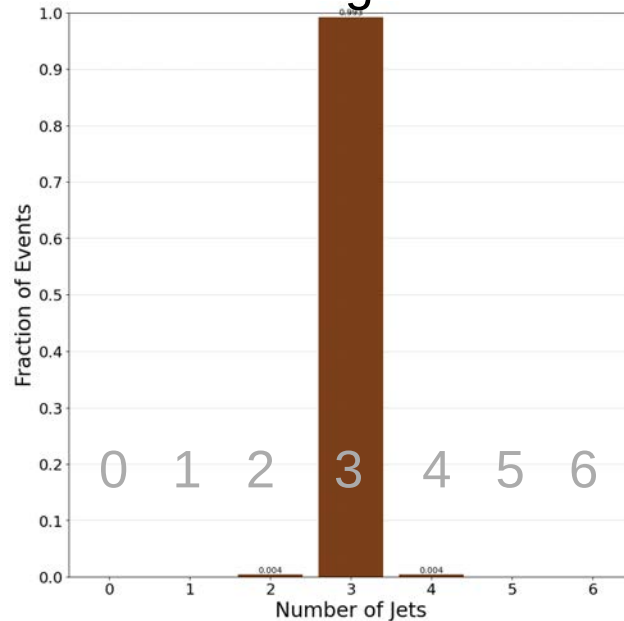


Parton-level truth and hemisphere method

Reconstructed mass



Jet assignment



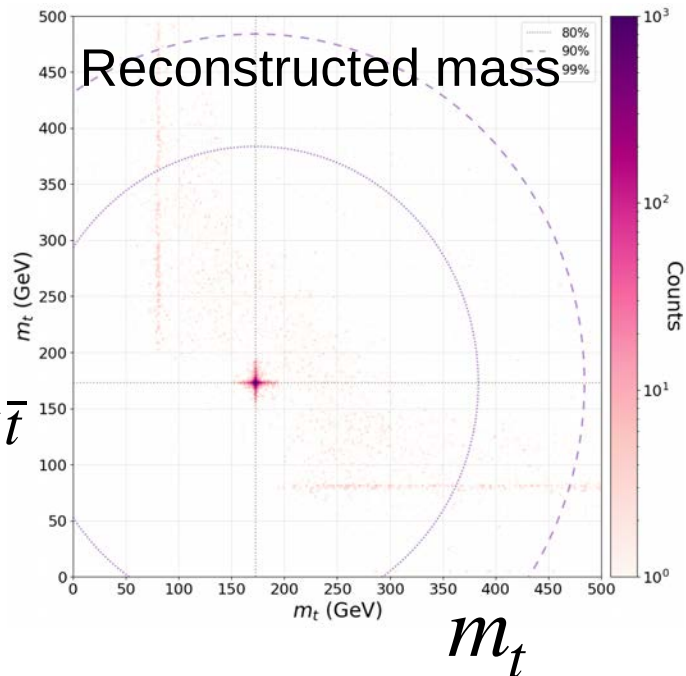
parton-level truth

79% overall efficiency

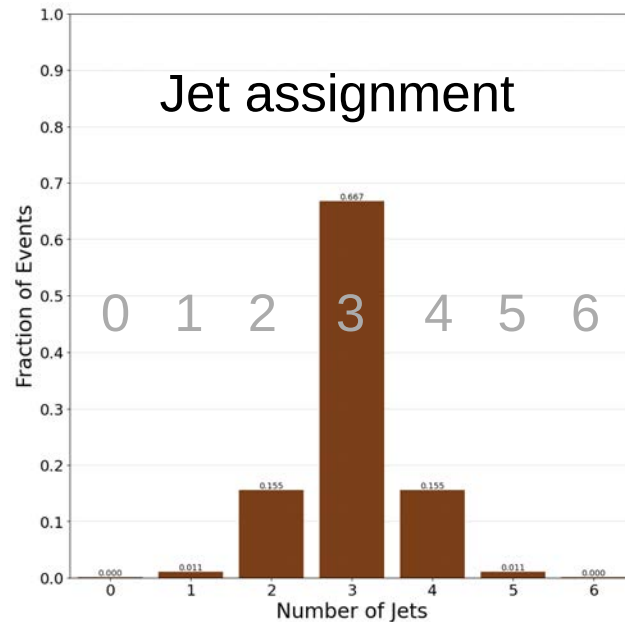
No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

Reconstructed mass



Jet assignment



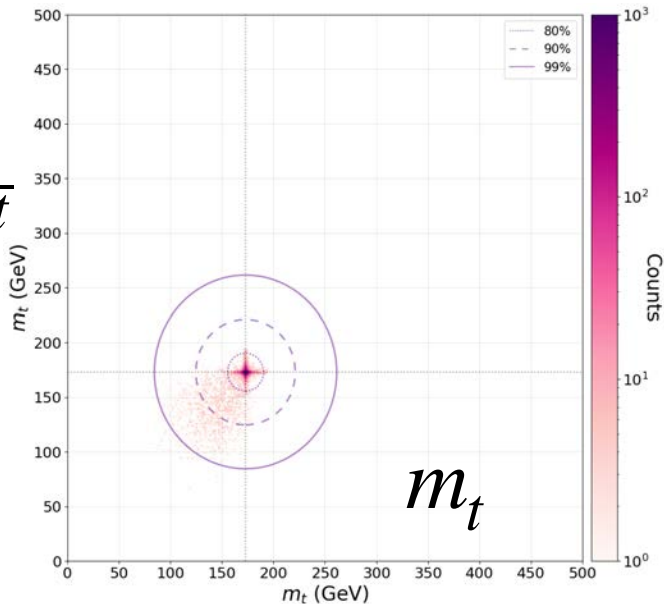
Hemisphere method

50% overall efficiency

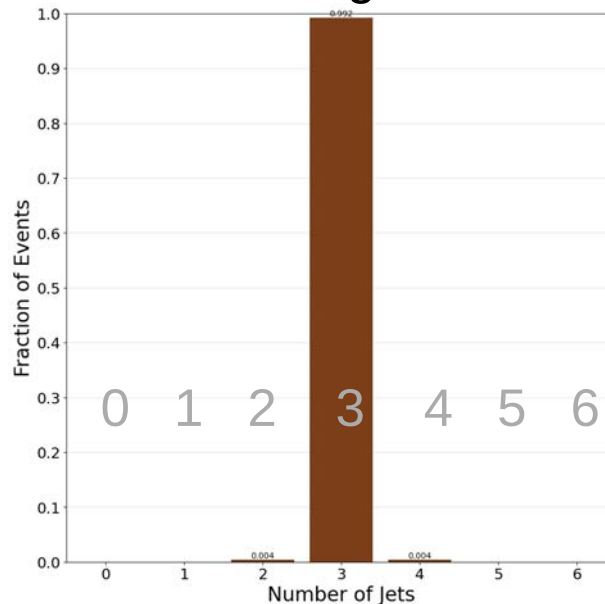
No mass information used.

ma-QAOA, VarQITE and FALQON

Reconstructed mass



Jet assignment



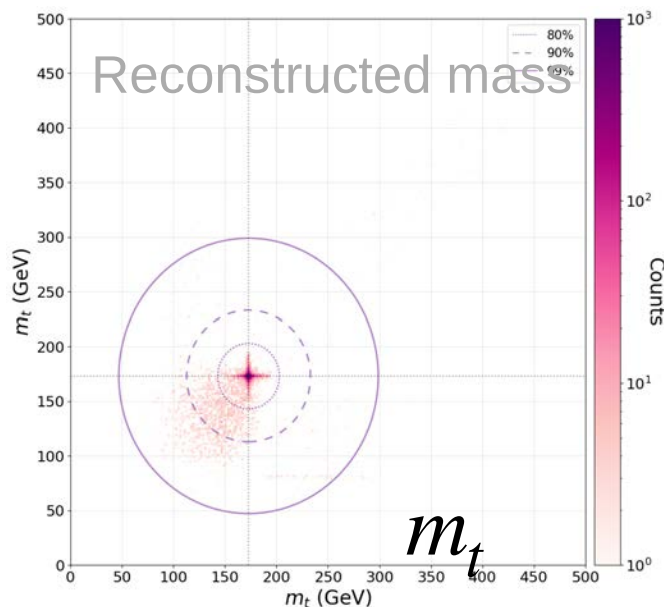
ma-QAOA / VarQITE

79% overall efficiency

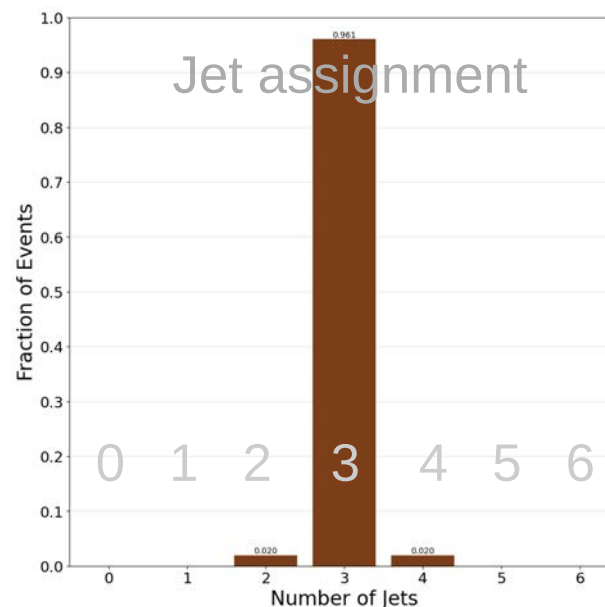
No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

Reconstructed mass



Jet assignment



FALQON

78% overall efficiency

No mass information used.

Is there a “Moore's law” for quantum computing?

- <https://arxiv.org/pdf/2303.15547>

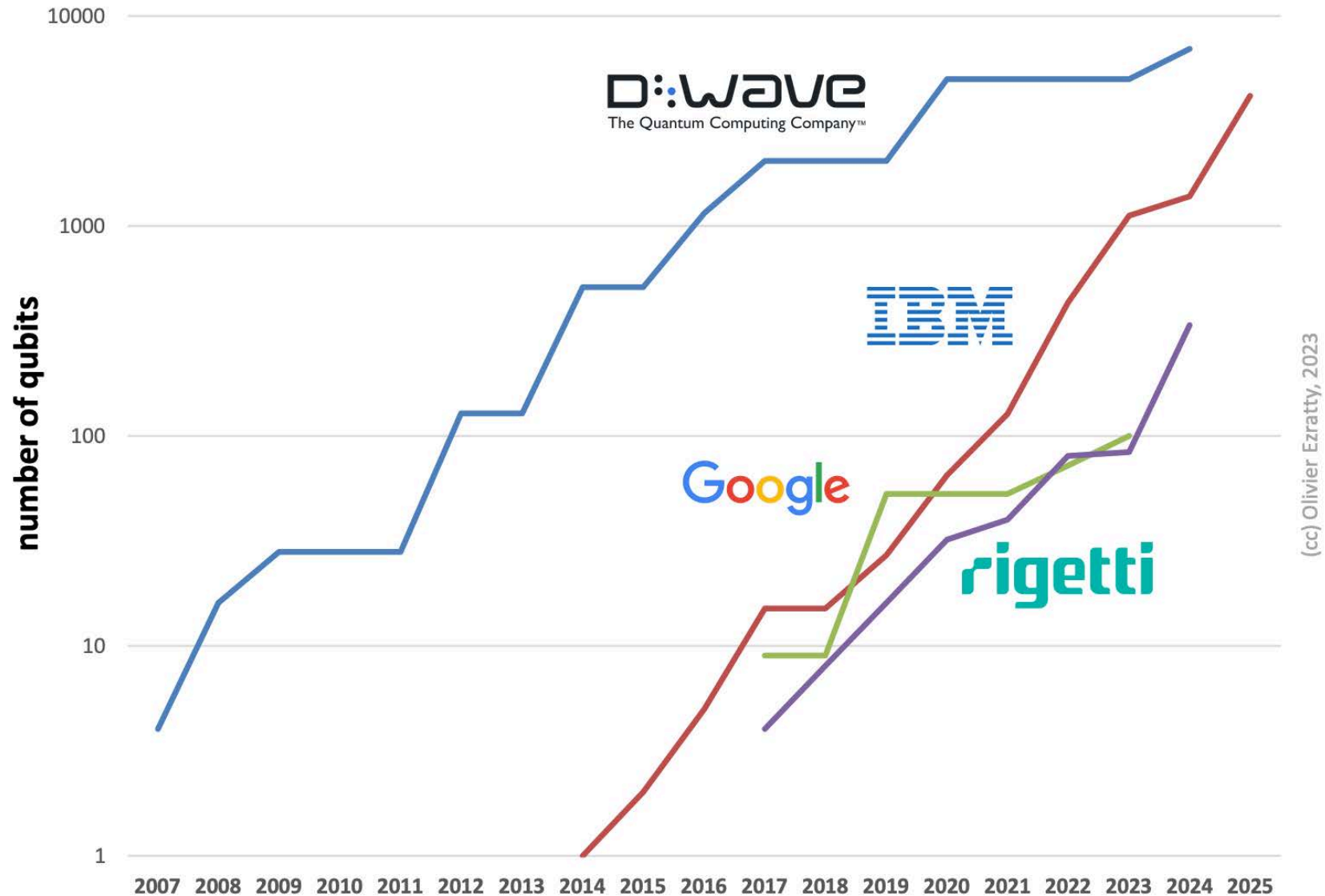


Figure 6: evolution of the number of physical qubits with D-Wave, IBM, Google and Rigetti. Compilation: Olivier Ezratty.

Is there a “Moore's law” for quantum computing?

- <https://arxiv.org/pdf/2303.15547>

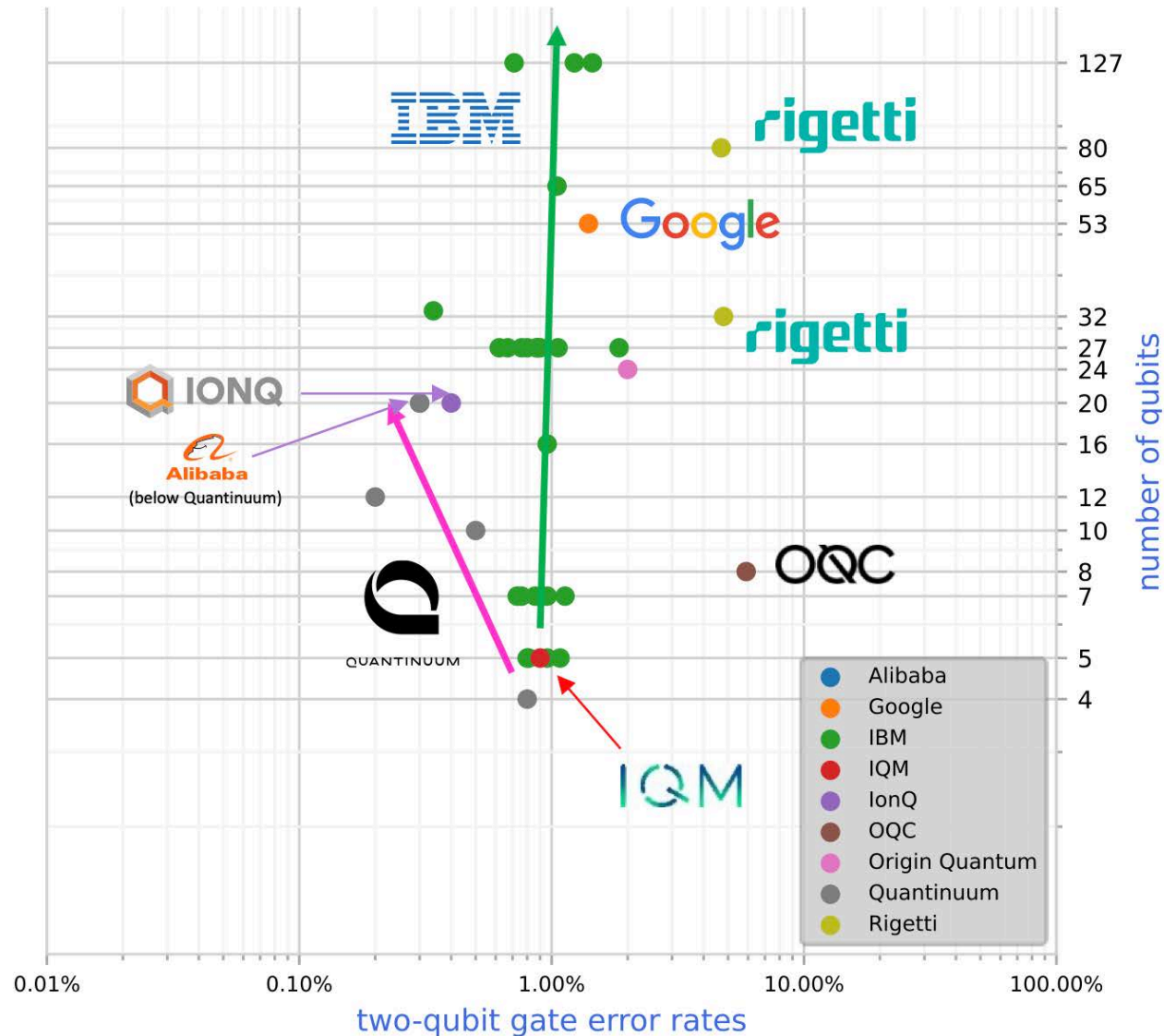


Figure 9: scatter plot of qubit numbers and two-qubit gate error rates for commercial vendors. Source: vendor fidelities numbers compiled by Olivier Ezratty as of March 2023.

Willow System Metrics	
Number of qubits	105
Average connectivity	3.47 (4-way typical)

Quantum Error Correction (Chip 1)		Random Circuit Sampling (Chip 2)	
Single-qubit gate error ¹ (mean, simultaneous)	0.035% ± 0.029%	Single-qubit gate error ¹ (mean, simultaneous)	0.036% ± 0.013%
Two-qubit gate error ¹ (mean, simultaneous)	0.33% ± 0.18% (CZ)	Two-qubit gate error ¹ (mean, simultaneous)	0.14% ± 0.052% (iswap-like)
Measurement error (mean, simultaneous)	0.77% ± 0.21% (repetitive, measure qubits)	Measurement error (mean, simultaneous)	0.67% ± 0.51% (terminal, all qubits)
Reset options	Multi-level reset (1> state and above) Leakage removal (2> state only)	Reset options	Multi-level reset (1> state and above) Leakage removal (2> state only)
T ₁ time (mean)	68 μs ± 13 μs ²	T ₁ time (mean)	98 μs ± 32 μs ²
Error correction cycles per second	909,000 (surface code cycle = 1.1 μs)	Circuit repetitions per second	63,000
Application performance	Λ _{3,5,7} = 2.14 ± 0.02	Application performance	XEB fidelity depth 40 = 0.1%
		Estimated time on Willow vs classical supercomputer	5 minutes vs. 10 ²⁵ years

Adiabatic Theorem

- Schrodinger equation: $i\hbar \frac{d\psi(t)}{dt} = H(t) \psi(t)$
- Instantaneous eigenstate: $H(t) \psi_n(t) = E_n(t) \psi_n(t)$
- Initial condition: $\psi(t = 0) = \psi_0$
- If evolution is slow enough, $\psi(t) \approx e^{i\theta(t)} \psi_0$

Born and Folk 1928

$$\psi(t) = U(t) \psi(0)$$

$$U_I(t) = \sum_{q=0}^{\infty} (-i)^q \int_0^t dt_q \cdots \int_0^{t_2} dt_1 H_I(t_q) \cdots H_I(t_1)$$

Quantum Annealing

(Gradient-Free quantum optimization)

- H_p is the problem Hamiltonian whose ground state encodes the solution to the optimization problem
- H_0 is the initial Hamiltonian whose ground state is easy to prepare.
- Prepare a quantum system to be in the ground state of H_0 and evolve the system using the following time-dependent Hamiltonian,

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- The system will remain to its ground state at all times for a large T , which means for $t=T$, the system will be in the ground state of H_p , our problem Hamiltonian.
- D-wave has built Quantum Annealing that solves optimization problem by transferring the original optimization to a hardware, that allows nearest neighbor interaction of qubits.
- Compared to the processing time of $O(2^n)$ with the simplest but a robust brute-force scanning algorithm with a classical computer, a quantum annealer can have an enormous advantage in the computation complexity as $T_{\text{QUBO}}(n) \sim \mathcal{O}(n^2) \ll \mathcal{O}(2^n)$

Apolloni, Bianchi, De Falco 1988

Efficient vs inefficient algorithms

- Efficient: computation time scales polynomially with problem's size.

– (Ex) n^4 (tensor operations, i.e., $\langle ij | k\ell \rangle$):

- $n=1000$: $t = 1$ sec

- $n=1050$: $t = 1.2$ sec

$$\left(\frac{1050}{1000}\right)^4 \approx 1.2$$

- Inefficient: computation time scales exponentially.

– (Ex) 2^n (factoring)

- $n=1000$: $t = 1$ sec

- $n=1001$: $t = 2$ sec

- $n=1006$: $t = 1$ min

- $n=1012$: $t = 1$ hour

- $n=1050$: $t \sim 3.3$ million years

$$\left(\frac{2^{1006}}{2^{1000}}\right) = 2^6 = 64$$

$$\left(\frac{2^{1050}}{2^{1000}}\right) = 2^{50}$$

Adiabatic Theorem

- **Adiabatic theorem:** A physical system remains in its instantaneous eigenstate, if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum. (Max Born and Vladimir Folk 1928)
- Under a slowly changing Hamiltonian $H(t)$ with instantaneous eigenstate $|n(t)\rangle$ and the corresponding energy $E(t)$, a quantum system evolves from initial state $|\psi(0)\rangle = \sum_n c_n(0) |n(0)\rangle$ to final state $|\psi(t)\rangle = \sum_n c_n(t) |n(t)\rangle$ where $c_n(t) = c_n(0) e^{i\theta_n(t)} e^{i\gamma_n(t)}$ with the dynamical phase $\theta_n(t) = -\frac{1}{\hbar} \int_0^t E_n(t') dt'$ and geometrical phase $\gamma(t) = i \int_0^t \langle n(t') | \dot{n}(t') \rangle dt'$
- **Adiabatic approximation:** the rate of change of Hamiltonian $\dot{H}(t)$ is small and there is finite gap $E_m(t) - E_n(t) \neq 0$ between energies for $m \neq n \rightarrow \langle n(t') | \dot{n}(t') \rangle = -\frac{\langle m(t) | \dot{H}(t) | n(t) \rangle}{E_m(t) - E_n(t)} \rightarrow 0$
- $|c_n(t)|^2 = |c_n(0)|^2$ so if the system begins in an eigenstate of $H(0)$, it remains in an eigenstate of $H(t)$ during the evolution with a change of phase only.

Adiabatic Theorem

$$H(t) |n(t)\rangle = E_n(t) |n(t)\rangle \quad |n(t)\rangle : \text{ is eigenstates of Hamiltonian, basis}$$

$$|\psi(t)\rangle = \sum_n c_n(t) |n(t)\rangle \quad \text{satisfies time-dependent Schrödinger equation} \quad i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

$$\frac{d}{dt} : \quad \dot{H}(t) |n(t)\rangle + H(t) |\dot{n}(t)\rangle = \dot{E}_n(t) |n(t)\rangle + E_n(t) |\dot{n}(t)\rangle$$

Assume $m \neq n$ and perform inner product with $|m(t)\rangle$: $H(t) |m(t)\rangle = E_m(t) |m(t)\rangle$
 $\langle m(t) | n(t) \rangle = \delta_{mn}$

$$\langle m(t) | \dot{H}(t) | n(t) \rangle + \langle m(t) | H(t) | \dot{n}(t) \rangle = \dot{E}_n(t) \langle m(t) | n(t) \rangle + E_n(t) \langle m(t) | \dot{n}(t) \rangle$$

$$\langle m(t) | \dot{H}(t) | n(t) \rangle + E_m(t) \langle m(t) | \dot{n}(t) \rangle = E_n(t) \langle m(t) | \dot{n}(t) \rangle \quad \rightarrow \quad \langle m(t) | \dot{n}(t) \rangle = - \frac{\langle m(t) | \dot{H}(t) | n(t) \rangle}{E_m(t) - E_n(t)}$$

Adiabatic approximation: the rate of change in Hamiltonian $\dot{H}(t)$ is small and there is finite gap $E_m(t) - E_n(t) \neq 0$ between energies $\rightarrow \langle m(t) | \dot{n}(t) \rangle \approx 0$.

Adiabatic Theorem

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle \quad \rightarrow \quad i\hbar \sum_n \dot{c}_n(t) |n(t)\rangle + c_n(t) |\dot{n}(t)\rangle = \sum_n E_n(t) c_n(t) |n(t)\rangle$$

$$|\psi(t)\rangle = \sum_n c_n(t) |n(t)\rangle$$

Inner product with $|m(t)\rangle$: $\langle m(t) | \left[i\hbar \sum_n \dot{c}_n(t) |n(t)\rangle + c_n(t) |\dot{n}(t)\rangle = \sum_n E_n(t) c_n(t) |n(t)\rangle \right]$

Using $\langle m(t) | n(t) \rangle = \delta_{mn}$, we obtain $i\hbar \dot{c}_m(t) + i\hbar \sum_n c_n(t) \langle m(t) | \dot{n}(t) \rangle = c_m(t) E_m(t)$

In the adiabatic limit,
 $\langle m(t) | \dot{n}(t) \rangle \approx 0$ for $m \neq n$

$$i\hbar \dot{c}_m(t) + i\hbar c_m(t) \langle m(t) | \dot{n}(t) \rangle = c_m(t) E_m(t)$$

$$i \dot{c}_m(t) = \left(\frac{E_m(t)}{\hbar} - i \langle m(t) | \dot{n}(t) \rangle \right) c_m(t) \quad \rightarrow \quad \dot{c}_m(t) = i \left(-\frac{E_m(t)}{\hbar} + i \langle m(t) | \dot{n}(t) \rangle \right) c_m(t)$$

$$\frac{d}{dt} \ln c_m(t) = \frac{1}{c_m(t)} \frac{dc_m(t)}{dt} = \frac{\dot{c}_m(t)}{c_m(t)} = -\frac{i}{\hbar} E_m(t) + i \langle m(t) | \dot{n}(t) \rangle$$

$$c_m(t) = c_m(0) e^{i\theta_m(t)} e^{i\gamma_m(t)} \quad \theta_m(t) = -\frac{1}{\hbar} \int_0^t E_m(t') dt' \quad \gamma(t) = i \int_0^t \langle m(t') | \dot{n}(t') \rangle dt'$$

dynamical phase,
real, function of E

geometrical phase,
pure real

Adiabatic Theorem

$$c_m(t) = c_m(0)e^{i\theta_m(t)}e^{i\gamma_m(t)}$$

$$\theta_m(t) = -\frac{1}{\hbar} \int_0^t E_m(t') dt'$$

$$\gamma(t) = i \int_0^t \langle m(t') | \dot{m}(t') \rangle dt'$$

dynamical phase,
real, function of E

geometrical phase,
pure real,
Has something to do with
direction in the Hilbert space

$$0 = \frac{d}{dt} \langle m(t) | m(t) \rangle = \langle \dot{m}(t) | m(t) \rangle + \langle m(t) | \dot{m}(t) \rangle$$

$$= \langle m(t) | \dot{m}(t) \rangle^* + \langle m(t) | \dot{m}(t) \rangle$$

$$= 2 \operatorname{Re} \langle m(t) | \dot{m}(t) \rangle \quad \rightarrow \quad \gamma_m(t) : \text{ pure real}$$

$$\langle \phi | \psi \rangle^* = \langle \psi | \phi \rangle$$

Adiabatic Theorem

- Schrodinger equation: $i\hbar \frac{d\psi(t)}{dt} = H(t) \psi(t)$
- Instantaneous eigenstate: $H(t) \psi_n(t) = E_n(t) \psi_n(t)$
- Initial condition: $\psi(t = 0) = \psi_0$
- If evolution is slow enough, $\psi(t) \approx e^{i\theta(t)} \psi_0$

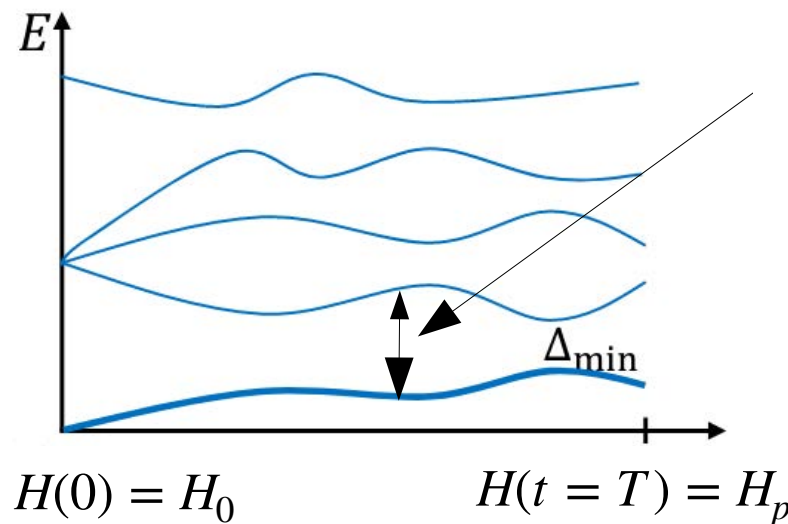
Born and Folk 1928

$$\psi(t) = U(t) \psi(0)$$

$$U_I(t) = \sum_{q=0}^{\infty} (-i)^q \int_0^t dt_q \cdots \int_0^{t_2} dt_1 H_I(t_q) \cdots H_I(t_1)$$

Limitation of Quantum Annealing

- Performance of quantum annealing are governed by the size of the gap.



$$\Delta t \gg \max_{0 \leq s \leq 1} \left| \frac{\langle 1(s) | \frac{dH(s)}{ds} | 0(s) \rangle}{\Delta(s)^2} \right|$$

$$T = \Delta s, \quad 0 \leq s \leq 1$$

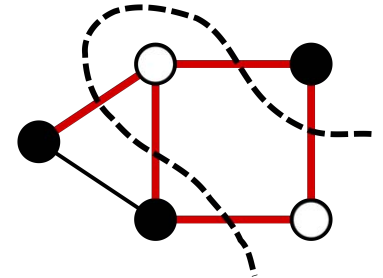
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- If the energy gap b/w the ground state and 1st excited state is small, T must be very large $\rightarrow$ computationally difficult.
- Performance is poor, when eigenvalues are degenerate.

Why should we care about QAOA?

- Solve optimization problems
 - Solves quadratic unconstrained problems with binary variables
- Near-term algorithm
 - Algorithm runs on small quantum computers
 - Low depth, robust to errors
 - Requires relatively few physical qubits to get to interesting practical problem sizes
- Adaptable algorithm
 - In principle, we can easily model the objective function that we are trying to solve
- **Expected** to be faster than classical
 - Classical approaches move through the search space one solution at a time
 - In quantum, we can create a superposition of states and operate in all states in parallel.

QUBO example: Max-cut Problem



- The cost function to be maximized:

$$C(x) = \sum_{(i,j) \in E} (x_i + x_j - 2x_i x_j) \quad \text{where } x_i \in \{0,1\}$$

$x_i + x_j - 2x_i x_j = 1$, if x_i and x_j belong in different sets .

$x_i + x_j - 2x_i x_j = 0$, if x_i and x_j belong in the same set .

$$s_i \in Z_2 = \{-1,1\}$$

- Introducing $x_i = \frac{s_i + 1}{2}$, the cost function can be rewritten

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} (1 - s_i s_j) \longrightarrow C(s) = \frac{1}{2} \sum_{(i,j) \in E} (1 - \sigma_i^z \sigma_j^z) \quad \begin{array}{l} (i,j) : \text{ the edge index} \\ i : \text{ vertex index} \end{array}$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{array}{l} \sigma^z |0\rangle = +1 |0\rangle \\ \sigma^z |1\rangle = -1 |1\rangle \end{array}$$

σ_i^z : Pauli's Z matrix acting on the i^{th} vertex

σ_j^z : Pauli's Z matrix acting on the j^{th} vertex

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrices = linear operators = observables

Eigenvalues = what are actually measured in experiments

Quantum Approximate Optimization Algorithm (QAOA)

Farhi et al 2014

$$H_P = C(s) = H_C = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right) : \text{Problem Hamiltonian}$$

(i, j) : the edge index

i : vertex index

$$H_M = B = H_B = \sum_j \sigma_j^X : \text{Mixer Hamiltonian (Initial Hamiltonian)}$$

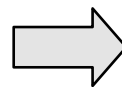
Full Hamiltonian: $H(t) = \left(1 - \frac{t}{T} \right) H_M + \frac{t}{T} H_P$

$$|\psi\rangle = \exp \left[-i \int_0^t H(t') dt' \right] |\psi_0\rangle = \exp \left[-i \sum_{j=1}^p H(j\Delta t) \Delta t \right] |\psi_0\rangle$$

$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left[\left(1 - \frac{j\Delta t}{T} \right) H_M + \frac{j\Delta t}{T} H_P \right] \right] |\psi_0\rangle$$

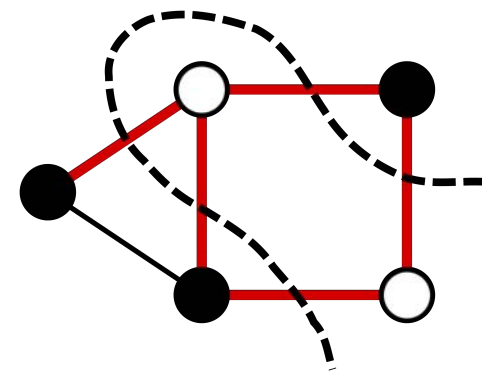
$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left(1 - \frac{j\Delta t}{T} \right) H_M \right] \exp \left[-i\Delta t \frac{j\Delta t}{T} H_P \right] |\psi_0\rangle$$

$$= \prod_{j=1}^p \underbrace{\exp \left[-i\beta_j H_M \right]}_{U(H_M, \beta_j)} \underbrace{\exp \left[-i\gamma_j H_P \right]}_{U(H_P, \gamma_j)} |\psi_0\rangle$$



$$|\gamma, \beta\rangle = \prod_{j=1}^p U(H_M, \beta_j) U(H_P, \gamma_j) |+\rangle^{\otimes n}$$

Works in the adiabatic limit or $p \rightarrow \infty$



Undirected Graph: $G = (V, E)$
 V : set of nodes
 E : set of edges

Quantum Approximate Optimization Algorithm (QAOA)

Farhi et al 2014

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right), \quad B = \sum_j \sigma_j^X$$

(i, j) : the edge index

i : vertex index

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

σ_i^z : Pauli's Z matrix acting on the i^{th} vertex

σ_j^z : Pauli's Z matrix acting on the j^{th} vertex

$$\sigma^z |0\rangle = +1 |0\rangle \quad \sigma^z |1\rangle = -1 |1\rangle \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U(C, \gamma) = e^{-i\gamma C} = \prod_{(i,j) \in E} e^{-i\gamma C_{ij}}, \quad U(B, \beta) = e^{-i\beta B} = \prod_{j=1}^n e^{-i\beta B_j}$$

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = \left[\prod_{i=1}^p U(B, \beta_i) U(C, \gamma_i) \right] H^{\otimes n} |0\rangle$$

$$e^{-i\beta \sigma_j^X} = \cos \beta - i \sigma_j^X \sin \beta$$

$$= U(B, \beta_p) U(C, \gamma_p) \cdots U(B, \beta_1) U(C, \gamma_1) \frac{1}{\sqrt{2}^n} \sum_{i=1}^{2^n-1} |i\rangle$$

2p angles (parameters): $\vec{\gamma} = (\gamma_1, \gamma_2, \dots, \gamma_p)$, and $\vec{\beta} = (\beta_1, \beta_2, \dots, \beta_p)$

Goal is to find minimum/maximum over angles:

$$M_p = \max_{\vec{\gamma}, \vec{\beta}} \langle \psi(\vec{\gamma}, \vec{\beta}) | C | \psi(\vec{\gamma}, \vec{\beta}) \rangle$$

Quantum Approximate Optimization Algorithm (QAOA)

Farhi et al 2014

$$H|\psi\rangle = i\frac{\partial}{\partial t}|\psi\rangle \quad \longrightarrow \quad |\psi\rangle = e^{-iHt}|\psi_0\rangle$$

$$H = C(s) + B$$

Problem Hamiltonian

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right),$$

Initial Hamiltonian

$$B = \sum_j \sigma_j^x$$

$$U(C, \gamma) = e^{-i\gamma C} = \prod_{(i,j) \in E} e^{-i\gamma C_{ij}}, \quad U(B, \beta) = e^{-i\beta B} = \prod_{j=1}^n e^{-i\beta B_j}$$

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = \left[\prod_{i=1}^p U(B, \beta_i) U(C, \gamma_i) \right] H^{\otimes n} |0\rangle \quad \text{For a large } p$$

(i, j) : the edge index
 i : vertex index

minimum/maximum over angles:

$$M_p = \max_{\vec{\gamma}, \vec{\beta}} \langle \psi(\vec{\gamma}, \vec{\beta}) | C | \psi(\vec{\gamma}, \vec{\beta}) \rangle$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

σ_i^z : Pauli's Z matrix acting on the i^{th} vertex

σ_j^z : Pauli's Z matrix acting on the j^{th} vertex

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

$$\sigma^z |0\rangle = +1 |0\rangle$$

$$\sigma^z |1\rangle = -1 |1\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Quantum Approximate Optimization Algorithm (QAOA)

- How do $U(C, \gamma)$ and $U(B, \beta)$ operate on $|\psi\rangle$?

$$\begin{aligned}
 U(C, \gamma) H^{\otimes n} |0 \dots 0\rangle &= e^{-i\gamma C} H^{\otimes n} |0 \dots 0\rangle = \exp\left[-i\gamma \frac{1}{2} \sum_{(i,j) \in E} (1 - \sigma_i^Z \sigma_j^Z)\right] H^{\otimes n} |0 \dots 0\rangle \\
 &= \prod_{(i,j) \in E} \exp\left[-i\gamma \frac{1}{2} (1 - \sigma_i^Z \sigma_j^Z)\right] H^{\otimes n} |0 \dots 0\rangle
 \end{aligned}$$

$$\exp\left[-i\gamma \frac{1}{2} (1 - \sigma_i^Z \sigma_j^Z)\right] H^{\otimes n} |0 \dots 0\rangle = \exp\left(-i\frac{\gamma}{2}\right) \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) H^{\otimes n} |0 \dots 0\rangle$$

Four different possibilities:

$$\begin{aligned}
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots \underset{i}{0} \dots \underset{j}{0} \dots\rangle &= \exp\left(+i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 0 \dots 0 \dots\rangle \\
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots \underset{i}{0} \dots \underset{j}{1} \dots\rangle &= \exp\left(-i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 0 \dots 1 \dots\rangle \\
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots \underset{i}{1} \dots \underset{j}{0} \dots\rangle &= \exp\left(-i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 1 \dots 0 \dots\rangle \\
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots \underset{i}{1} \dots \underset{j}{1} \dots\rangle &= \exp\left(+i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 1 \dots 1 \dots\rangle
 \end{aligned}$$

Quantum Approximate Optimization Algorithm (QAOA)

- If bits i and j are the same, $\exp\left[-i\gamma\frac{1}{2}\left(1 - \sigma_i^Z\sigma_j^Z\right)\right]|\dots\rangle = +1|\dots\rangle$
- If bits i and j are different, $\exp\left[-i\gamma\frac{1}{2}\left(1 - \sigma_i^Z\sigma_j^Z\right)\right]|\dots\rangle = e^{-i\gamma}|\dots\rangle$

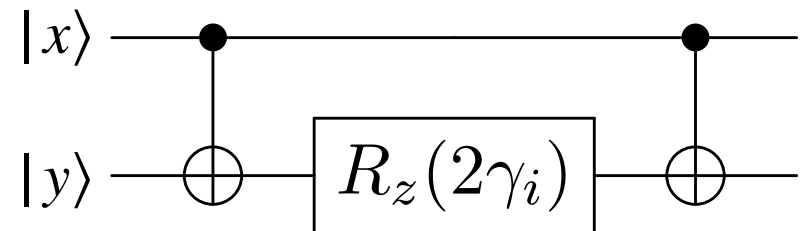
➡ • If bits i and j are different, rotate the output state around z axis by an angle γ .

$$\sigma_Z|a\rangle = (-1)^a|a\rangle \quad R_Z(\theta) = \exp\left(-i\frac{\theta}{2}\sigma_Z\right) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{+i\theta/2} \end{pmatrix}$$

In circuit: CNOT $|x\ y\rangle = |x\ x \oplus y\rangle$

$$I \otimes R_z(2\gamma) |x\ x \oplus y\rangle = \exp\left(-i\gamma(-1)^{x \oplus y}\right) |x\ x \oplus y\rangle$$

$$\text{For } U(B, \beta) = e^{-i\beta B} = \prod_{j=1}^n e^{-i\beta\sigma_j^X} = \prod_{j=1}^n R_x^j(2\beta)$$



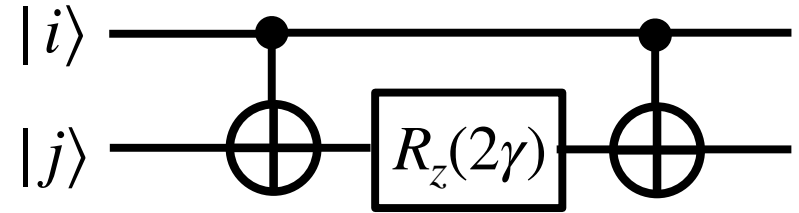
- Rotation of all n -qubits about x -axis with angle 2β

Quantum Approximate Optimization Algorithm (QAOA)

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = U(B, \beta_p) U(C, \gamma_p) \cdots U(B, \beta_1) U(C, \gamma_1) \frac{1}{\sqrt{2}^n} \sum_{i=1}^{2^n-1} |i\rangle$$

Farhi et al 2014

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = \left[\prod_{i=1}^p U(B, \beta_i) U(C, \gamma_i) \right] H^{\otimes n} |0\rangle$$



$$e^{-i\beta\sigma_j^x} = \cos \beta - i \sin \beta \sigma_j^x$$

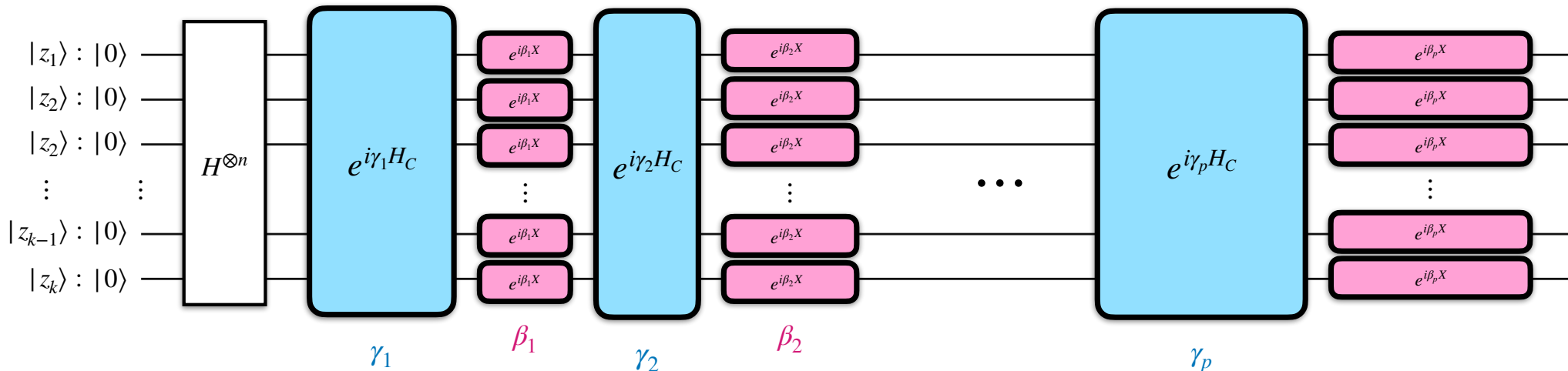
Rotate qubit j around x-axis by β

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right), \quad B = \sum_j \sigma_j^x$$

C=problem Hamiltonian

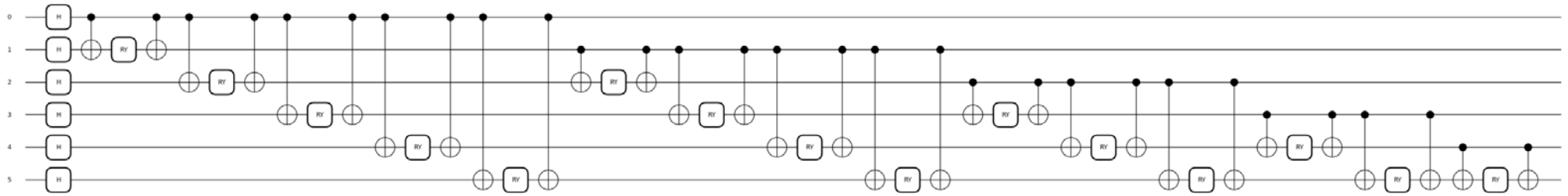
B = Initial Hamiltonian

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

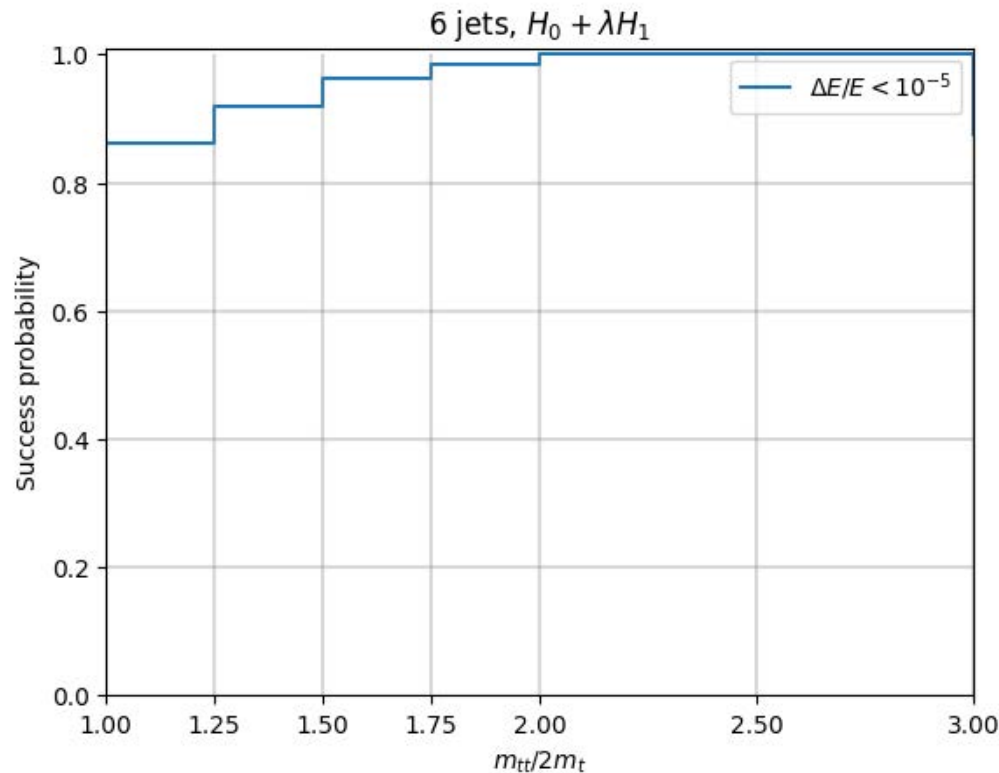


VarQITE Ansatz (with IonQ)

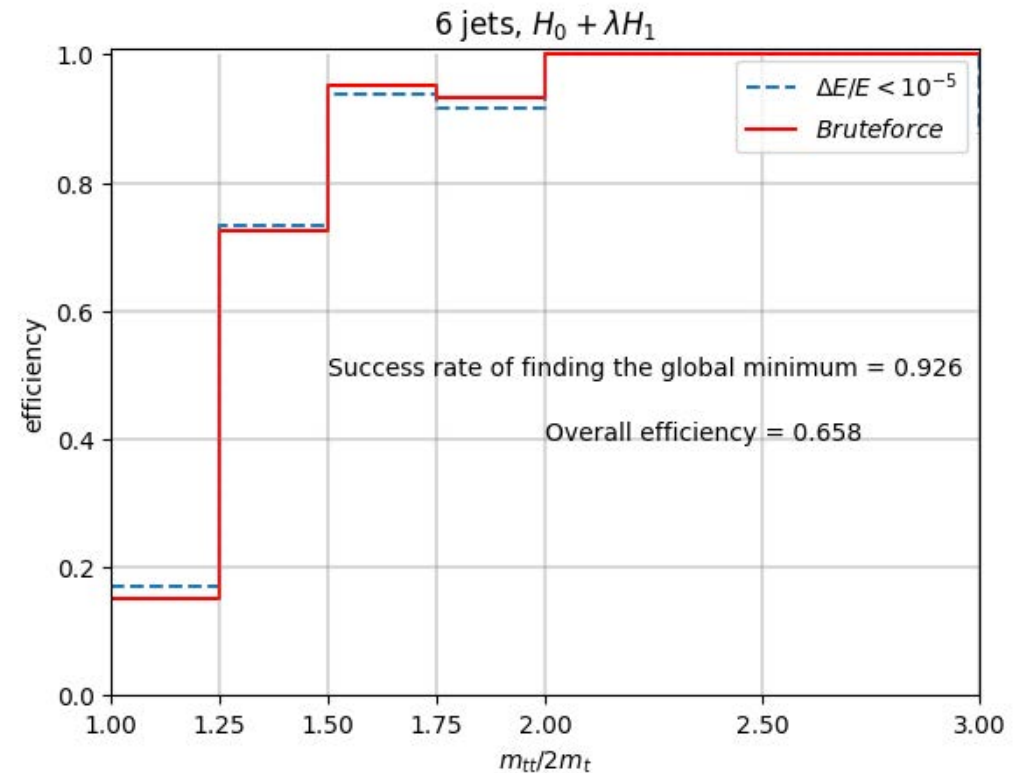
15 parameters
no classical optimizers needed



Performant near-term quantum combinatorial optimization : 2404.16135



Success probability of
finding the global minimum



Efficiency of resolving the
combinatorial problem

Feedback-based ALgorithm for Quantum Optimization (FALQON)

Feedback-based ALgorithm Quantum Optimization (FALQON)

- 2103.08619, https://pennylane.ai/qml/demos/tutorial_falqon.html
- Consider a quantum system whose dynamics is governed by

$$i \frac{d}{dt} |\psi(t)\rangle = (H_p + H_d \beta(t)) |\psi(t)\rangle$$

- Goal is to minimize: $\langle H_p \rangle = \langle \psi(t) | H_p | \psi(t) \rangle$

H_p : drift Hamiltonian (Problem H) $\beta(t)$: time – dependent control function

H_d : control Hamiltonian

- One can minimize $\langle H_p \rangle$ by designing $\beta(t)$ such that

$$\frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle(t) \leq 0, \quad \forall t \geq 0$$

$$\begin{aligned} \frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle &= i \langle \psi(t) | (H_p + H_d \beta(t)) H_p | \psi(t) \rangle - i \langle \psi(t) | H_p (H_p + H_d \beta(t)) | \psi(t) \rangle \\ &= \langle \psi(t) | i [H_d, H_p] | \psi(t) \rangle \beta(t) \equiv A(t) \beta(t) \end{aligned}$$

Feedback-based ALgorithm Quantum Optimization (FALQON)

$$\frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle = \langle \psi(t) | i [H_d, H_p] | \psi(t) \rangle \beta(t) \equiv A(t) \beta(t)$$

- We can choose any $\beta(t)$.
- Consider $\beta(t) = -w f(t, A(t))$ for $w > 0$, where $f(t, A(t))$ is any continuous function with $f(t, 0) = 0$ and $A(t)f(t, A(t)) > 0$ for all $A(t) \neq 0$.
- Take $w = 1$ and $f(t, A(t)) = A(t)$ such that $\beta(t) = -A(t)$ for simplicity.
- Consider alternating (rather than concurrent) applications of H_p and H_d , leading to a time evolution:

$$U = U_d(\beta_\ell) U_p \cdots U_d(\beta_1) U_p$$

$$U_p = e^{-iH_p\Delta t} \quad k = 1, 2, \dots, \ell \quad \beta_k = \beta(k\tau - \Delta t)$$

$$U_d(\beta_k) = e^{-i\beta_k H_d \Delta t} \quad \tau = 2\Delta t \quad = \beta((k-1)\Delta t)$$

- For small Δt , this unitary evolution yields Trotterized approximation to the continuous time evolution of the system.

Feedback-based ALgorithm Quantum Optimization (FALQON)

- During the time evolution when H_p is applied, $\frac{d}{dt}\langle H_p \rangle = 0$, but eigenstate of H_p accumulates phase changes. (H_p is time-independent.)
- For the time evolution when H_d is applied, we recover $\frac{d}{dt}\langle H_p \rangle = A(t)\beta(t)$
- Set $\beta_{k+1} = -A_k$, where $A_k = \langle \psi_k | i[H_d, H_p] | \psi_k \rangle$
- In this setting, it is always possible to choose Δt small enough such that $\frac{d}{dt}\langle \psi(t) | H_p | \psi(t) \rangle \leq 0$. If Δt is chosen to be too large, the inequality will be violated.
- FALQON is a constructive, optimization free procedure for assigning values to each β_k according to a feedback law.
- By design, the quality of the solution to the combinatorial optimization problem improves monotonically with respect to depth of the circuit, k .

FALQON vs QAOA

- Circuits used in QAOA has the same alternative structure as those in FALQON with additional parameters $\vec{\gamma} = (\gamma_1, \dots, \gamma_\ell)$ that enter into U_p such that $U_{QAOA} = U_d(\beta_\ell)U_p(\gamma_\ell)\dots U_d(\beta_1)U_p(\gamma_1)$.
- Solution to the original combinatorial optimization is found by minimizing $\langle \psi(\vec{\gamma}, \vec{\beta}) | H_p | \psi(\vec{\gamma}, \vec{\beta}) \rangle$ over 2ℓ parameters, using classical optimization.
($|\psi(\vec{\gamma}, \vec{\beta})\rangle = U_{QAOA} |\psi_0\rangle$)
- FALQON minimizes $\langle H_p \rangle$ over a sequence of quantum circuit layers, guided by qubit measurement-based feed back without classical optimization.

FALQON for MaxCut problem

- MaxCut: $H_p = - \sum_{(i,j) \in E} \frac{1}{2} (1 - Z_i Z_j)$ and $H_d = \sum_{j=1}^n X_j$
- $i [H_d, H_p] = \sum_{(i,j) \in E} Y_i Z_j + Z_i Y_j$ where X_j , Y_j and Z_j are Pauli's matrices.

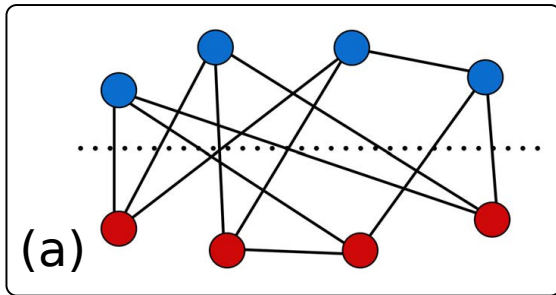
FALQON for MaxCut problem

Approximation ration:

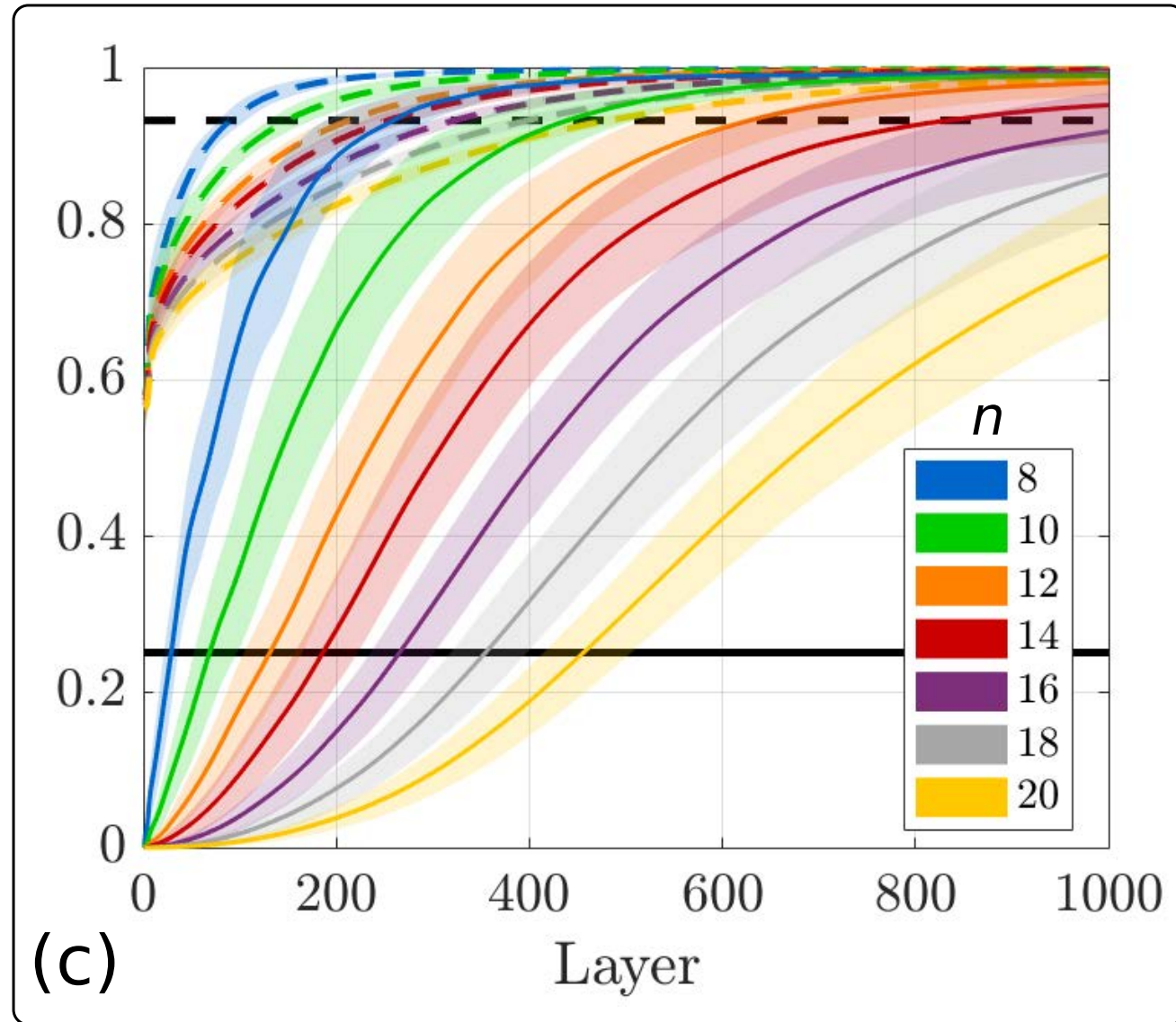
$$r_A = \langle H_p \rangle / \langle H_p \rangle_{\min}$$

The largest known approximation ratio $r_A = 0.932$ by algorithm of Goemans and Williamson.

approximation ratio (dashed curves)
and the success probability of
measuring the degenerate ground
state (solid curves)

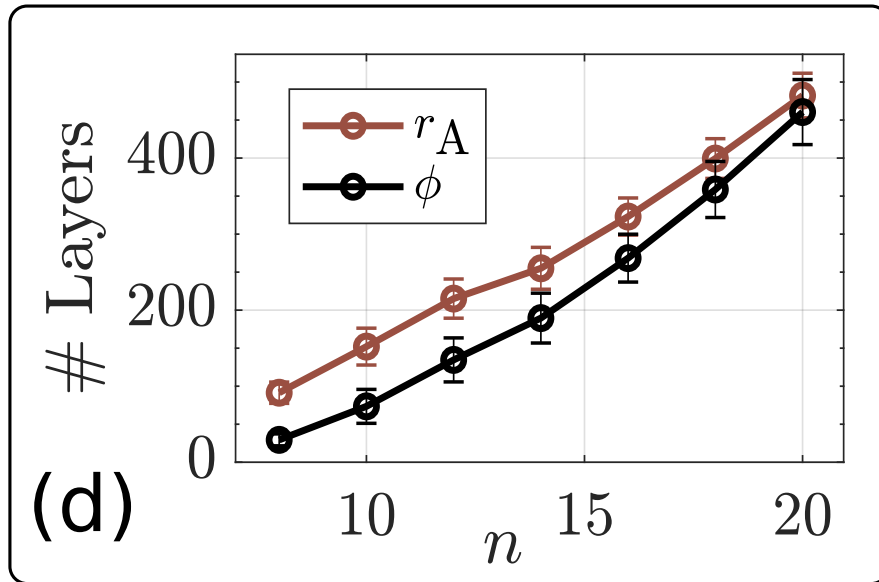


(a) Pictorial representation of MaxCut on a 3-regular graph with 8 vertices.

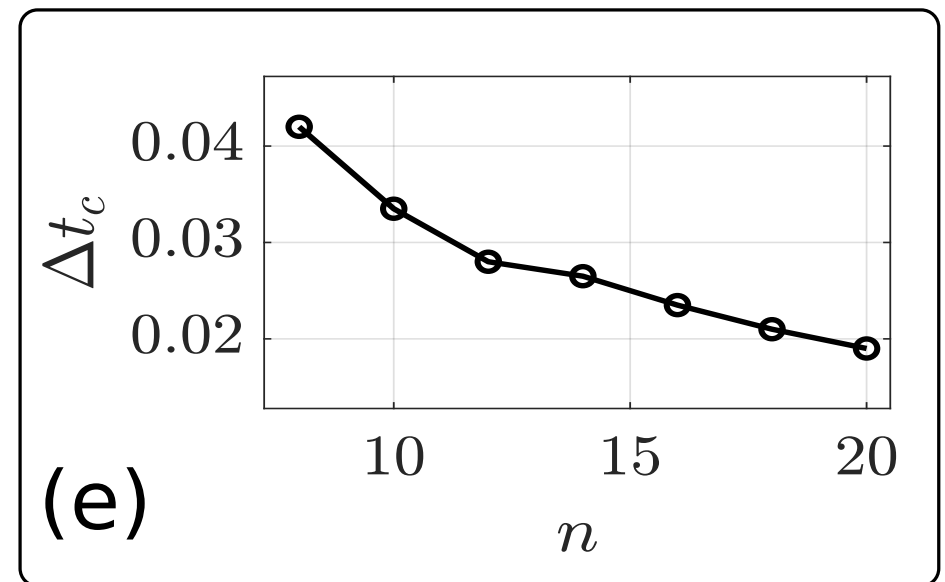


n = the number of vertices

FALQON for MaxCut problem



The mean number of layers needed to achieve the reference values of $r_A = 0.932$ (dashed curve) and $\phi = 0.25$ (solid curve) is shown; error bars report the associated standard deviation.

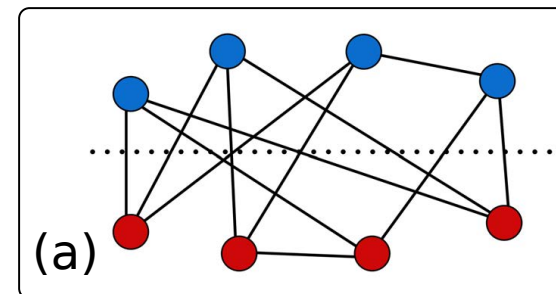


The critical Δt values for different problem sizes are plotted.

The only free parameter is time step Δt , which is tuned to be as large as possible.

$$r_A = \langle H_p \rangle / \langle H_p \rangle_{\min} = \text{approximation ratio}$$

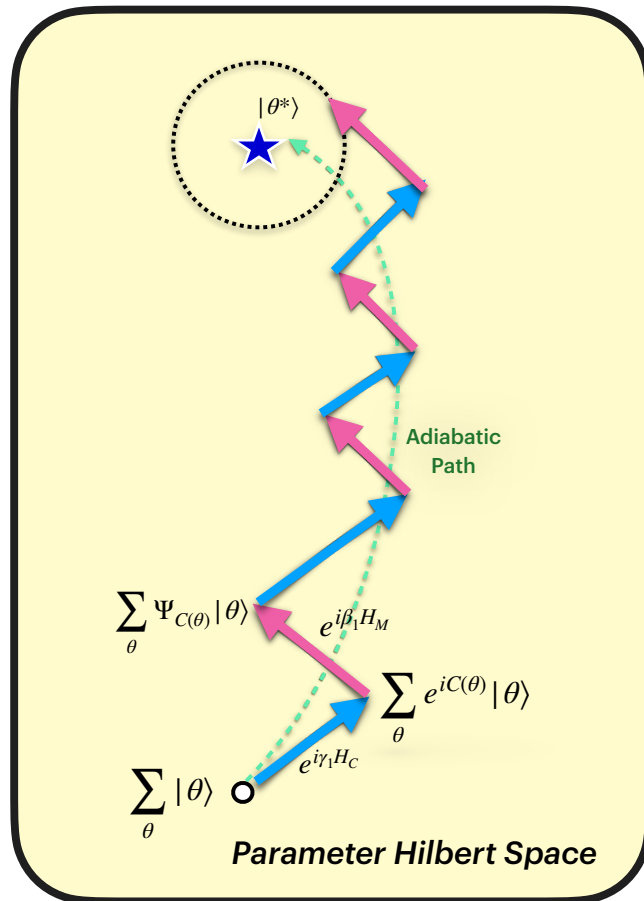
$$\phi = \sum_i |\langle \psi | q_{0,i} \rangle|^2 = \text{the success probability of measuring the (potentially degenerate) ground state(s) } \{ |q_{0,i}\rangle \},$$



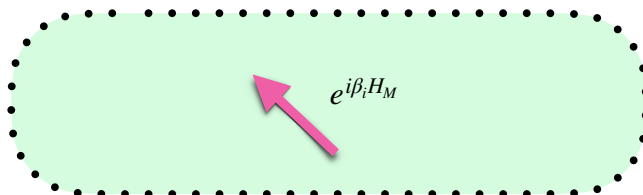
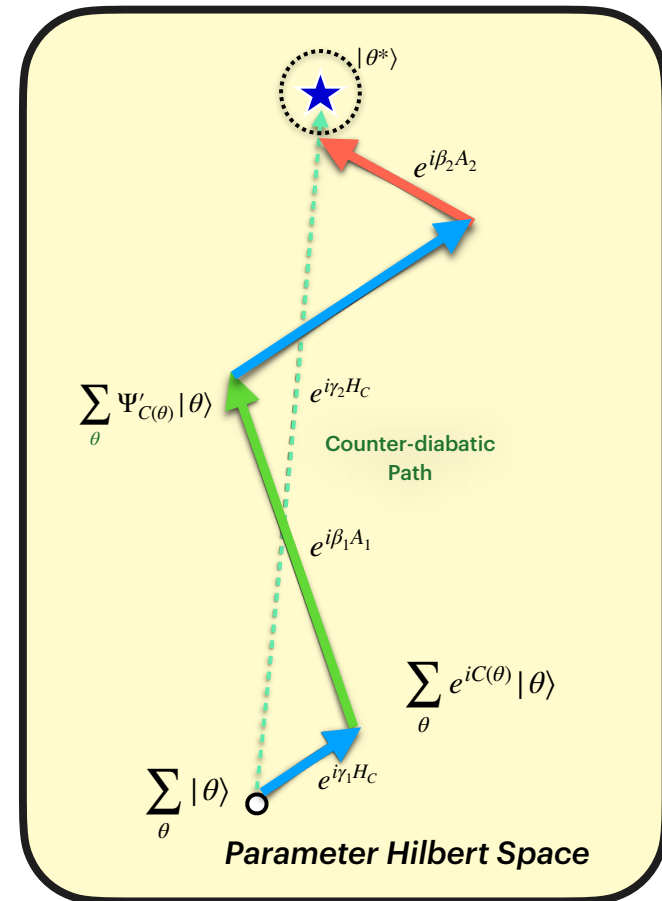
Pictorial representation of MaxCut on a 3-regular graph with 8 vertices.

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)

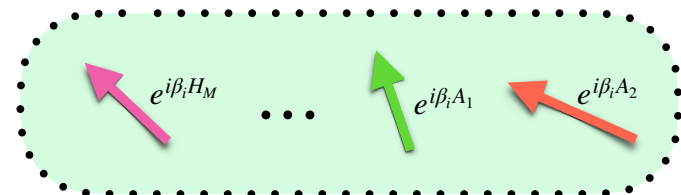
QAOA



Adaptive QAOA

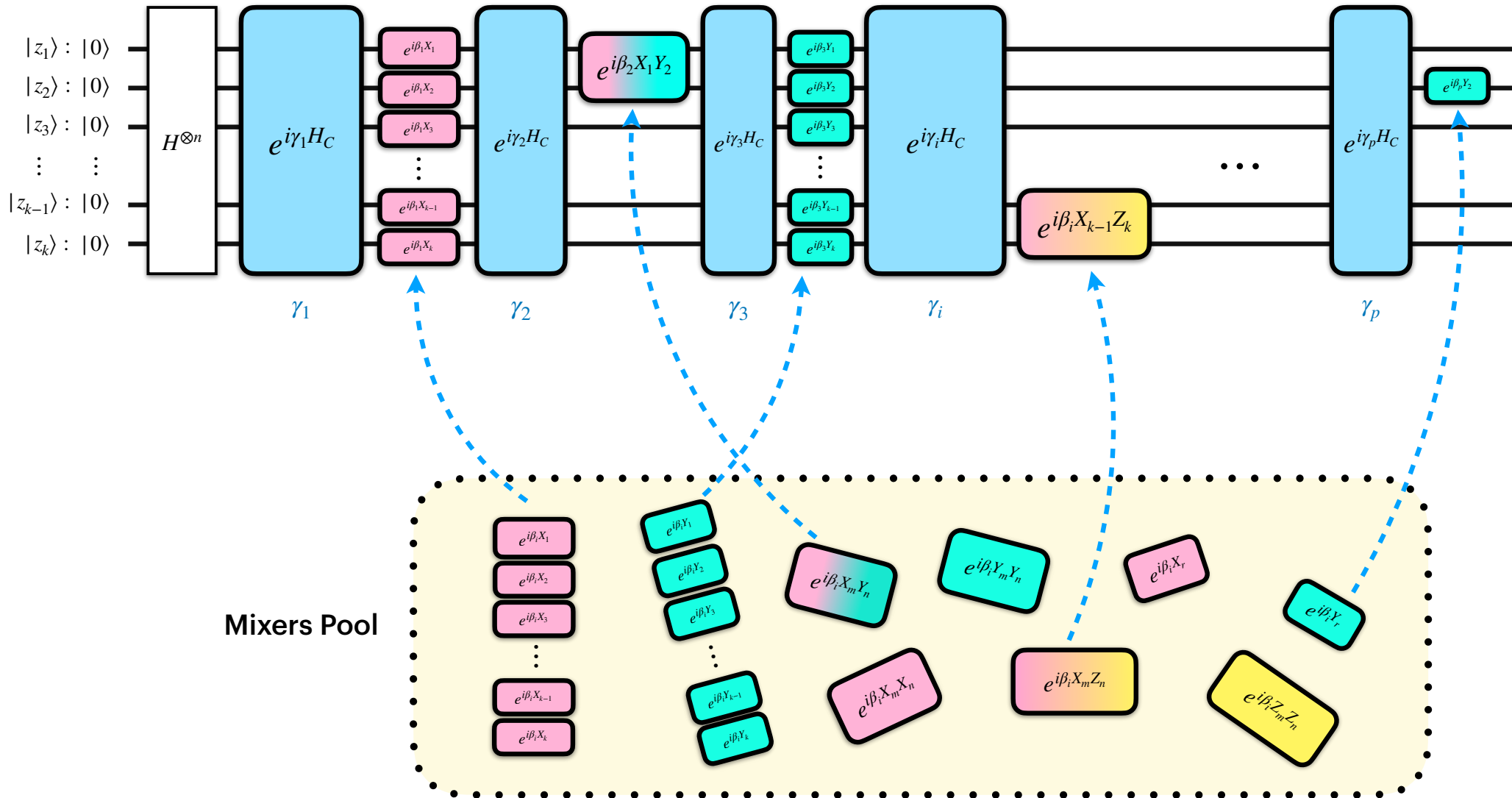


Mixers Pool

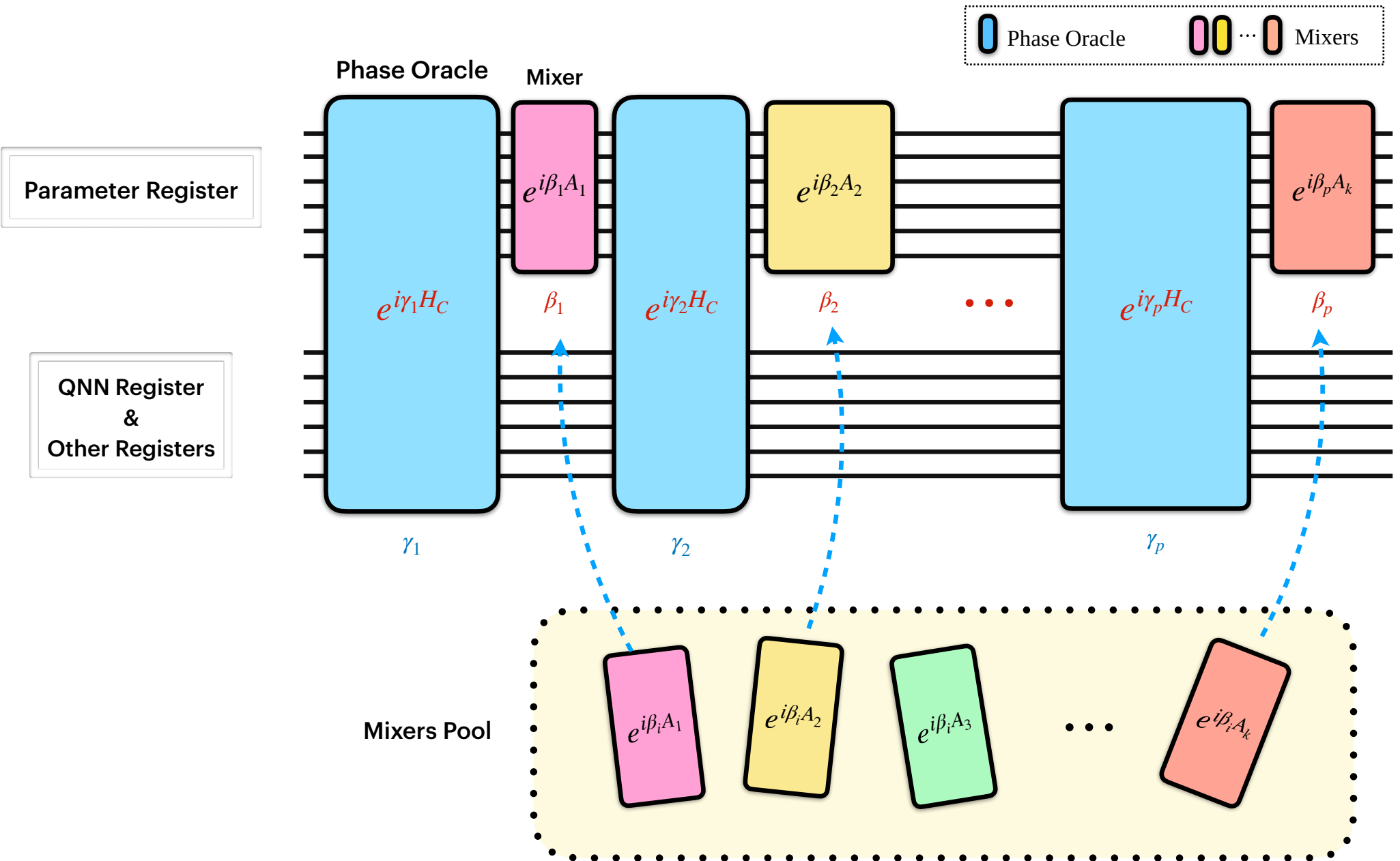


ADAPT-QAOA

Zhu et al 2020



Quantum Neural Networks



Adaptive Derivative Assembled Problem Tailored - Quantum Approximate Optimization Algorithm (ADAPT-QAOA)

- <https://arxiv.org/pdf/2005.10258.pdf>

ADAPT-QAOA

$$|\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iH_M \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle \Rightarrow |\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iA_k \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle$$

$$|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$$

n=number of qubits

mixer pool = set of A_j : $\{A_j\}$

Algorithm 1 ADAPT-QAOA

Initial state: $|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$

Predefined: Number of layers p ; Cost Hamiltonian H_C ;

Initial parameter for optimization: γ_0 ; Operator pool with m operators A_j , $j \in [1, m]$

for $k = 1 \dots p$ **do**

 //From operator pool select operator

for $j = 1 \dots m$ **do**

 //Get max measured gradient operator $A_{\text{max}}^{(k)}$:

 Set $\gamma_k = \gamma_0$

 Define $|\psi^{(k)}\rangle_t = e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$

$A_{\text{max}}^{(k)} = \text{argmax} \left(-i \langle \psi^{(k)} | [H_C, A_j] | \psi^{(k)} \rangle_t \right)$

end for

 //Add $A_{\text{max}}^{(k)}$ to current ansatz:

$|\psi^{(k)}\rangle = e^{-iA_{\text{max}}^{(k)} \beta_k} e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$

 // Optimization

$\min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle \rightarrow \vec{\beta}, \vec{\gamma}$

 output.add($\vec{\beta}, \vec{\gamma}, A_{\text{max}}^{(k)}, \min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle$)

end for

return output

ADAPT-QAOA

$$|\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iH_M \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle \Rightarrow |\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iA_k \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle$$

$$|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$$

n=number of qubits

mixer pool = set of A_j : $\{A_j\}$

$$\text{Define } |\psi^{(k)}\rangle_t = e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$$

$$A_{\text{max}}^{(k)} = \text{argmax} \left(-i {}_t\langle \psi^{(k)} | [H_C, A_j] | \psi^{(k)} \rangle_t \right)$$

end for

//Add $A_{\text{max}}^{(k)}$ to current ansatz:

$$|\psi^{(k)}\rangle = e^{-iA_{\text{max}}^{(k)} \beta_k} e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$$

// Optimization

$$\min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle \rightarrow \vec{\beta}, \vec{\gamma}$$

$$\text{output.add}(\vec{\beta}, \vec{\gamma}, A_{\text{max}}^{(k)}, \min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle)$$

end for

return output

$$\begin{aligned} \left. \frac{\partial}{\partial \beta_k} \langle \psi_k | H_P | \psi_k \rangle \right|_{\beta_k=0} &= \left. \frac{\partial}{\partial \beta_k} \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} H_P e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \right|_{\beta_k=0} \\ &= \left. \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} (iA_j H_P - iH_P A_j) e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \right|_{\beta_k=0} \\ H_P &= H_C \\ &= -i \langle \psi_{k-1} | e^{i\gamma_k H_P} [H_P, A_j] e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \end{aligned}$$

ADAPT-QAOA

$$|\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iA_k\beta_k} e^{-iH_C\gamma_k}] \right) |\psi_{\text{ref}}\rangle$$

$$|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$$

mixer pool = set of $A_j : \{A_j\}$

$\mathcal{O}(1)$ elements $P_{\text{QAOA}} = \left\{ \sum_{i \in Q} X_i \right\}$

$\mathcal{O}(n)$ elements $P_{\text{single}} = \cup_{i \in Q} \{X_i, Y_i\} \cup \left\{ \sum_{i \in Q} Y_i \right\} \cup P_{\text{QAOA}}$

$\mathcal{O}(n^2)$ elements $P_{\text{multi}} = \cup_{i,j \in Q \times Q} \{B_i C_j | B_i, C_j \in \{X, Y, Z\}\} \cup P_{\text{single}}$

$$P_{\text{QAOA}} \subset P_{\text{single}} \subset P_{\text{multi}} \quad \longrightarrow \text{Best performance}$$

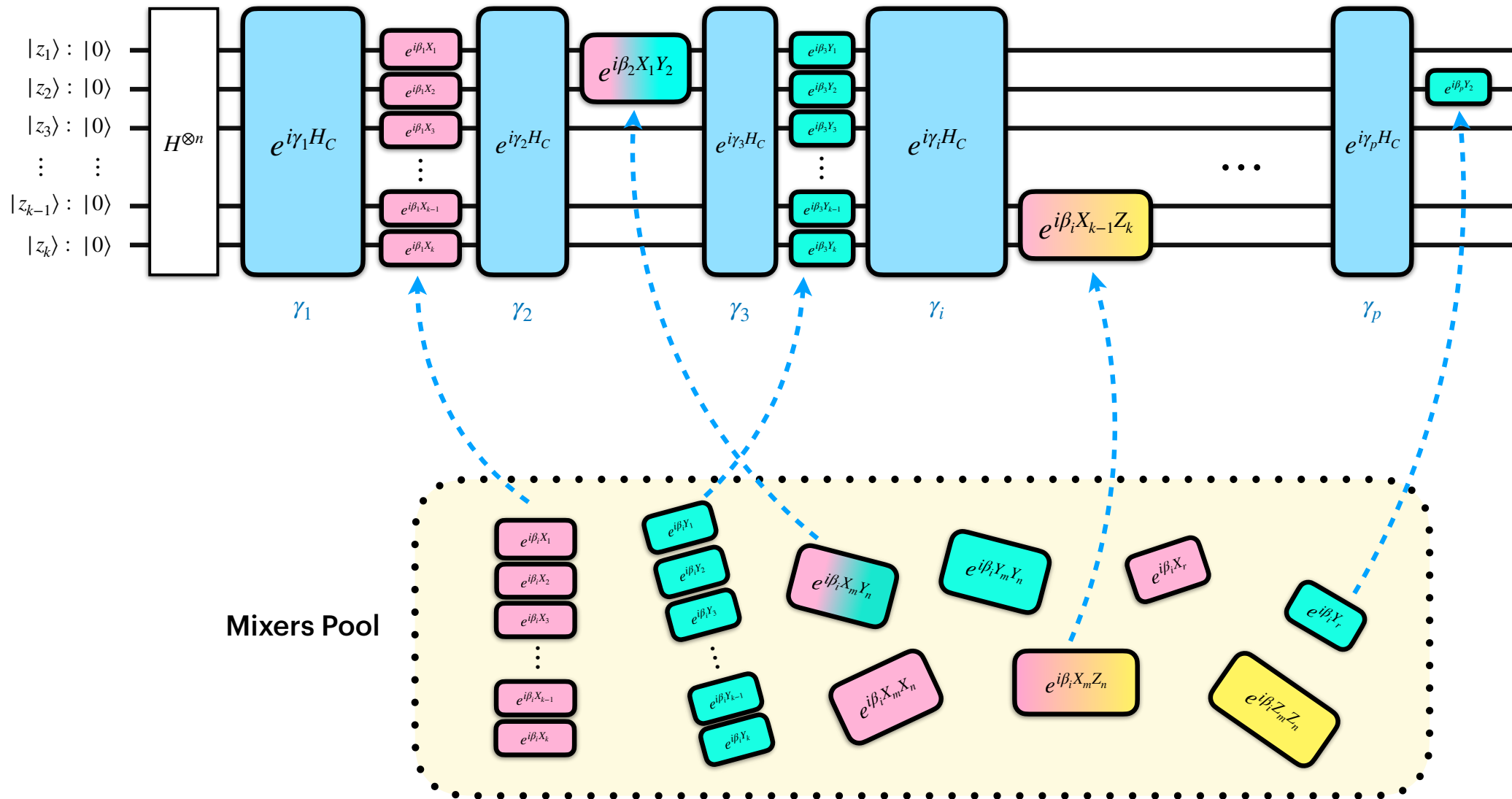
$$H_C = -\frac{1}{2} \sum_{i,j} w_{i,j} (I - Z_i Z_j)$$

- $H_C = H_P$ has a Z_2 symmetry associated with the operator $F = \otimes_i X_i$. Since $[F, H_C] = 0$, one can show that the gradient is only nonzero for $[F, A_j] \neq 0$. The A_j that commutes with F are Pauli strings that have an even number of Y or Z operators.

ADAPT-QAOA

2005.10258

Zhu et al 2020



ADAPT-QAOA applied to Mascot

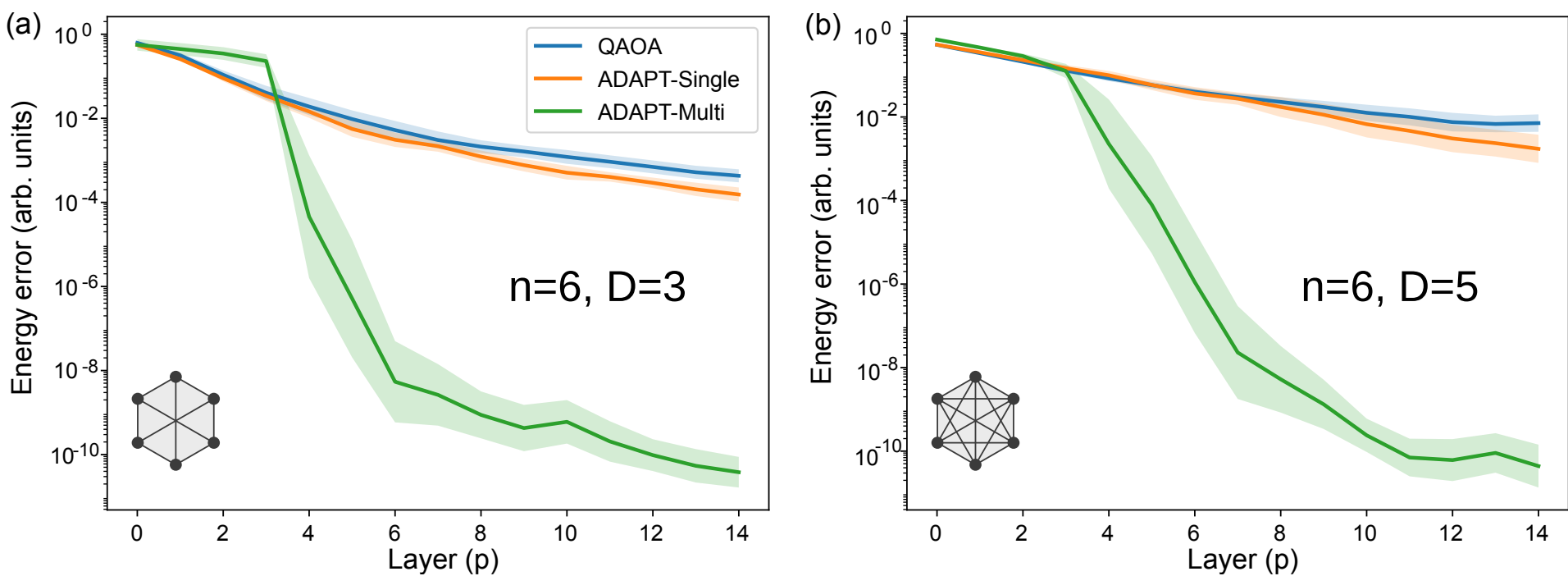


FIG. 1. Comparison of the performance of standard QAOA (blue) with ADAPT-QAOA for the single-qubit (orange) and multi-qubit (green) pools. The algorithms are run on the Max-Cut problem for the regular graphs shown in the figure, which have $n=6$ vertices and are of degree $D=3$ (a) and $D=5$ (b). The energy error (the difference between the energy estimate obtained by the algorithm and the exact ground state energy of H_C) is shown as a function of the number of layers in the ansatz. Results are shown for 20 different instances of edge weights, which are randomly sampled from the uniform distribution $U(0,1)$. The shaded regions indicate 95% confidence intervals.

$\gamma_0 = 0.01$

- Nelder-Mead for optimization
- = downhill simplex method
- = amoeba method
- = polytope method

- How much does the ADAPT-QAOA ansatz differ from the standard QAOA ansatz?
- When the single-qubit mixer pool is used, the single-qubit operators X_i are chosen instead of the standard mixer approximately **36.6%** of the time for $n=6, D=3$ graphs and **25%** of the time for $n=6, D=5$ graphs.
- For the multi-qubit mixer pool, the algorithm chooses operators other than the standard mixer approximately **75%** of the time for $n=6, D=3$ graphs and **80%** of the time for $n=6, D=5$ graphs.
- This trend supports the intuitive idea that a more connected graph requires more entanglement for a rapid convergence to the solution.

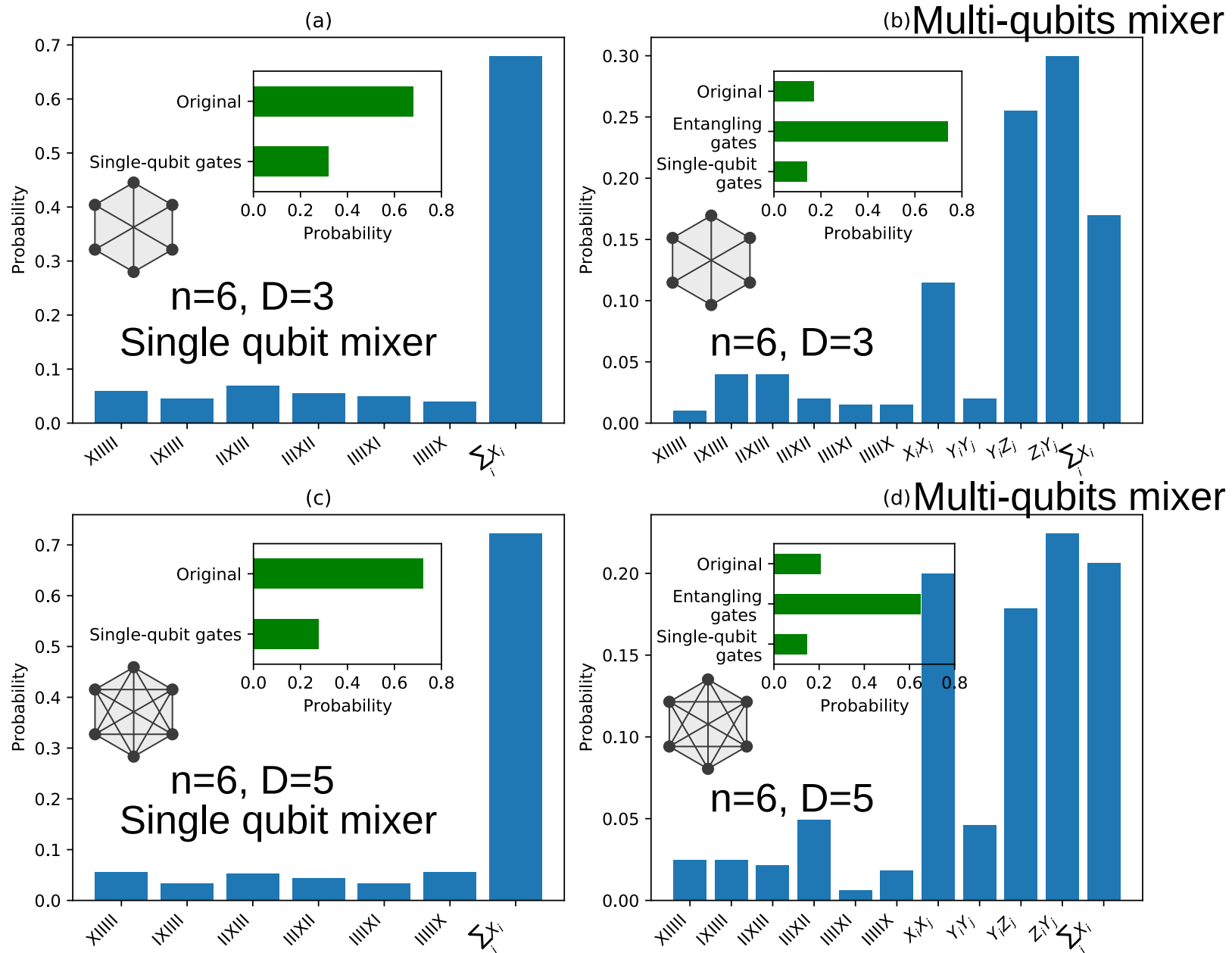
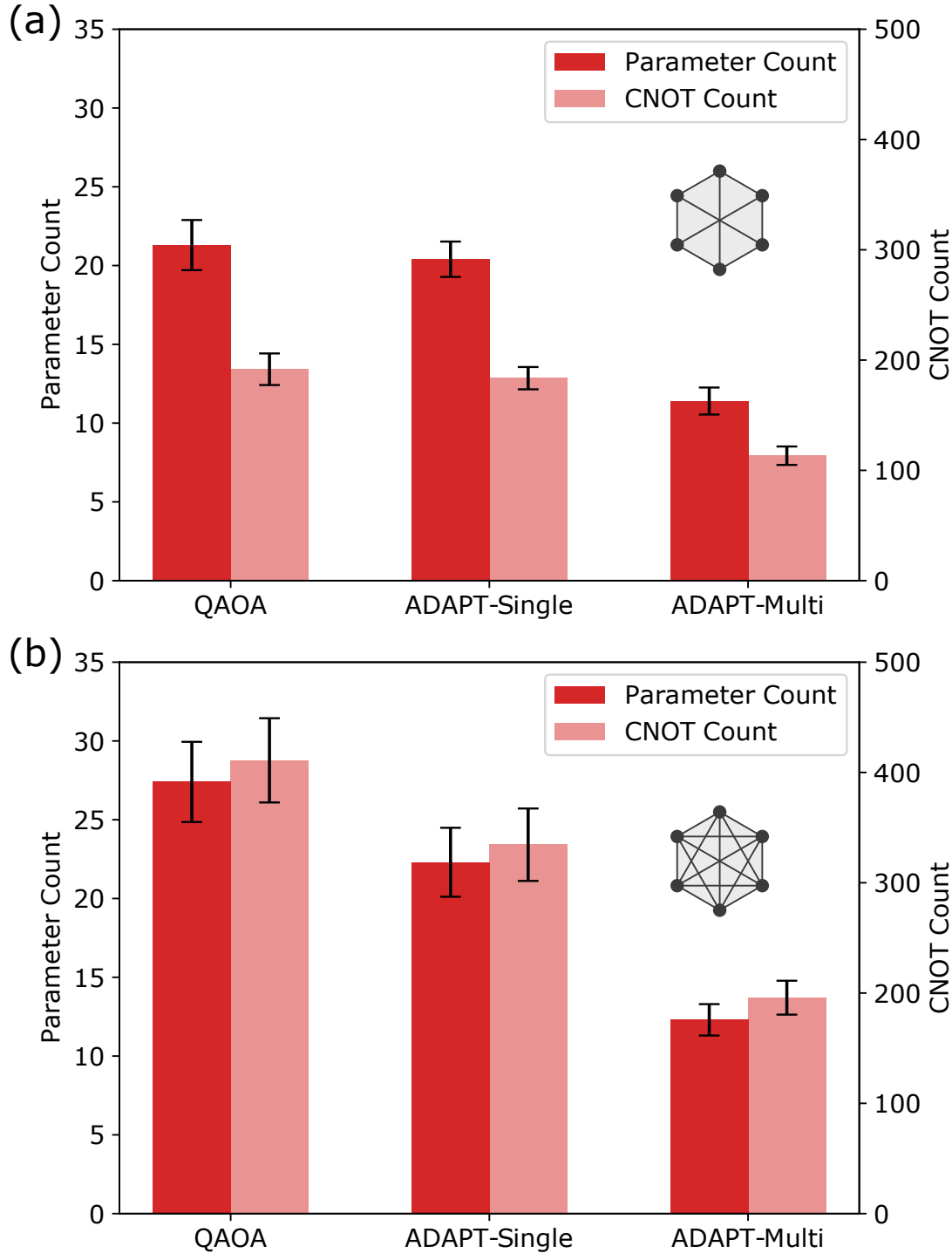


FIG. 6. Probability of operators picked by the original QAOA, ADAPT-QAOA with the single-qubit mixer and ADAPT-QAOA with multi-qubit pool for the Max-Cut problem on regular graphs with $n=6$ vertices with degree $D=3$ (a)(b) and $D=5$ (c)(d) with random edge weights sampled from a uniform distribution $U(0, 1)$. The blue bars show the probability of each particular operator used for ansatz, and green bars show the probability of the original mixer, sum over all single-qubit gates and sum over all entangling gates used in ansatz. The results from 20 instances of random edge weights.

2005.10258



- ADAPT-QAOA provides a systematic way to both improve performance and reduce the number of parameters and CNOTs.

FIG. 2. Resource comparison of the standard QAOA, ADAPT-QAOA with the single-qubit mixer pool, and ADAPT-QAOA with the multi-qubit mixer pool for the Max-Cut problem on regular graphs with $n=6$ vertices and random edge weights. Panels (a) and (b) show the comparison for graphs of degree $D=3$ and $D=5$, respectively. For all cases except the standard QAOA applied to $D=5$ graphs never reaches this error threshold, we instead count the CNOT gates and parameters at the end of the simulation (15 layers). The dark (light) red bars show variational parameter (CNOT gate) counts. The error bars show variances obtained by sampling over 20 different instances of edge weights.

Why ADAPT-QAOA performs better?

- Considering that the standard QAOA ansatz has a structure dictated by the adiabatic theorem, a possible explanation is related to **Shortcuts to adiabaticity (STA)**.
- **STA (counter-diabatic or transitionless driving)** was introduced by Demirplak and Rice and later, independently, by Berry.
- If we want to drive a system such that it remains in the instantaneous ground state at all times, then by adding a certain term H_{CD} to the Hamiltonian, we can achieve this without paying the price of a slow evolution.
- Although the instantaneous eigenstates of the original Hamiltonian only solve the time-dependent Schrodinger equation in the adiabatic limit, they become exact solutions when the Hamiltonian is updated to include H_{CD} .
- The advantage of STA is that the evolution can be achieved non-adiabatically.

Gauge transformation in electrodynamics

$$\nabla \cdot \vec{E} = \rho \quad \text{Gauss' law}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{Gauss' law}$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \text{Faraday's law}$$

$$\nabla \times \vec{E} - \frac{\partial \vec{E}}{\partial t} = \vec{J} \quad \text{Ampere-Maxwell's law}$$

$$\vec{F}_{\text{Lorentz}} = q \left(\vec{E} + \vec{v} \times \vec{B} \right) \quad \text{Lorentz force}$$

$$\vec{F} = m\vec{a} \quad \text{Newton's equation of motion}$$

$$\frac{\partial \rho(\vec{x}, t)}{\partial t} + \nabla \cdot \vec{J}(\vec{x}, t) = 0 \quad \text{Continuity equation}$$

- The electromagnetic fields $\vec{E} = -\nabla\phi + \frac{\partial \vec{A}}{\partial t}$ and $\vec{B} = \nabla \times \vec{A}$ are invariant under the following Gauge transformation

$$\phi \longrightarrow \phi - \frac{\partial \Lambda(\vec{x}, t)}{\partial t}$$

$$\vec{A} \longrightarrow \vec{A} + \nabla \Lambda(\vec{x}, t)$$

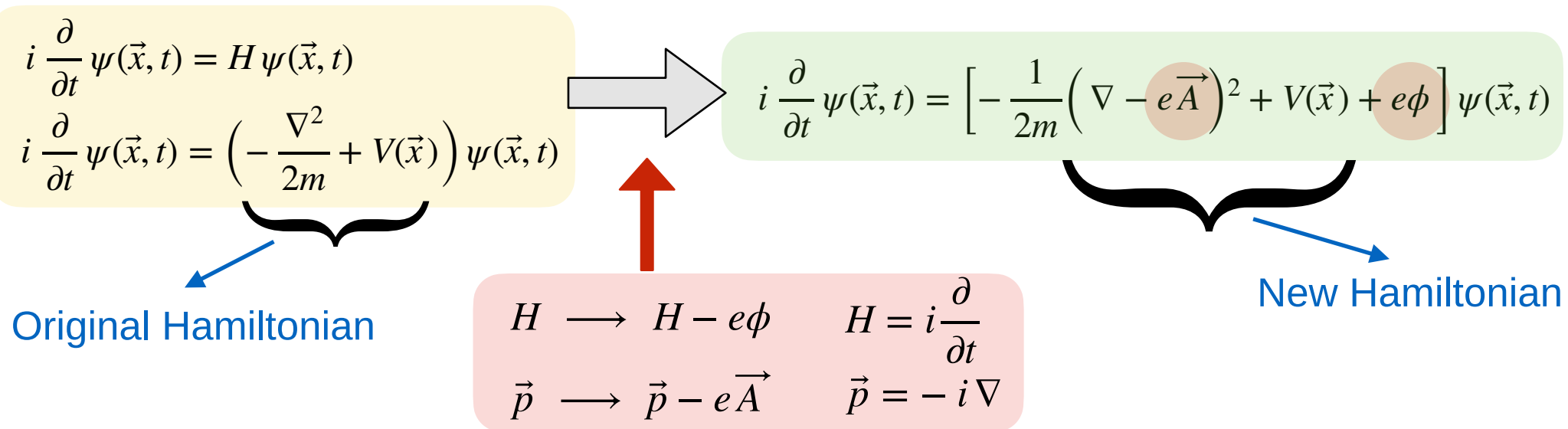
- Therefore the Lorentz force and the corresponding trajectory of an object under the electromagnetic remain the same.
- The gauge symmetry implies the conservation of electric charge.
- Perhaps $(\phi, \vec{A})$ may be **more fundamental** physical quantities than the electromagnetic fields $(\vec{E}, \vec{B})$. $A_\mu = (\phi, \vec{A})$ is called photon field.

Gauge transformation in electrodynamics

- The electromagnetic field $\vec{E} = -\nabla\phi + \frac{\partial\vec{A}}{\partial t}$ and $\vec{B} = \nabla \times \vec{A}$ are invariant (and therefore the classical trajectory of an object) under the following Gauge transformation

$$\phi \longrightarrow \phi - \frac{\partial\Lambda(\vec{x}, t)}{\partial t} \qquad \vec{A} \longrightarrow \vec{A} + \nabla\Lambda(\vec{x}, t)$$

- In quantum mechanics, we need to modify Hamiltonian (for action of quantum state) to be invariant under the gauge transformation.



- Quanta of electromagnetic fields are photons, $A_\mu = (\phi, \vec{A})$

Shortcuts to Adiabaticity (transitionless driving protocols)

$$i \partial_t |\psi\rangle = H(\theta(t)) |\psi\rangle \quad \longrightarrow \quad i \partial_t |\tilde{\psi}\rangle = (\tilde{H} - \dot{\theta} \tilde{A}_\theta) |\tilde{\psi}\rangle$$

$$\begin{aligned} |\psi\rangle &\longrightarrow |\tilde{\psi}\rangle = U^\dagger |\psi\rangle & H_{CD} &= \dot{\theta} A_\theta \\ H &\longrightarrow \tilde{H} = U^\dagger H U & \tilde{A}_\theta &= i U^\dagger \partial_\theta U \\ i \partial_t &\longrightarrow i \partial_t - \dot{\theta} \tilde{A}_\theta & A_\theta &= U \tilde{A}_\theta U^\dagger \end{aligned}$$

- Suppose that we consider a unitary transformation $U(\theta(t))$ to move the Hamiltonian $H(\theta(t))$ from the initial basis to its instantaneous eigenbasis, where $\tilde{H}(\theta) = U^\dagger(\theta) H(\theta) U(\theta)$ is diagonal at all times.
- The Schrodinger equation in the instantaneous eigenbasis is $i \partial_t |\tilde{\psi}\rangle = (\tilde{H} - \dot{\theta} \tilde{A}_\theta) |\tilde{\psi}\rangle$, where $\tilde{A}_\theta = i U^\dagger A_\theta U$ is the **adiabatic gauge potential** in the rotated frame. It is evident that the term $-\dot{\theta} \tilde{A}_\theta$ drives transitions between the energy levels of the original Hamiltonian H . Therefore, one can add the counterdiabatic term $H_{CD} = \dot{\theta} A_\theta$ to $H(\theta)$, with $A_\theta = U \tilde{A}_\theta U^\dagger$, to eliminate such transitions in the rotated frame. **This is the core of transitionless driving protocols.**
- Ref. [40] proposes an approximate gauge potential:

$$\mathcal{A}_\theta^{(p)} = i \sum_{k=1}^p a_k [\mathcal{H}, \partial_\theta \mathcal{H}]_{2k-1} \quad [X, Y]_{k+1} = [X, [X, Y]]_k$$

Connection between ADAPT-QAOA and STA

- Apply the above formalism using the Hamiltonian

$$H = \frac{t}{T} H_C + \left(1 - \frac{t}{T}\right) \sum_{i=1}^n X_i, \text{ and set}$$

$\theta = t$. T is the duration of the evolution from the initial state $|\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$ to the ground state of the cost Hamiltonian H_C .

- In all cases, the mixer operator at the first layer is also an element of the set $\mathcal{O}_{CD}$.
- Going to higher order in the H_{CD} approximation increases the probability of finding the mixers in the set $\mathcal{O}_{CD}$.
- ADAPT-QAOA finds the appropriate rotation axes in Hilbert space for faster convergence to the solution, and that these axes may in some sense be universal across all possible choices of $H(t)$ that interpolate between the initial and target states. This suggests that STA can be used as a tool to construct operator pools for ADAPT-QAOA.

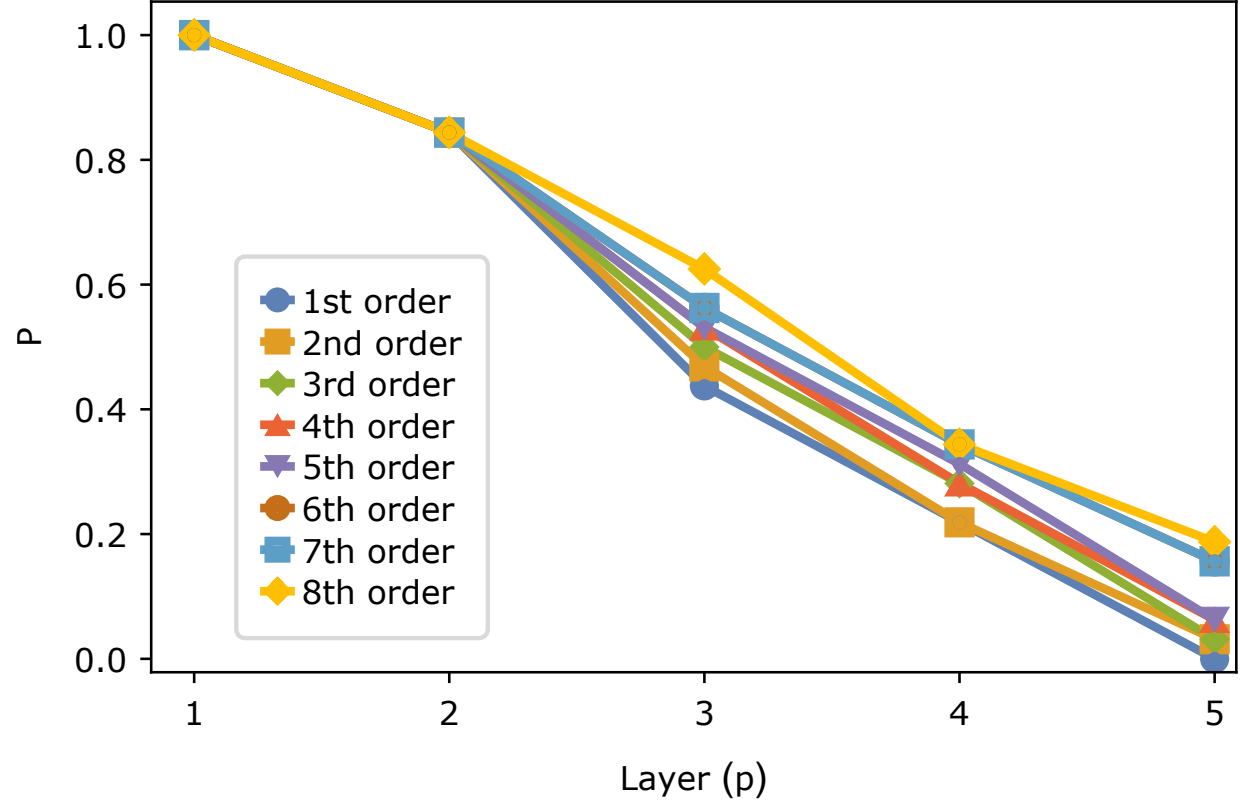
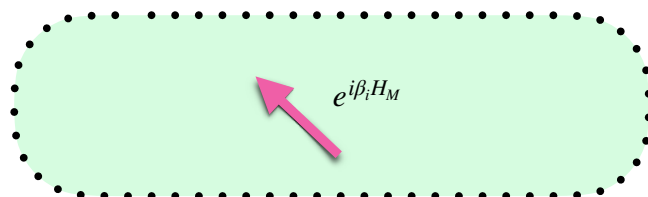
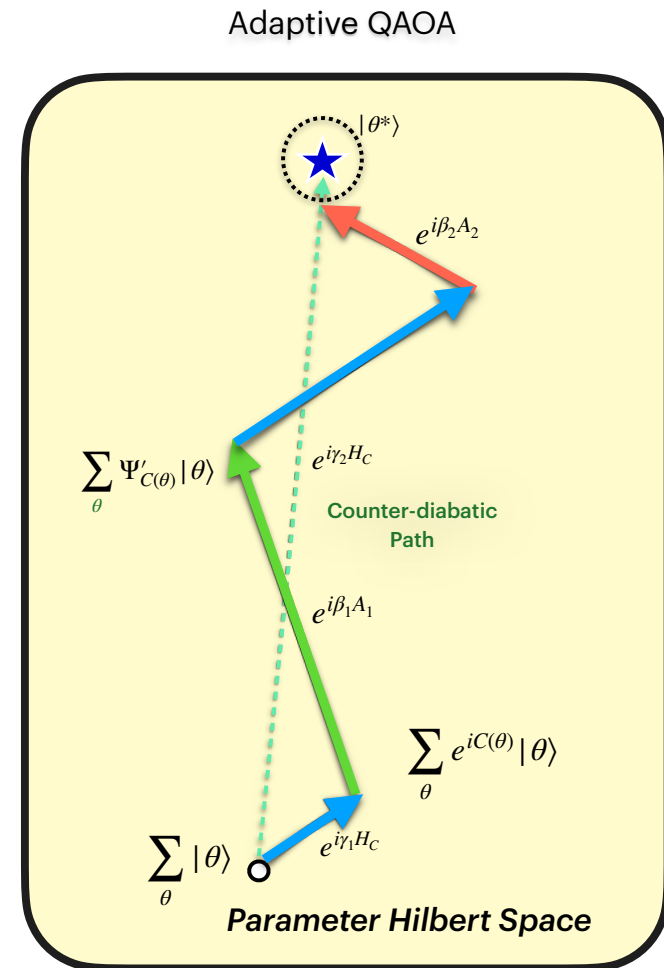
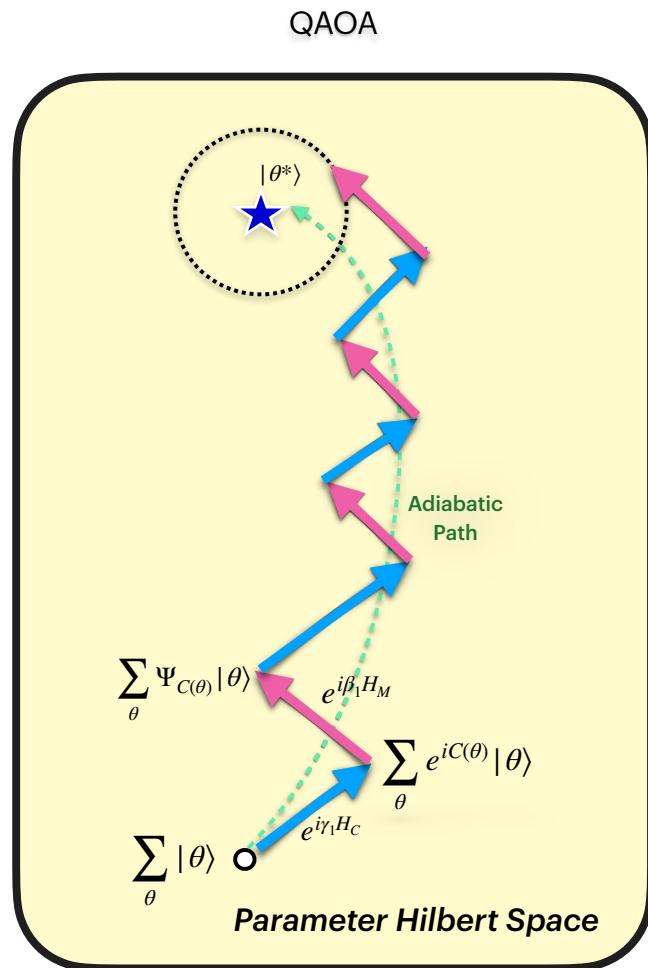
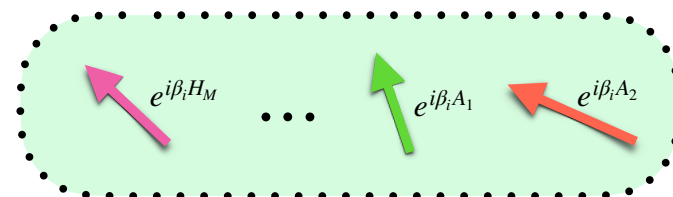


FIG. 3. Probability P of the operator at layer p of the ADAPT-QAOA ansatz to be among the Pauli strings with the largest coefficient in $\mathcal{H}_{CD}$ averaged over 32 graphs with $n = 6$, $D = 3$. The different curves correspond to different orders of the approximation.

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)



Mixers Pool



Combinatorial problems at the LHC

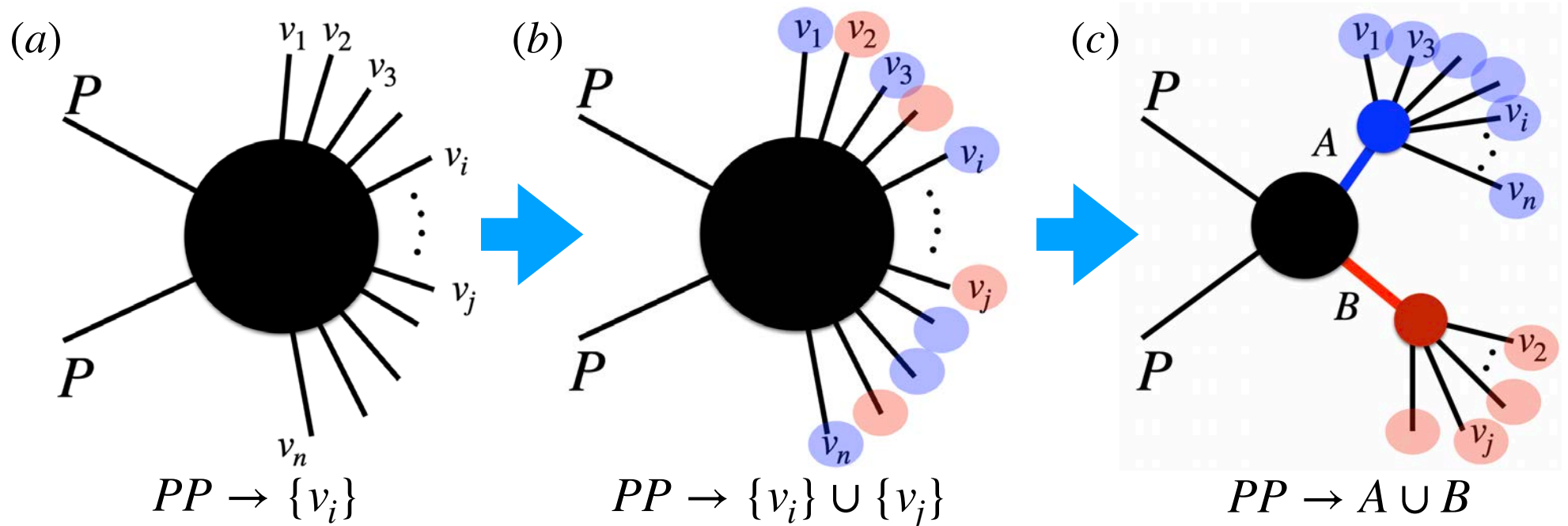


FIG. 1. (a) n -observed particles (b) Dividing n particles into two groups for $2 \rightarrow 2$ process (c) Identified event-topology with A and B .



NATIONAL QUANTUM INITIATIVE

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Welcome to *quantum.gov*, the home of the National Quantum Initiative and its ongoing activities to explore and promote Quantum Information Science (QIS). The [National Quantum Initiative Act](#) provides for the continued leadership of the United States in QIS and its technology applications. It calls for a coordinated Federal program to accelerate quantum research and development for the economic and national security of the United States. The United States strategy for QIS R&D and related activities is described in the [National Strategic Overview for QIS](#) and [supplementary documents](#).

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The Nobel Prize in Physics 2022



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Alain Aspect

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Anton Zeilinger

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The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Yuri Milner founded the Breakthrough prize in 2012.

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BB84, quantum cryptography, factoring algorithms, Deutsch's algorithm

Basic Group Theory

- **Group:** a set G with an associative binary operation $\bullet$ satisfying
 - For any two elements g_1 and $g_2 \in G$, $g_1 \bullet g_2 \in G$ (closure)
 - $\exists e \in G$ such that $e \bullet g = g \bullet e = g$ for $\forall g \in G$ (identity)
 - $\exists g^{-1} \in G$ such that $g^{-1} \bullet g = g \bullet g^{-1} = e$ for $\forall g \in G$ (inverse)
 - $g_1 \bullet (g_2 \bullet g_3) = (g_1 \bullet g_2) \bullet g_3$ (associativity)
- $(\mathbb{Z}_n, + \pmod n) = \{0, 1, 2, \dots, n-1\}$ forms a group under addition modulo n .
- Set of k -bit string, $\mathbb{Z}_2^k$ forms a group under bitwise addition modulo 2.
- For a prime p , $\{1, 2, \dots, p-1\}$ forms a group $\mathbb{Z}_p^*$ under multiplication modulo p .
- $U(n)$: all unitary operators on an n -dimensional vector space V .
- Order = # of elements = $|G|$
- Finite group if $|G| < \infty$. Otherwise G is an infinite group.
- The order of an element g = the size of the subgroup of G that it generates.
 - The order of an element g must divide the order of the group.
- A set of generators of a group G is a subset of G such that all elements of G can be written as a finite product of the generators and their inverse.

Basic Group Theory

- A set of generators of a group is independent, if no generator can be written as a product of the other generators.
- A group is finitely generated if a finite group of generators exists.
- If a group can be generated by a single element, it is cyclic.
- The centralizer $Z(H)$ of a subgroup H of G is the set of elements of G that commute with all elements of H : $Z(H) = \{g \in G \mid gh = hg \text{ for all } h \in H\}$.
- For $H < G$, $Z(H)$ is a subgroup of G .
- If $g_1 \bullet g_2 = g_2 \bullet g_1$, G is Abelian or commutative.
- Every finite Abelian group is isomorphic to a product of one or more cyclic groups, $\mathbb{Z}_n$.
- If $n = pq$ and p and q are relatively prime, $\mathbb{Z}_n \cong \mathbb{Z}_p \times \mathbb{Z}_q$ ($\mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_q$).
- Any Abelian group A has the unique decomposition (up to ordering of factors) into cyclic groups of prime power order: $A \cong \mathbb{Z}_{c_1} \times \mathbb{Z}_{c_2} \times \cdots \times \mathbb{Z}_{c_K}$
 - $|A| = \prod_i c_i$ (prime factorization) where $c_i = p_i^{s_i}$ and p_i are distinct primes
- Product group $G \times H$ with operations $\bullet$ and $\circ = \{(g, h) \mid g \in G, h \in H\}$ with $(g_1, h_1) \star (g_2, h_2) = (g_1 \bullet g_2, h_1 \circ h_2)$.

Discrete Logarithm Problem

- All standard public key encryption system and digital signature schemes are based on either factoring or discrete logarithm problem.
- $\mathbb{Z}_p^*$: group of integers $\{1, 2, \dots, p-1\}$ under multiplication modulo p .
 - b : generator of $\mathbb{Z}_p^*$ (any b relatively prime to $p-1$ will work)
 - The discrete logarithm of $y \in \mathbb{Z}_p^*$ with respect to base b is the element $x \in \mathbb{Z}_p^*$ such that $b^x = y \pmod{p}$.
- **Discrete logarithm problem**: Given a prime p , a base $b \in \mathbb{Z}_p^*$ and an arbitrary element $y \in \mathbb{Z}_p^*$, find an $x \in \mathbb{Z}_p^*$ such that $b^x = y \pmod{p}$
 - Find the discrete logarithm of $y \in \mathbb{Z}_p^*$ with respect to base b such that $b^x = y \pmod{p}$
 - For a large p , this problem is computationally difficult to solve.
 - It is a special case of Abelian hidden subgroup problem.
 - Can be generalized to arbitrary finite cyclic groups.

Hidden Subgroup Problem

- **Hidden subgroup problem:** Let G be a group. Suppose a subgroup $H < G$ is implicitly defined by a function f on G in that f is constant and distinct on every coset of H . Find a set of generators of H .
 - Aim is to find an algorithm that computes a set of generators for H in $\mathcal{O}((\log |G|)^k)$ steps for some integer k .
- **Finite Abelian hidden subgroup problem:** Let G be a finite Abelian group with cyclic decomposition $G = \mathbb{Z}_{n_0} \times \cdots \times \mathbb{Z}_{n_L}$. Suppose G contains a subgroup $H < G$ that is implicitly defined by a function f on G that is constant and distinct on every coset of H . Find a set of generators.
- **Period finding as a hidden subgroup problem:** f is a periodic function on $\mathbb{Z}_N$ with period r that divides N . The subgroup $H < \mathbb{Z}_N$ generated by r is the hidden subgroup. Once a generator h for H has been found, the period r can be found by taking the greatest common divisor of h and N .
- H is a subgroup of G . The left coset is defined as $gH = \{gh \mid h \in H, \text{ for all } g \in G\}$

Ansatz	Main Idea	Enhancement & Applications
ma-QAOA [77] 2021	Multi-angle ansatz with a unique parameter for each element of cost and mixer Hamiltonians	Improves approximation ratio for MaxCut while reducing circuit depth
QAOA+ [78]	Augments traditional QAOA with an additional multi-parameter problem-independent layer	Higher approximation ratios for MaxCut on random regular graphs
DC-QAOA [79, 80]	Adds a problem-dependent counterdiabatic driving term to the QAOA ansatz	Improves the convergence rate of the approximation ratio while reducing circuit depth
ab-QAOA [81]	Incorporates local fields into the operators to reduce computation time	Computation time reduction for combinatorial optimization
ADAPT-QAOA [82] 2020	Iterative version of QAOA with systematic selection of mixers based on gradient criterion	Can be problem-specific and addresses hardware constraints
Recursive QAOA [83]	Non-local variant of QAOA that iteratively reduces problem size by eliminating qubits	Overcomes locality constraints and achieves better performance
QAOAnsatz [84]	Extends the original formulation with broader families of operators and allows for encoding of constraints	Adaptable to a wider range of optimization problems with hard and soft constraints
GM-QAOA [85]	Uses Grover-like selective phase shift mixing operators	Solves k -Vertex Cover, Traveling Salesperson Problem, Discrete Portfolio Rebalancing

Th-QAOA [86]	Replaces standard phase separator with a threshold function	Solves MaxCut, Max k -Vertex Cover, Max Bisection
Constraint Preserving Mixers [87]	Constructs mixers that enforce hard constraints	Solves optimization problems with hard constraints
WS-QAOA [88]	Modifies the initial state and mixer Hamiltonian based on the optimal solution to the relaxed QUBO problem	Solutions guaranteed to retain the GW bound for the MaxCut problem
FALQON [66]	Uses qubit measurements for feedback-based quantum optimization, avoiding classical optimizers	Produces monotonically improving approximate solutions as circuit depth grows while bypassing classical optimization loops
2021		
FALQON+ [89]	Combines FALQON's initialization with QAOA for better parameter initialization	Improves initialization of standard QAOA for non-isomorphic graphs with 8 to 14 vertices
FQAOA [90]	Utilizes fermion particle number preservation to intrinsically impose constraints in QAOA process	Improves performance in portfolio optimization, applicable to Grover adaptive search and quantum phase estimation
Quantum Dropout [91]	Selectively drops out clauses defining the quantum circuit while keeping the cost function intact	Improves QAOA performance on hard cases of combinatorial optimization problems
ST-QAOA [92]	Uses an approximate classical solution to construct a problem instance-specific circuit	Achieves same performance guarantee as the classical algorithm, outperforms QAOA at low depths for MaxCut problem
Modified QAOA [31]	Modifies cost Hamiltonian with conditional rotations	Improves approximation ratio for MaxCut at $p = 1$

ADAPT-QAOA

$$|\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iH_M \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle \Rightarrow |\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iA_k \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle$$

$$|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$$

n=number of qubits

mixer pool = set of A_j : $\{A_j\}$

$$\text{Define } |\psi^{(k)}\rangle_t = e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$$

$$A_{\text{max}}^{(k)} = \text{argmax} \left(-i {}_t\langle \psi^{(k)} | [H_C, A_j] | \psi^{(k)} \rangle_t \right)$$

end for

//Add $A_{\text{max}}^{(k)}$ to current ansatz:

$$|\psi^{(k)}\rangle = e^{-iA_{\text{max}}^{(k)} \beta_k} e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$$

// Optimization

$$\min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle \rightarrow \vec{\beta}, \vec{\gamma}$$

$$\text{output.add}(\vec{\beta}, \vec{\gamma}, A_{\text{max}}^{(k)}, \min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle)$$

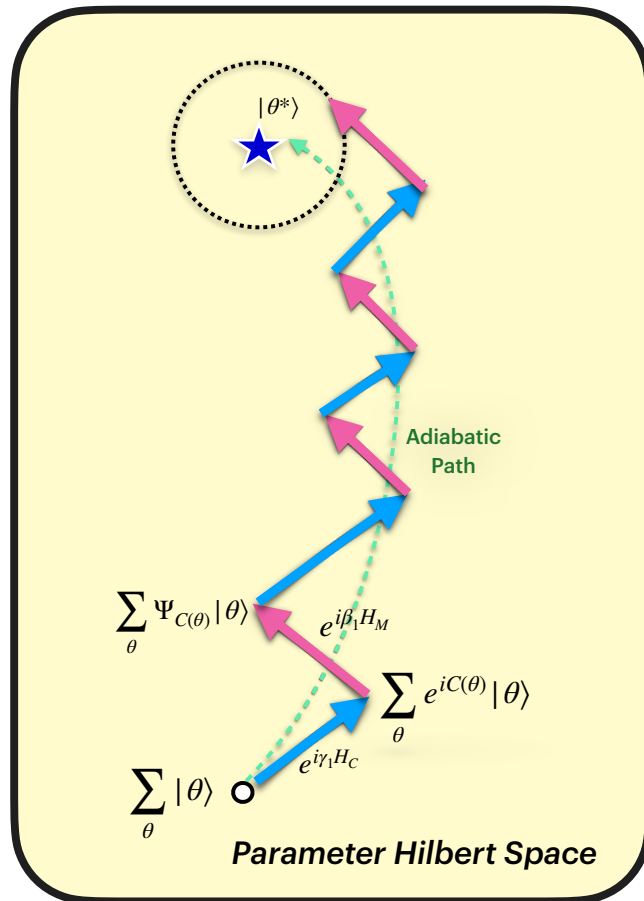
end for

return output

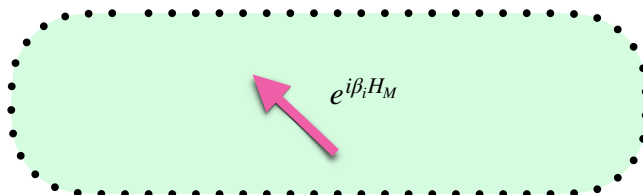
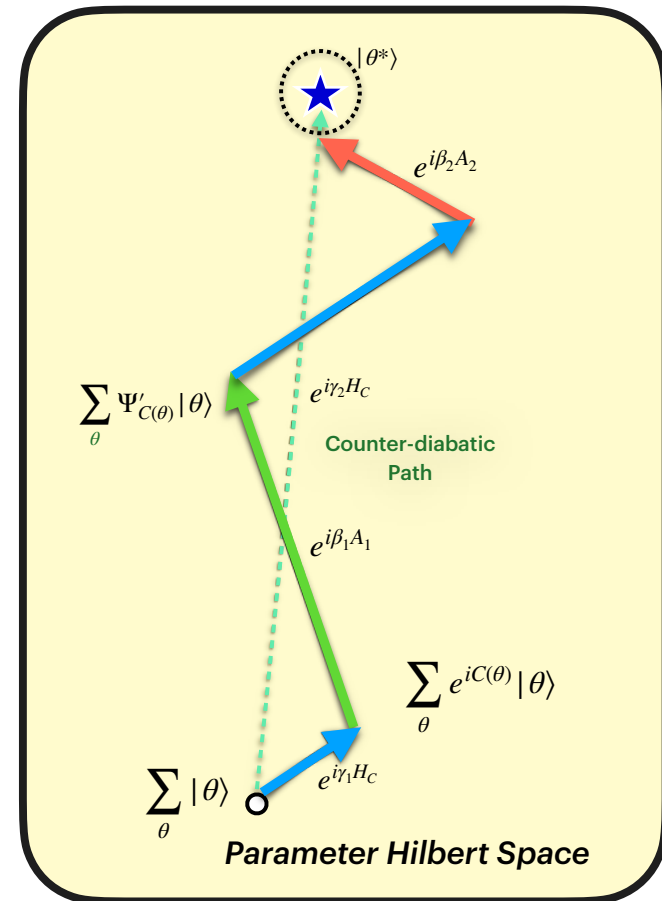
$$\begin{aligned} \frac{\partial}{\partial \beta_k} \langle \psi_k | H_P | \psi_k \rangle \Big|_{\beta_k=0} &= \frac{\partial}{\partial \beta_k} \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} H_P e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \Big|_{\beta_k=0} \\ &= \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} (iA_j H_P - iH_P A_j) e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \Big|_{\beta_k=0} \\ H_P &= H_C \\ &= -i \langle \psi_{k-1} | e^{i\gamma_k H_P} [H_P, A_j] e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \end{aligned}$$

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)

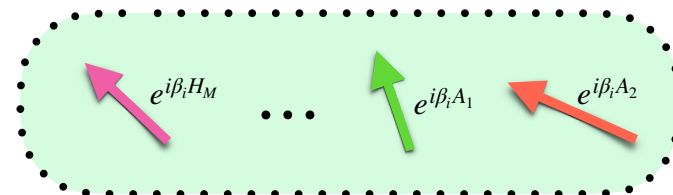
QAOA



Adaptive QAOA

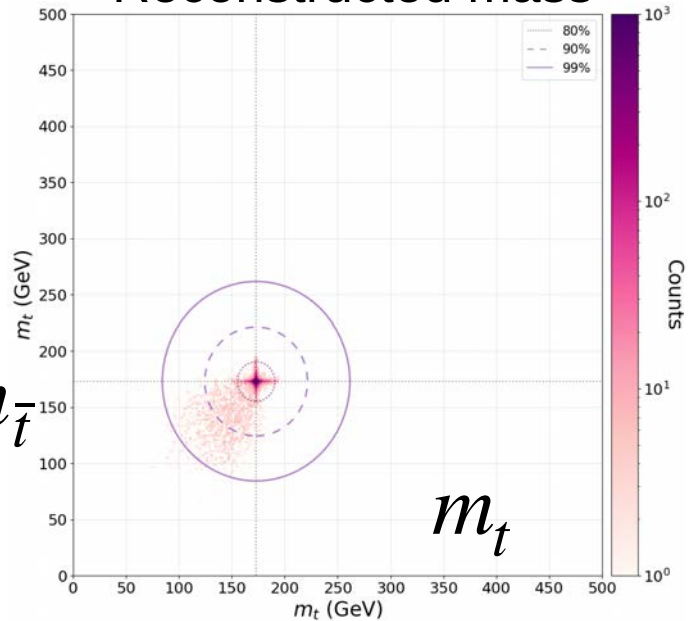


Mixers Pool

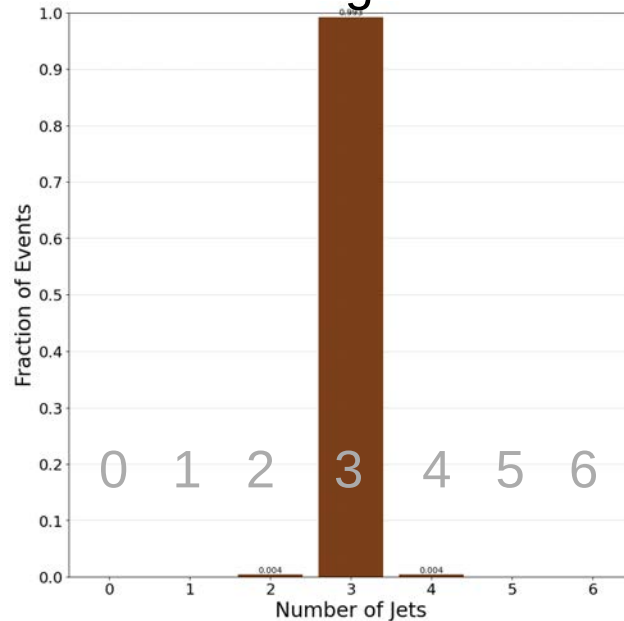


Parton-level truth and hemisphere method

Reconstructed mass



Jet assignment



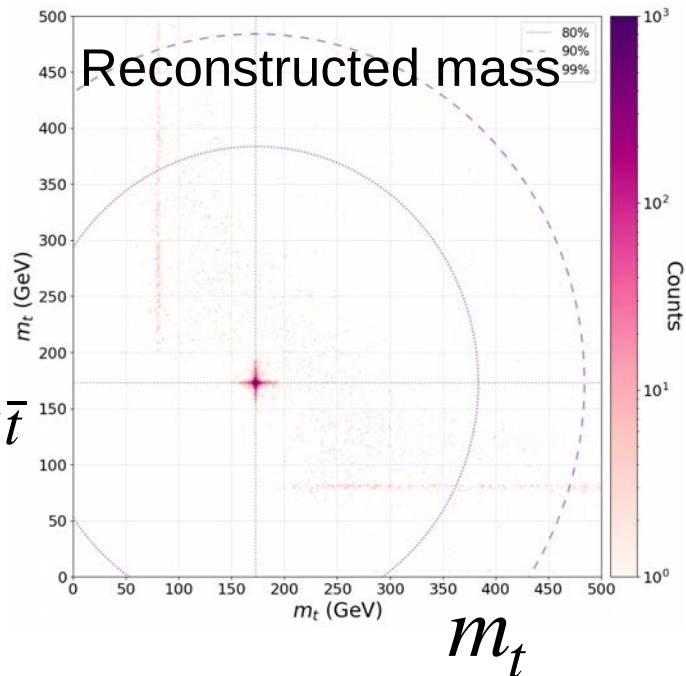
parton-level truth

79% overall efficiency

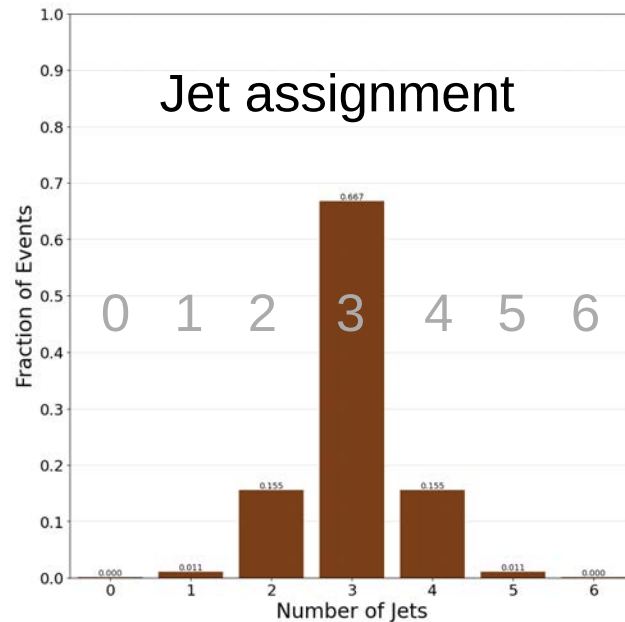
No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

Reconstructed mass



Jet assignment



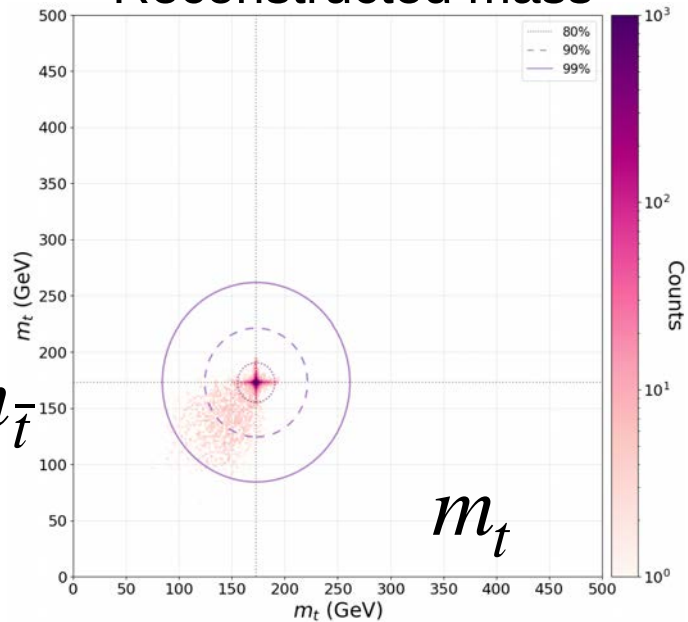
Hemisphere method

50% overall efficiency

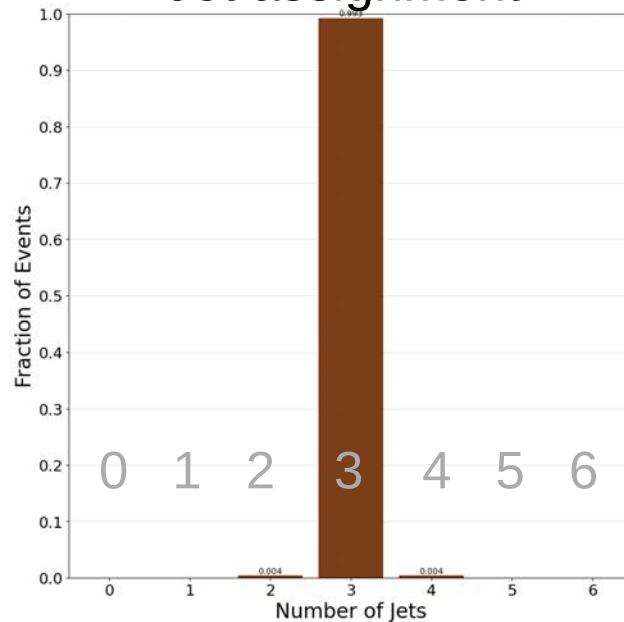
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QAOA method

Reconstructed mass



Jet assignment



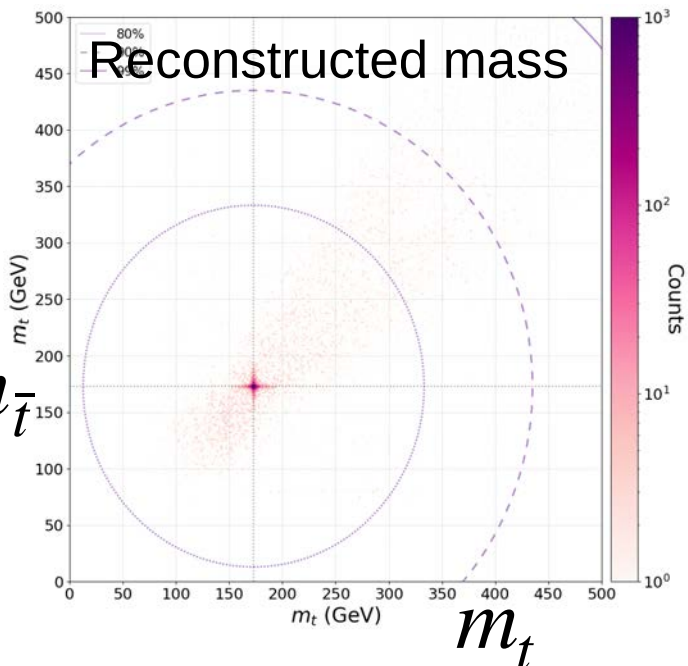
parton-level truth

79% overall efficiency

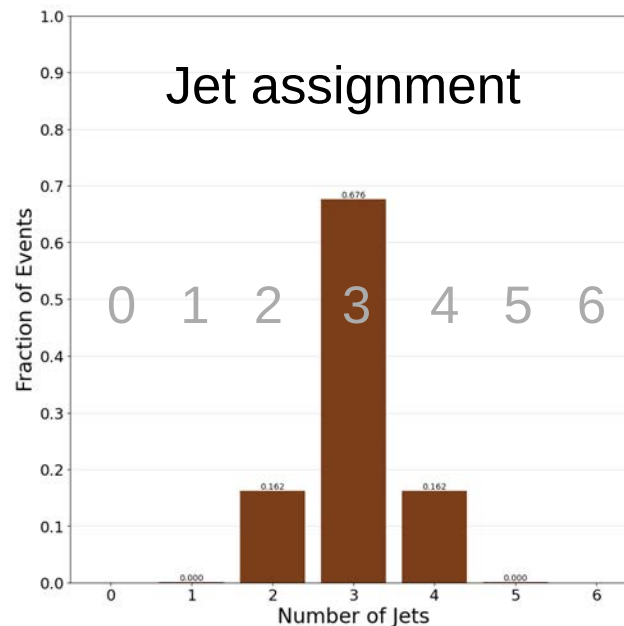
No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

Reconstructed mass



Jet assignment



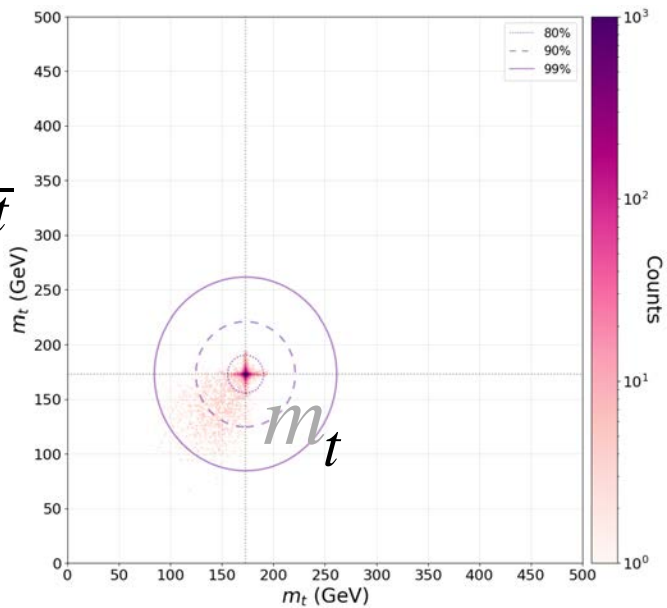
QAOA

43% overall efficiency

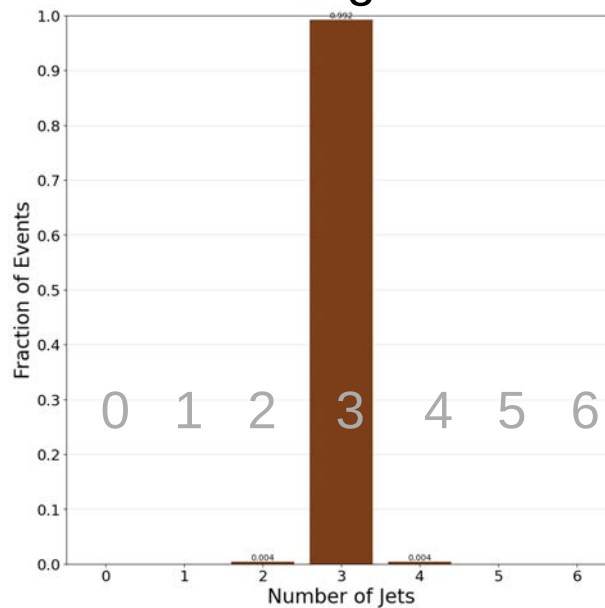
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ma-QAOA and FALQON

Reconstructed mass



Jet assignment

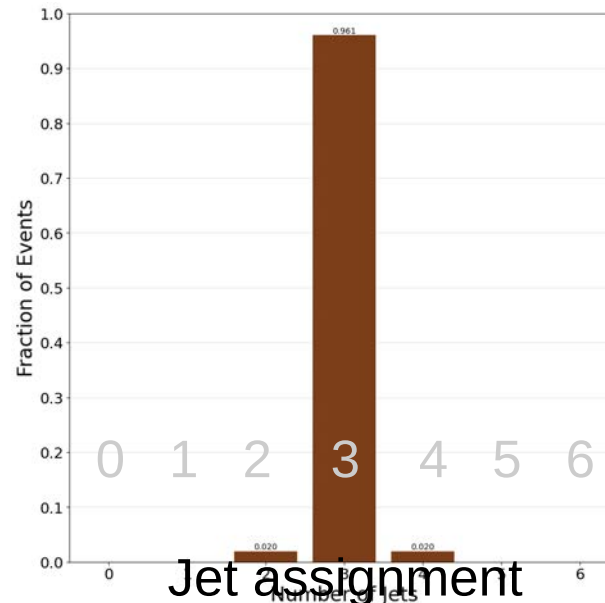
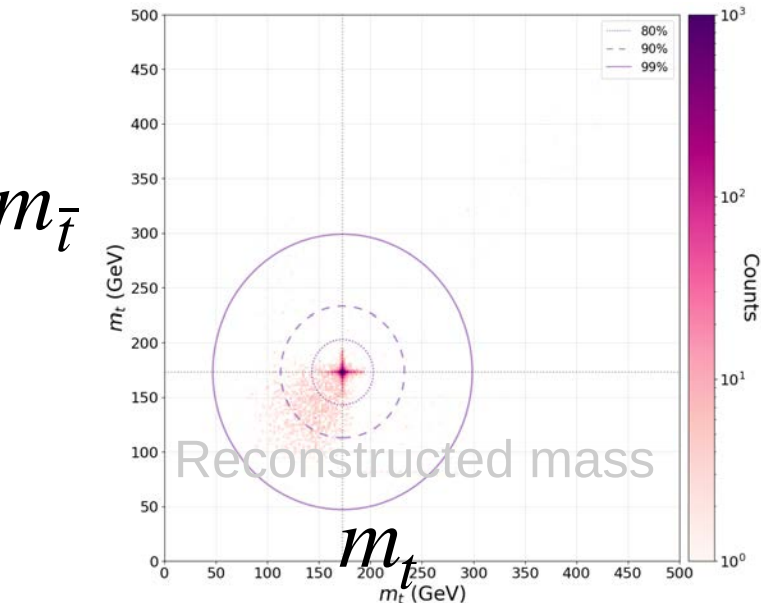


ma-QAOA / VarQITE

79% overall efficiency

No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

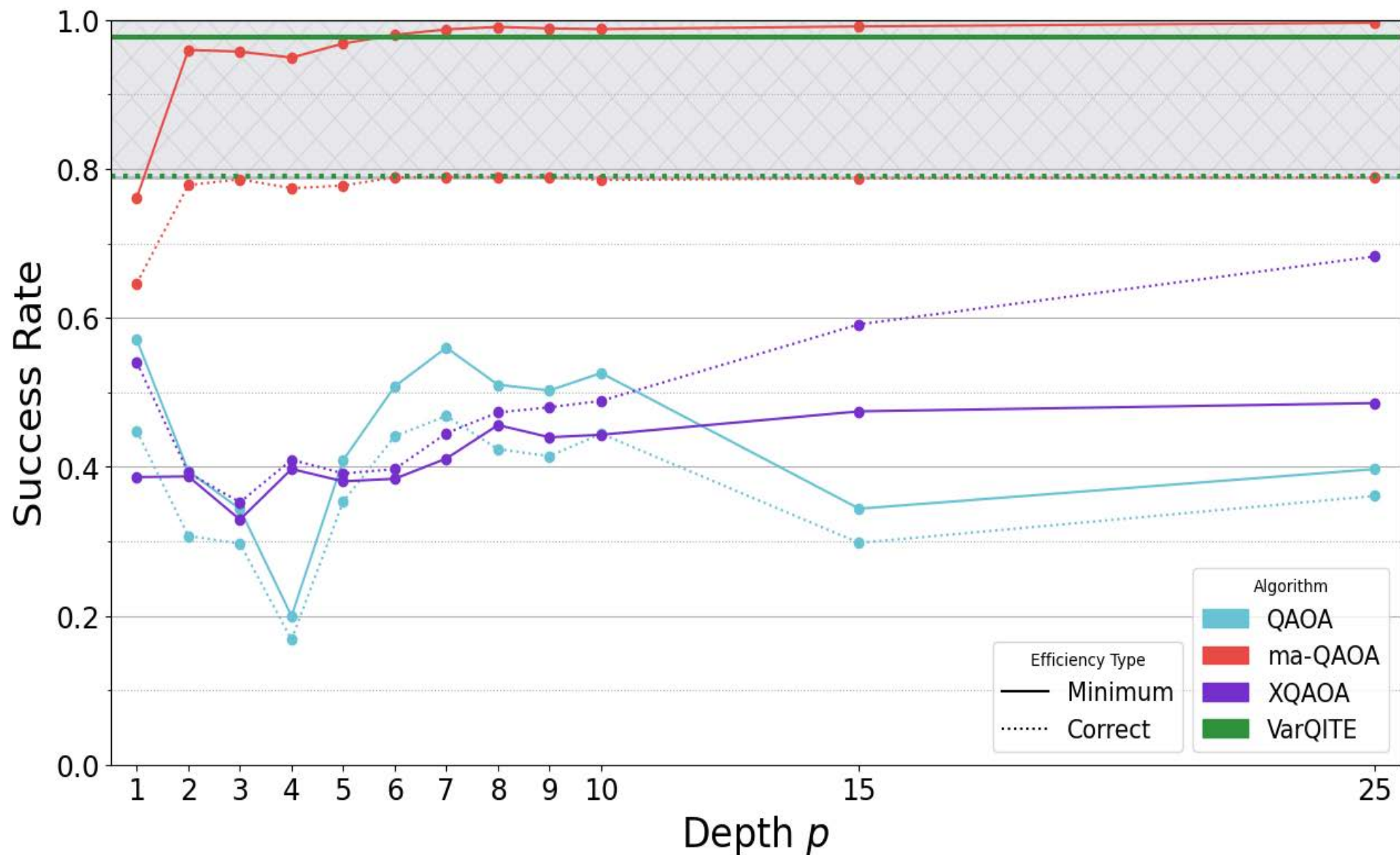


FALQON

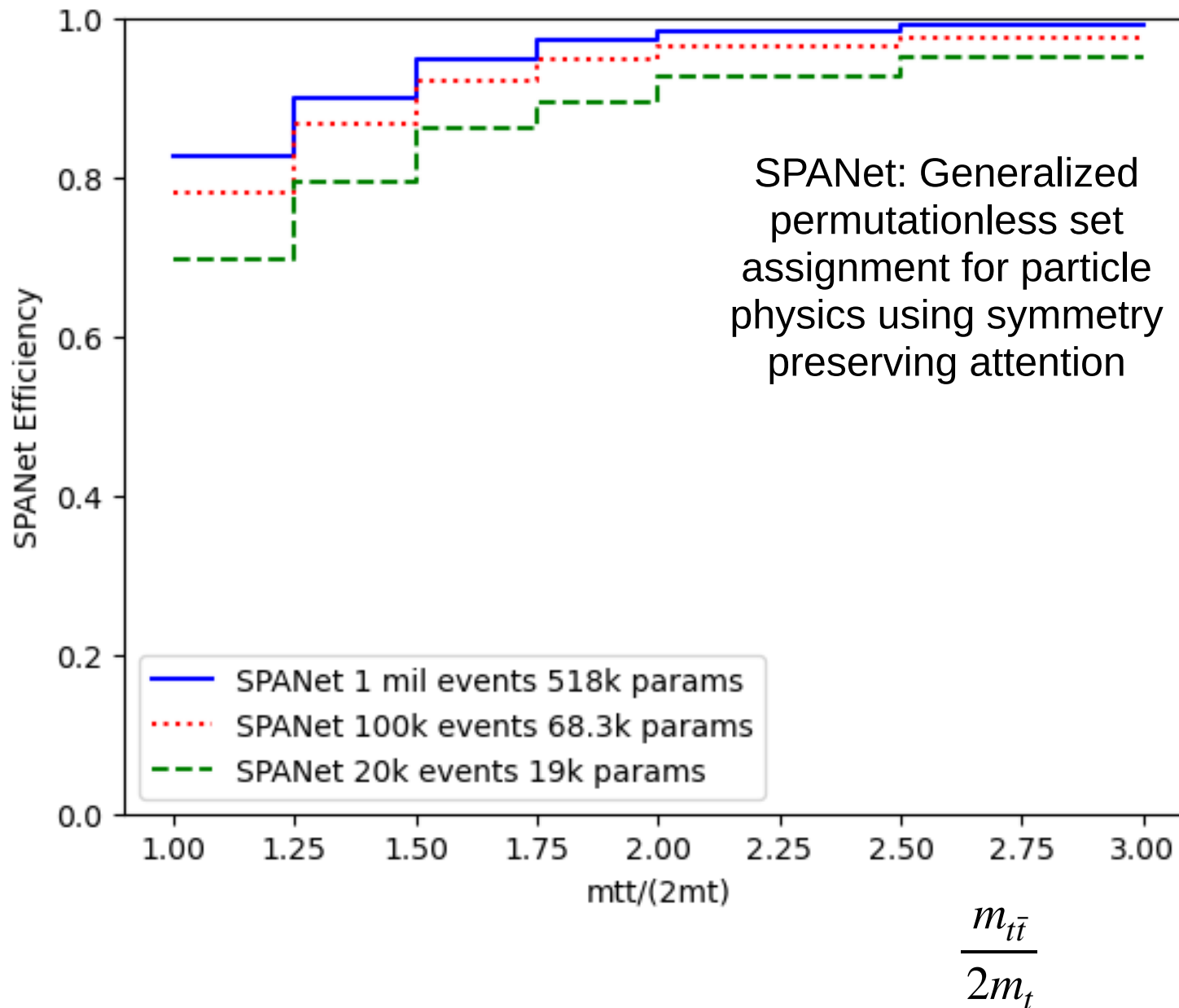
78% overall efficiency

No mass information used.

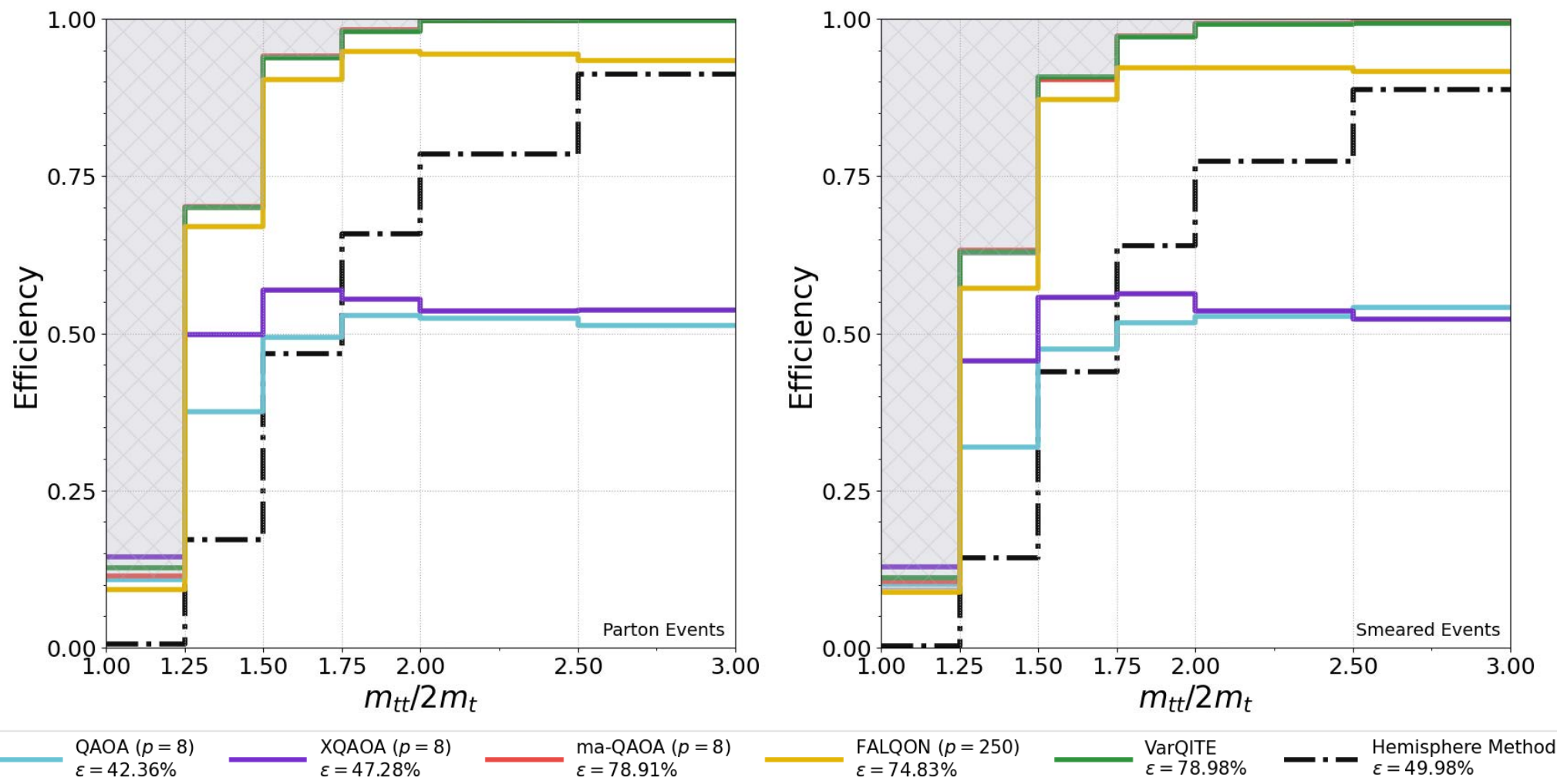
Combinatorial problems in the top quark production



Combinatorial problems in the top quark production



Detector effects

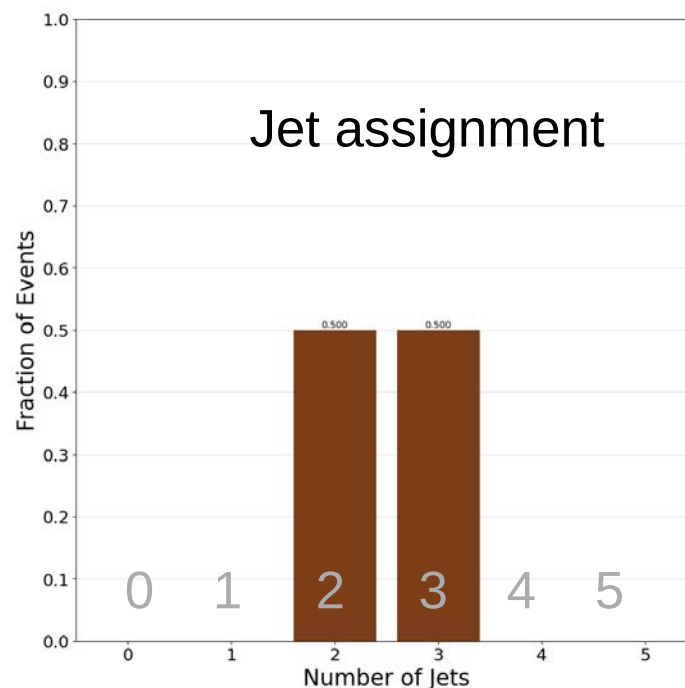
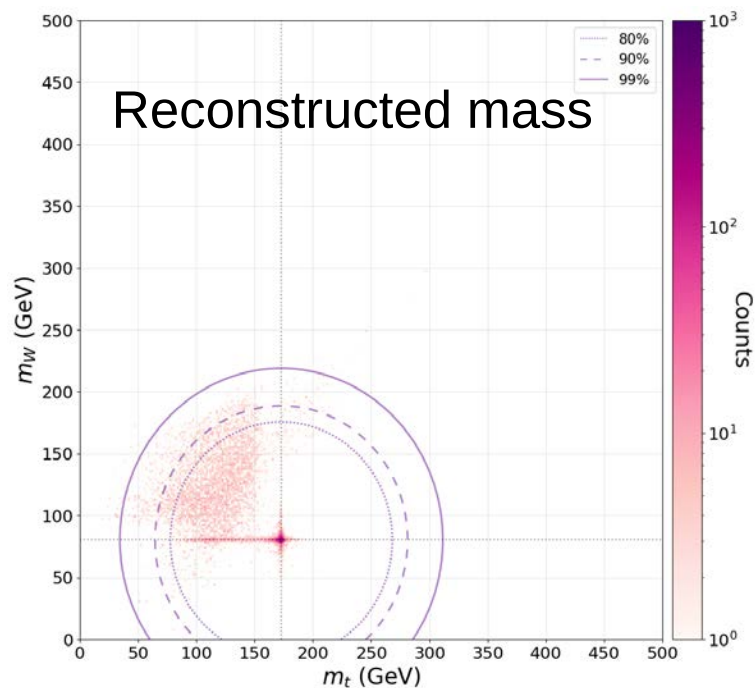


$$\frac{\sigma}{E} = \sqrt{\left(\frac{N}{E}\right)^2 + \left(\frac{S}{\sqrt{E}}\right)^2 + C^2},$$

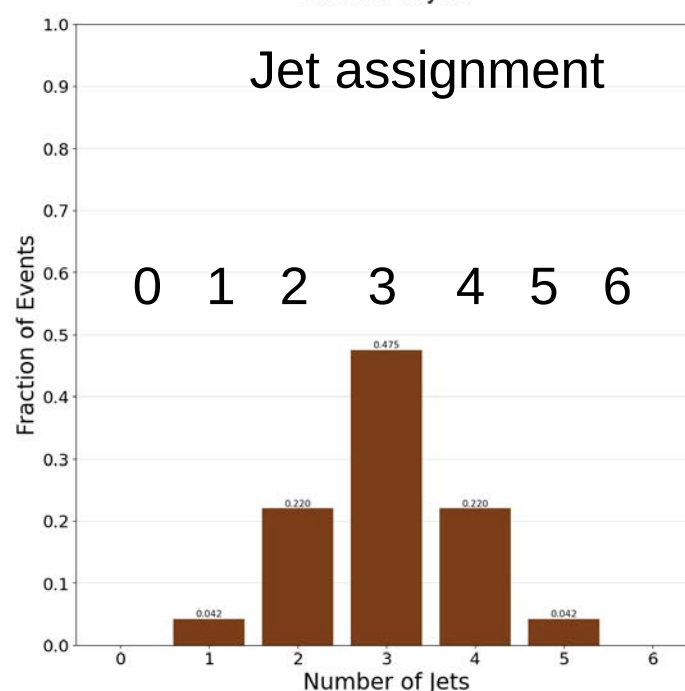
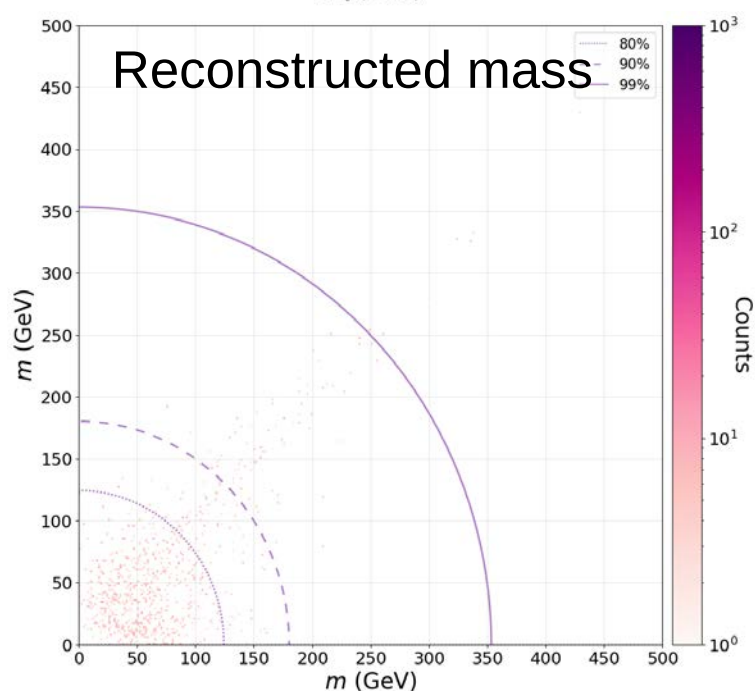
$$N=5.3, S=0.74, C=0.05$$

tW and multi-jet production

ma-QAOA with $p=8$
parton-level events

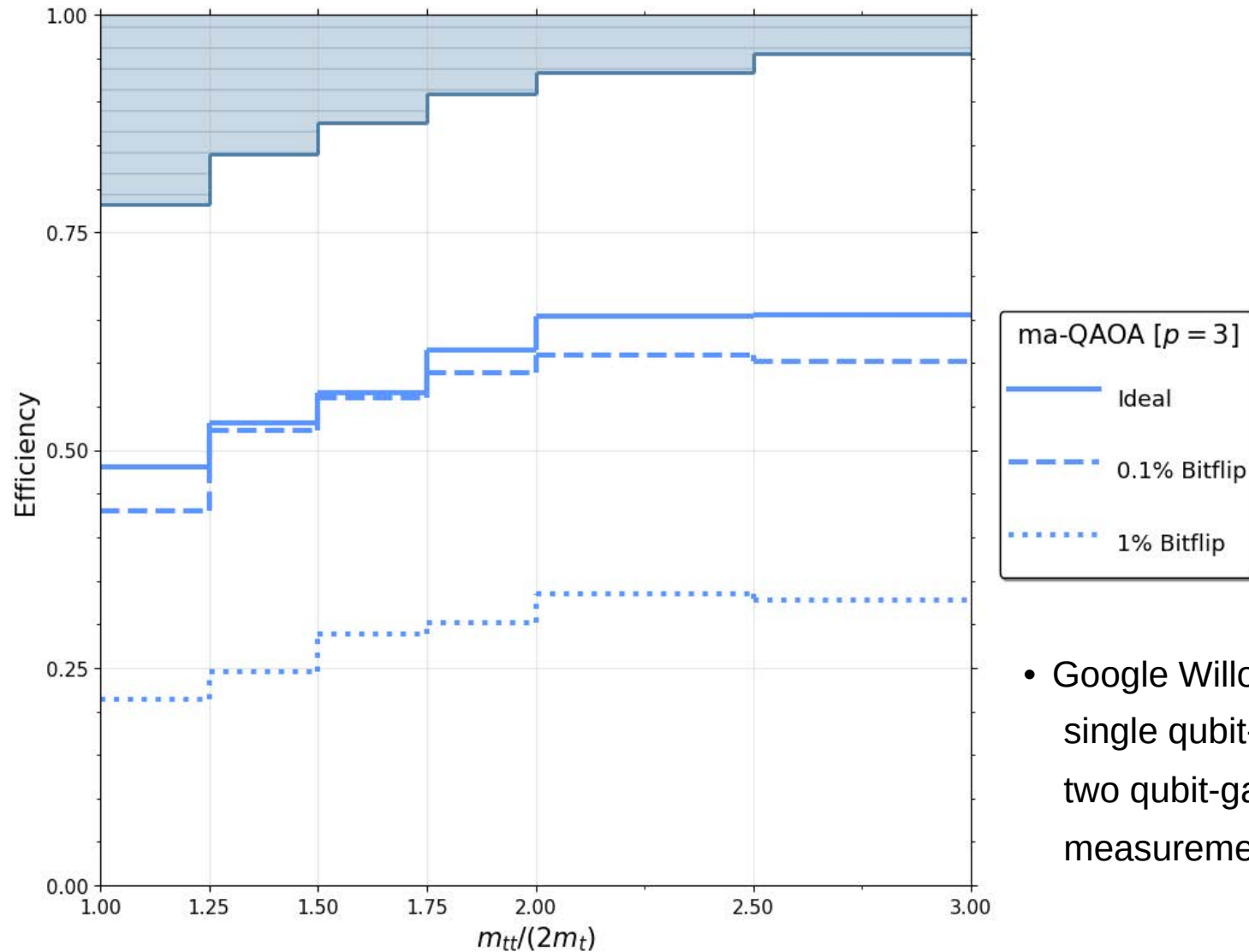


tW (5 jets)



multi-jet (6 jets)

Quantum noises



- Google Willow
single qubit-gate error $\sim 3 \times 10^{-4}$
two qubit-gate error $\sim 3 \times 10^{-3}$
measurement error $\sim 7 \times 10^{-3}$

Quantum Enhancement of the Signal Rate

U_{DM} induces pure phase rotation of its eigenstates

E.g. for $\alpha = 0$: $U_{\text{DM}} \simeq \begin{pmatrix} 1 & i\delta \\ i\delta & 1 \end{pmatrix}$ with $\delta \equiv \eta\tau$

$$\Rightarrow U_{\text{DM}}|\pm\rangle = e^{\pm i\delta}|\pm\rangle \quad \text{with} \quad |\pm\rangle \equiv \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$$

$$\Rightarrow U_{\text{DM}}^{\otimes N_q} |+\rangle^{\otimes N_q} = e^{iN_q\delta} |+\rangle^{\otimes N_q}$$

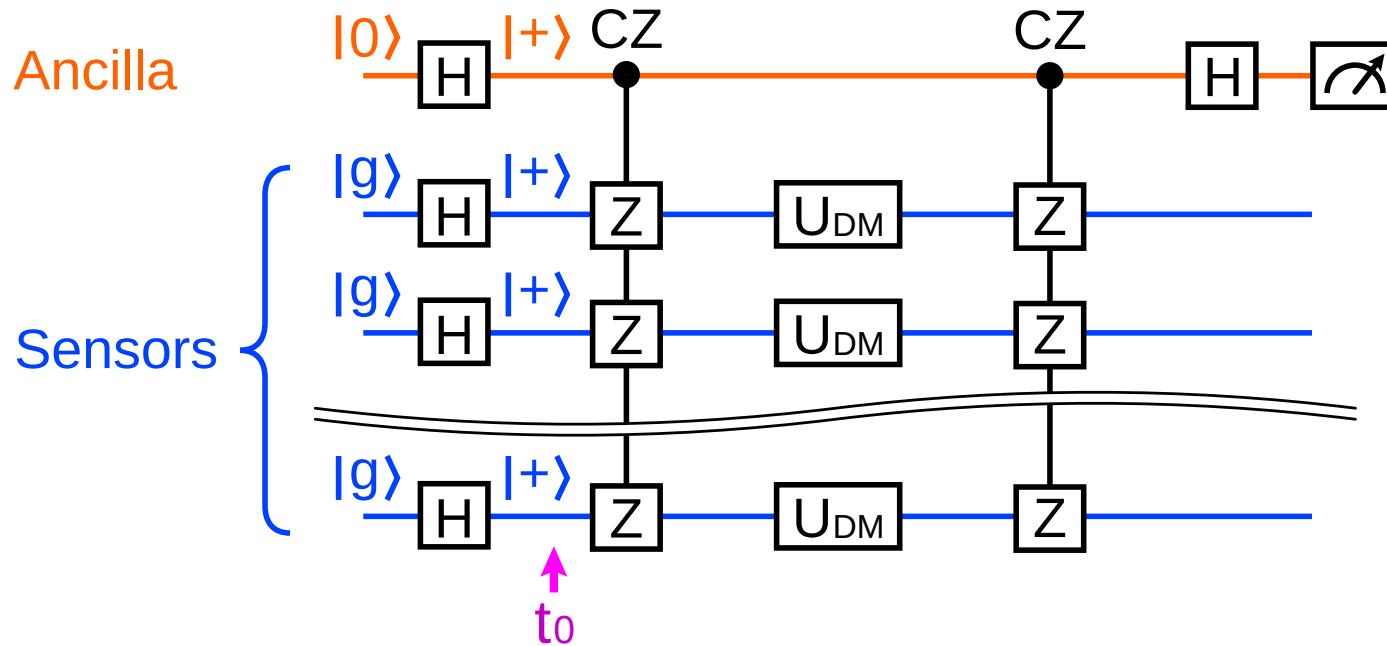
We can design quantum operations to enhance the signal

$\Rightarrow$ Quantum enhanced parameter estimation

[Giovannetti, Lloyd, Maccone ('04)]

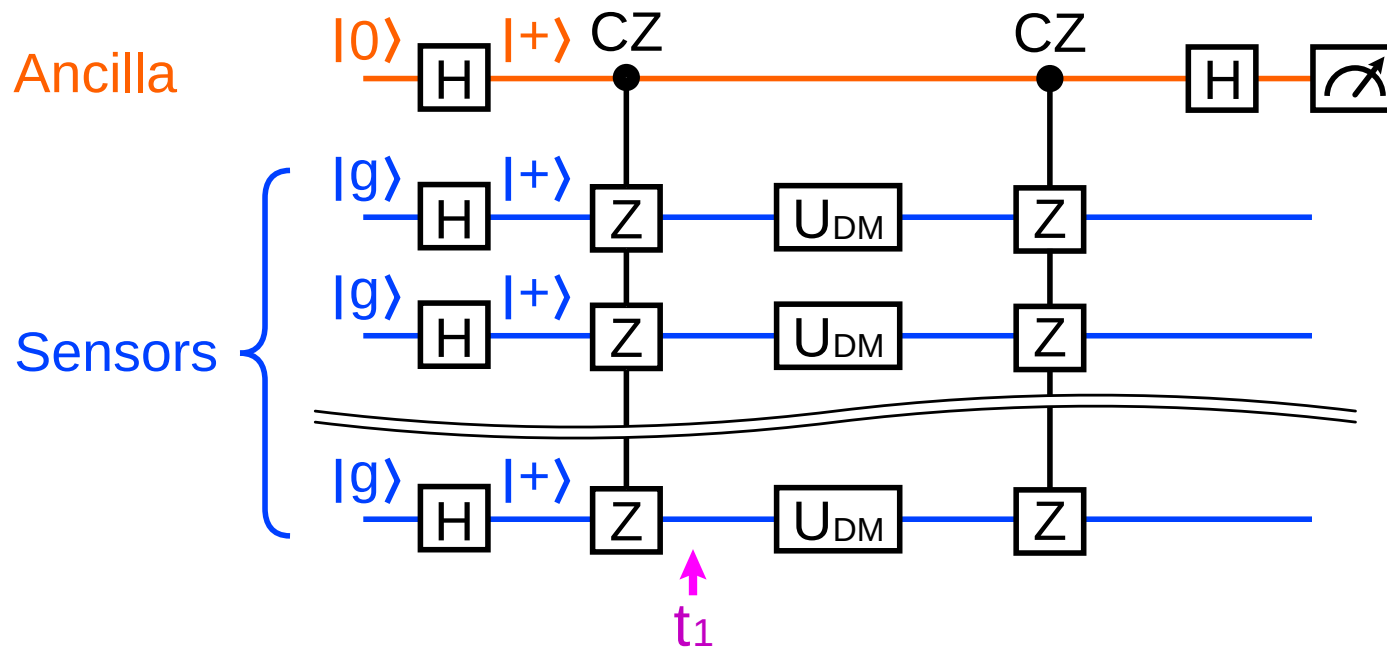
Quantum Enhancement of the Signal Rate

One measurement cycle for the signal enhancement



$$|\Psi(t_0)\rangle = |+\rangle \otimes |+\rangle^{\otimes N_q} = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}}|1\rangle \otimes |+\rangle^{\otimes N_q}$$

One measurement cycle for the signal enhancement



$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle^{\otimes N_q}$$

- Controlled Z gate

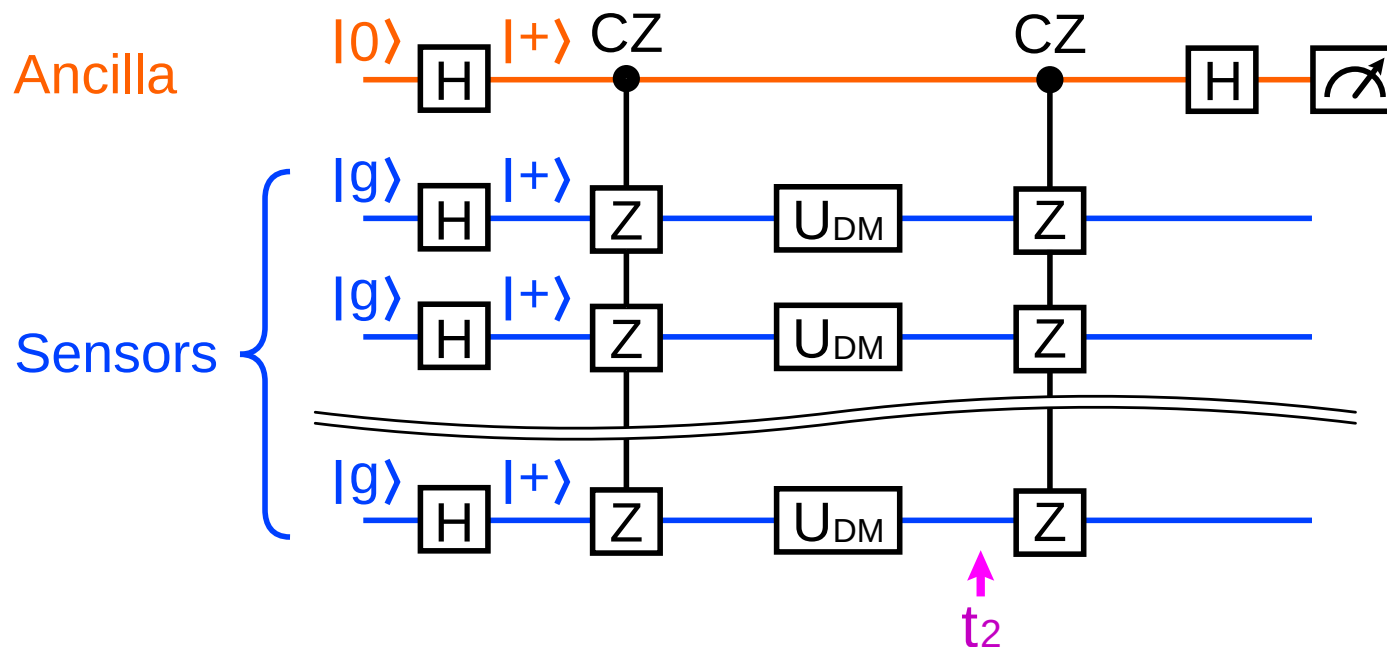
$$CZ = |0\rangle\langle 0| \otimes \mathbf{1} + |1\rangle\langle 1| \otimes Z$$

$$\Rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |+\rangle \xrightarrow{CZ} \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle$$

Shion Chen, Hajime Fukuda, Toshiaki
Inada, Takeo Moroi, Tatsumi Nitta,
Thanaporn Sichanugrist, 2024 PRL

Quantum Enhancement of the Signal Rate

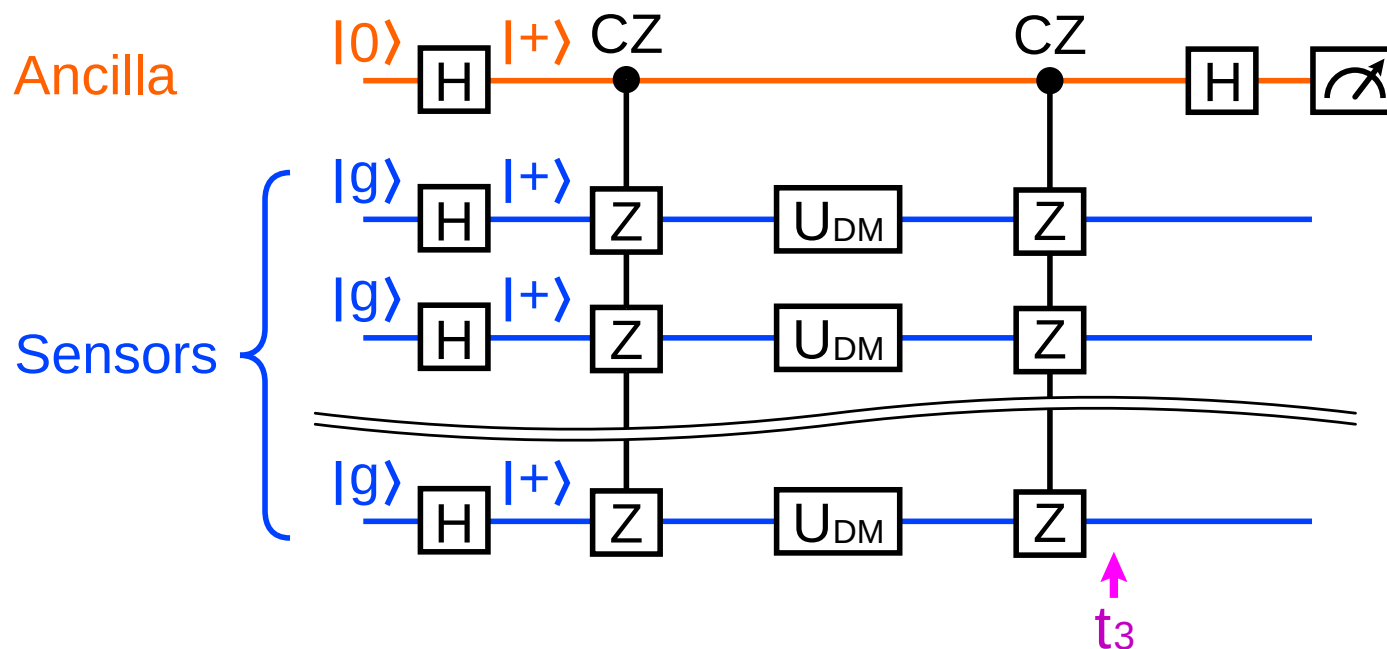
One measurement cycle for the signal enhancement



$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}} e^{iN_q\delta} |0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta} |1\rangle \otimes |-\rangle^{\otimes N_q}$$

Quantum Enhancement of the Signal Rate

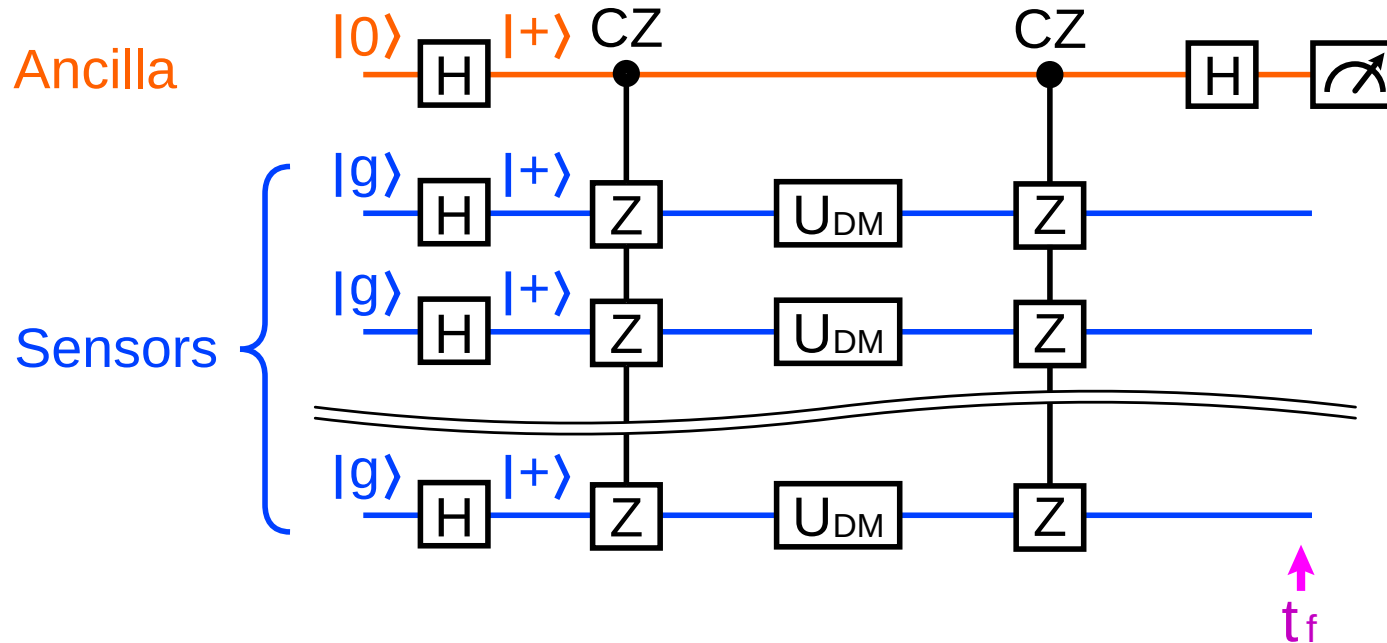
One measurement cycle for the signal enhancement



$$\begin{aligned}
 |\Psi(t_3)\rangle &= \frac{1}{\sqrt{2}} e^{iN_q\delta} |0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta} |1\rangle \otimes |+\rangle^{\otimes N_q} \\
 &= \left(\cos N_q\delta |+\rangle + i \sin N_q\delta |-\rangle \right) \otimes |+\rangle^{\otimes N_q}
 \end{aligned}$$

Quantum Enhancement of the Signal Rate

One measurement cycle for the signal enhancement



$$|\Psi(t_f)\rangle = \left(\cos N_q \delta |0\rangle + i \sin N_q \delta |1\rangle \right) \otimes |+\rangle^{\otimes N_q}$$

$\Rightarrow$ Ancilla qubit can be excited: $P_{0 \rightarrow 1} \simeq \sin^2 N_q \delta \simeq N_q^2 \delta^2$

A simplified Setup: signal-sensor interaction

- Core setup for many quantum sensing protocols in HEP (Details vary, but the essential physics is often similar)
- **Sensor: A Two-Level Quantum System**
 - A qubit as a simple example
 - Two states: Ground $|0\rangle$ and Excited $|1\rangle$
 - Can be a physical qubit, a cavity mode ($n = 0, n = 1$) etc.
- **Signal: A Weak Classical Field**
 - E.g., wave-like DM, high-frequency GWs
 - Drive transitions from $|0\rangle$ to $|1\rangle$
 - Interaction Hamiltonian:
$$H_I = \epsilon (\cos \alpha \sigma_X + \sin \alpha \sigma_Y)$$
 - ϵ : Signal strength (the quantity to be estimated)
 - α : Signal phase (often stochastic/random)
- **The Goal to estimate the signal strength ϵ .**
 1. Evolve: System evolves from $|0\rangle$ under H_I . For a weak signal $\epsilon t \ll 1$,
$$|\psi(t)\rangle \approx |0\rangle - ie^{i\alpha} \epsilon t |1\rangle$$
 2. Measure: Project onto $|0\rangle$ and $|1\rangle$. Probability of finding $|1\rangle$: $p_1 = \left| \langle 1 | \psi(t) \rangle \right|^2 = \epsilon^2 t^2$

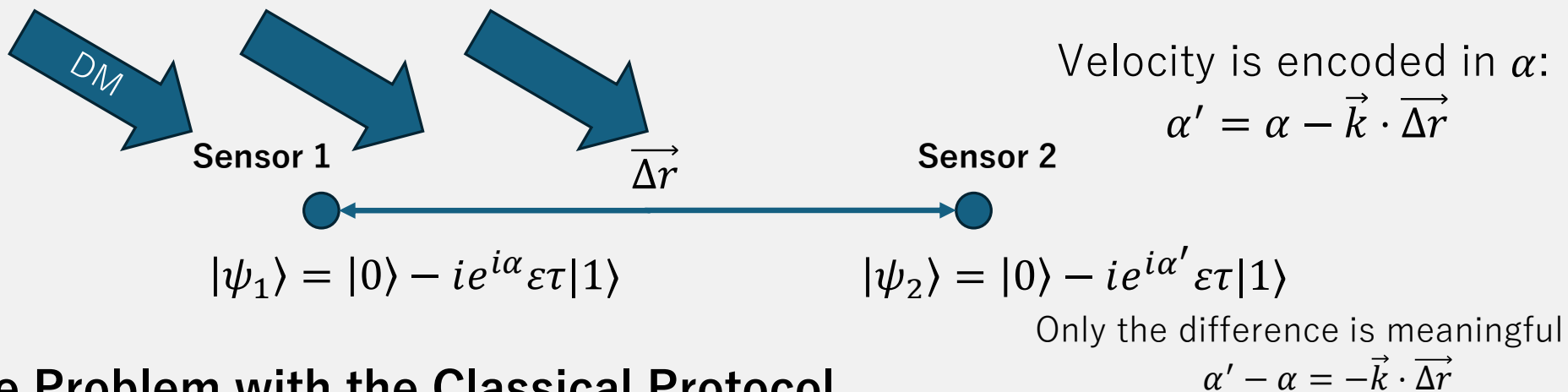
Quantum networks for dark matter flows

Measuring Dark Matter Velocity

Classical Protocol [Derevianko '18]

For simplicity, I consider only two sensors here

1. **Interaction:** The DM signal field interacts with two sensors
2. **Measurement:** Measure an observable σ_x , for each sensor independently:
 $\langle \sigma_x \rangle = 2\varepsilon\tau \sin \alpha$
3. **Correlation:** Look for **coincident events** where both sensors show a response, and correlate the outcomes.



The Problem with the Classical Protocol

- α fluctuates randomly, preventing simple time averaging
- To extract correlation, we need a **coincident detection event** (i.e., both sensors "click").
- The probability of a single click is $p_1 \sim (\varepsilon\tau)^2 \Rightarrow p_1 p_2 \sim (\varepsilon\tau)^4$

Measuring Dark Matter Velocity

Quantum Protocol: Interference with distributed sensors

Step 1: Entangling Measurement (Projection)

- Two-sensor state: $|\psi_1 \otimes \psi_2\rangle = (|0\rangle - ie^{i\alpha}\varepsilon\tau|1\rangle) \otimes [|0\rangle - ie^{i(\alpha - \vec{k} \cdot \vec{\Delta r})}\varepsilon\tau|1\rangle]$
- Measure $P = |10\rangle\langle 10| + |01\rangle\langle 01|$, which succeeds with probability $\langle P \rangle = 2\varepsilon^2\tau^2$
- State after measurement: $P|\psi_1 \otimes \psi_2\rangle \sim |10\rangle + e^{-ik\Delta r}|01\rangle \equiv |\Psi\rangle$

Step 2: Information Readout

- Measure $M = -i|01\rangle\langle 10| + i|10\rangle\langle 01|$
- $\langle M \rangle = -\sin k\Delta r$

Measuring a Projection Operator P

$$|\phi\rangle \rightarrow \begin{cases} P|\phi\rangle & \text{with probability } p = \langle \phi | P | \phi \rangle \\ (1 - P)|\phi\rangle & \text{with probability } 1 - p \end{cases}$$

e.g. Measure $P_1 = |1\rangle\langle 1|$ for one qubit state $|\varphi\rangle$
After measurement: $|1\rangle$ with $p = |\langle \varphi | 1 \rangle|^2$ or $|0\rangle$

State of the art: Scalable Quantum Network

4

Color center-based Quantum repeater network by two leaders, R. Hanson in QuTech, Netherlands and M. Lukin in MIT, USA

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Entanglement of nanophotonic quantum memory nodes in a telecom network

[C. M. Knaut](#), [A. Suleymanzade](#), [Y.-C. Wei](#), [D. R. Assumpcao](#), [P.-J. Stas](#), [Y. Q. Huan](#), [B. Machielse](#), [E. N. Knall](#), [M. Sutula](#), [G. Baranes](#), [N. Sinclair](#), [C. De-Eknamkul](#), [D. S. Levonian](#), [M. K. Bhaskar](#), [H. Park](#), [M. Lončar](#) & [M. D. Lukin](#) ✉

[Nature](#) **629**, 573–578 (2024) | [Cite this article](#)

Nature **629**, 573 (2024)

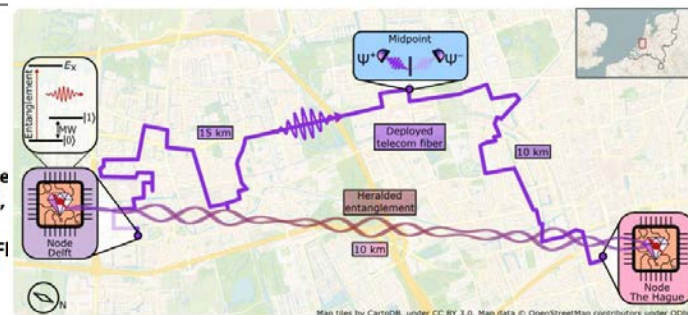
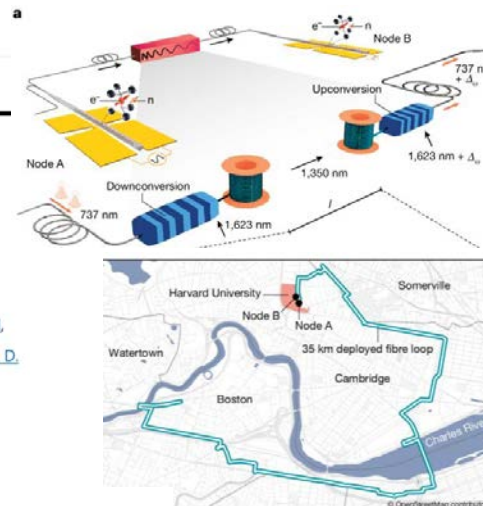
SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Metropolitan-scale heralded entanglement of solid-state qubits

[Arian J. Stolk](#)^{1†}, [Kian L. van der Enden](#)^{1†}, [Marie-Christine Slater](#)^{1‡}, [Ingmar te Raa-De Pieter Botma](#)¹, [Joris van Rantwijk](#)¹, [J. J. Benjamin Biemond](#)^{1,2}, [Ronald A. J. Hagen](#)^{1,2}, [Rodolf W. Herfst](#)^{1,2}, [Wouter D. Koek](#)^{1,2}, [Adrianus J. H. Meskers](#)^{1,2}, [René Vollmer](#)^{1,2}, [Erwin J. van Zwet](#)^{1,2}, [Matthew Markham](#)³, [Andrew M. Edmonds](#)³, [J. Fabian Geus](#)^{4,5}, [Fl. Bernd Jungbluth](#)⁴, [Constantin Haefner](#)^{4,5}, [Christoph Tresp](#)⁶, [Jürgen Stuhler](#)⁶, [Stephan Ritter](#)⁶, [Ronald Hanson](#)^{1*}

Sci. Adv. **10**, eadp6442(2024).

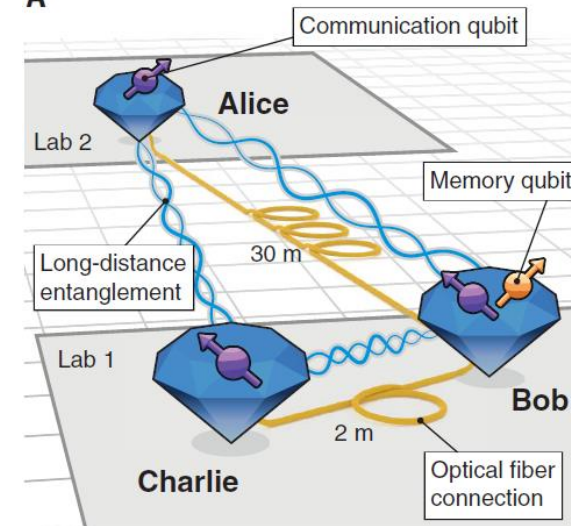


QUANTUM NETWORKS

Realization of a multinode quantum network of remote solid-state qubits

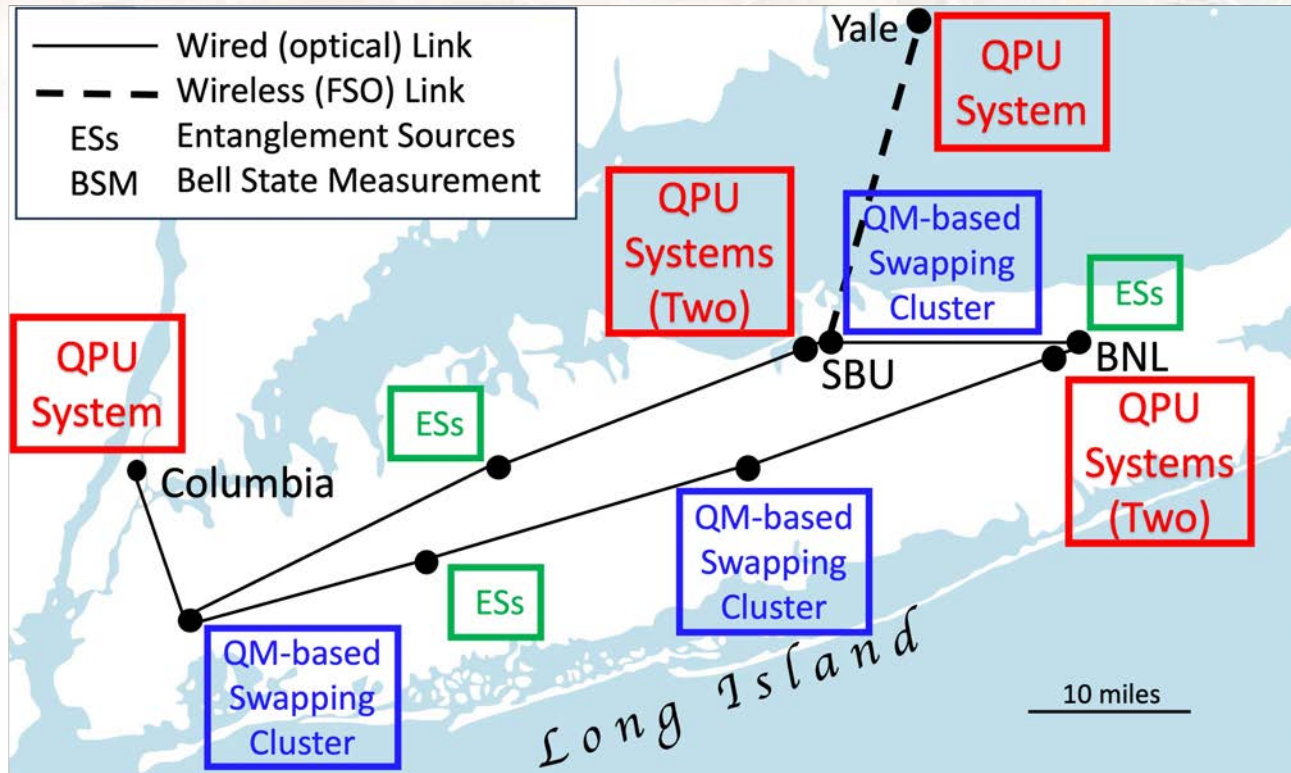
[M. Pompili](#)^{1,2†}, [S. L. N. Hermans](#)^{1,2†}, [S. Baier](#)^{1,2†}, [H. K. C. Beukers](#)^{1,2}, [P. C. Humphreys](#)^{1,2§}, [R. N. Schouten](#)^{1,2}, [R. F. L. Vermeulen](#)^{1,2}, [M. J. Tiggeleman](#)^{1,2¶}, [L. dos Santos Martins](#)^{1,2}, [B. Dirkse](#)^{1,2}, [S. Wehner](#)^{1,2}, [R. Hanson](#)^{1,2*}

A



Science **372**, 259 (2021)

SCY QNet Requirement 3: Interconnecting QPUs using QRs



Technical readiness:

- Building QPUs based of atomic traps for Rb, Sr and Yb.
- Demonstrated QFC between telecom and IR photons.

SCY-QNet goals:

- Create and transmit atom-photon entanglement from the QPUs at O-band.
- Perform entanglement swapping between the QPU qubits and the QM Banks.

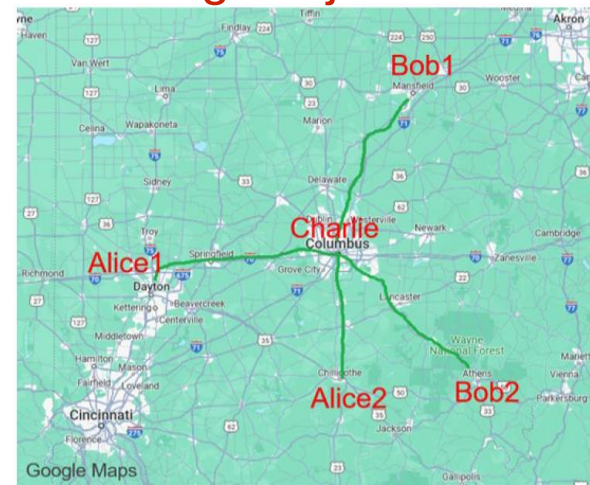
CQISE Quantum Networking Project



P2P QKD



MDI QKD CONCEPT



STAR NETWORK MDI QKD CONCEPT

Emerging partnerships

- AFIT
- NGRC
- Cisco
- Honda
- Huntington Bank
- Columbus State
- Starlab

CQISE Quantum Networking Projects

Creating an intercity network for quantum information science and engineering in the State of Ohio

- Congressionally Funded Research Project (CFRP), Department of Education, 2023 - 2026, \$1M
- PI: Ronald M. Reano, Co-PI: Ezekiel Johnston-Halperin, OARnet: Pankaj Shah, Mark Fulmer

Creating long distance quantum networks for cybersecurity in the state of Ohio

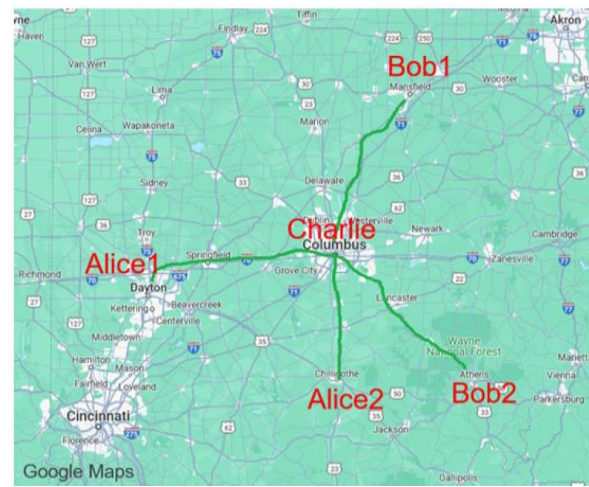
- Ohio Third Frontier Research Incentive, Ohio Department of Higher Education (ODHE), 2024 – 2025, \$750k
- PI: Ronald M. Reano, Co-PI: Ezekiel Johnston-Halperin, OARnet: Pankaj Shah, Mark Fulmer



P2P QKD



MDI QKD CONCEPT



STAR NETWORK MDI QKD CONCEPT

Emerging partnerships

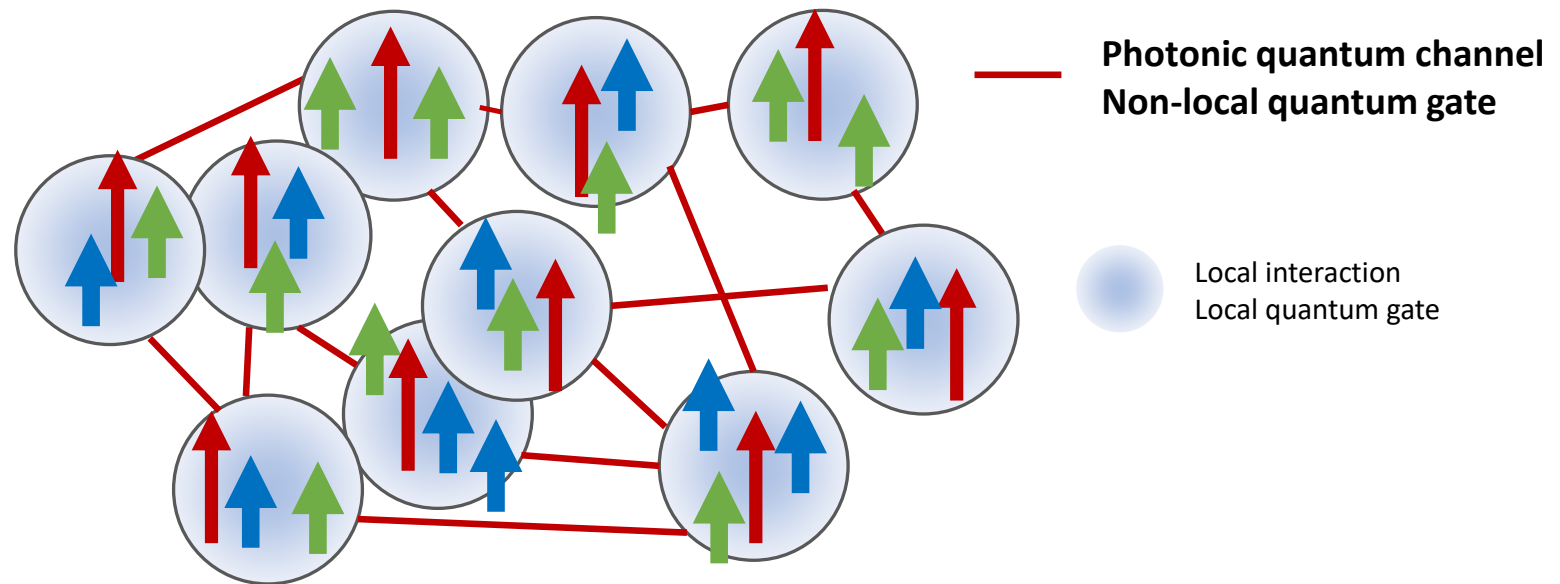
- AFIT
- NGRC
- Cisco
- Honda
- Huntington Bank
- Columbus State
- Starlab



THE OHIO STATE UNIVERSITY

CENTER FOR QUANTUM INFORMATION
SCIENCE AND ENGINEERING

Quantum network: From one big noisy chip to many quiet chips with photonic links.



- ❑ Distributed quantum computing & Quantum computing modular architecture
- ❑ Quantum communication (quantum internet, quantum repeater network)

Colloquium: Quantum Networks with Trapped Ions. Rev. Mod. Phys. (2010)

Scaling the Ion Trap Quantum Processor. Science (2013)

Technical roadmap for fault-tolerant quantum computing, NQIT Technical Roadmap (2016)

Material platforms for spin-based photonic quantum technologies, Nat. Rev. Mater. (2018)

Quantum Internet: A vision for the road ahead, Science (2018)

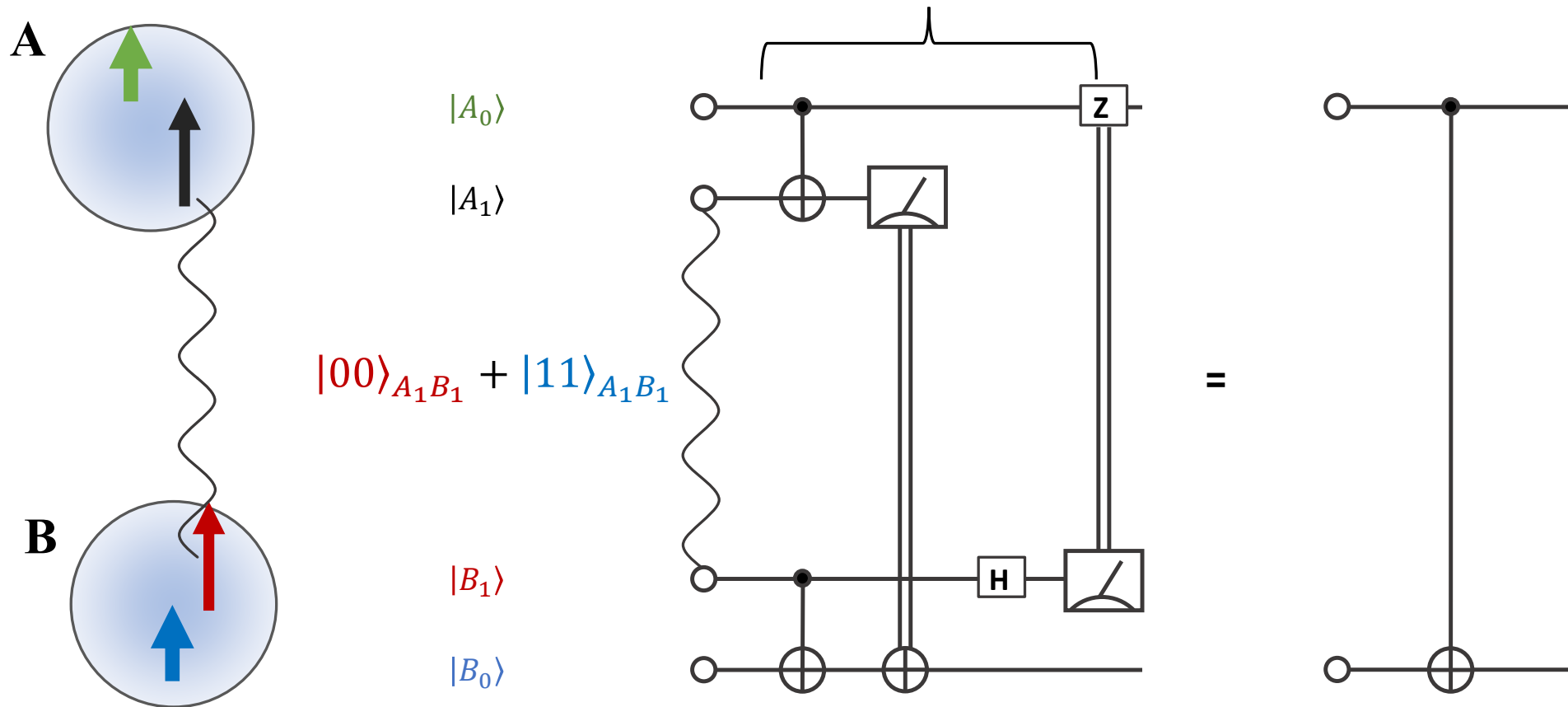
Problems with one big chip

- ❑ As N grows, control lines $\propto N$, noise/crosstalk $> N$, uniformity $\downarrow$
- ❑ I/O fan-out $\uparrow$
- ❑ Crosstalk $\uparrow$
- ❑ Calibration overhead $\uparrow$
- ❑ classical/quantum noise $\uparrow$

Nonlocal two qubit gate

7

Remotely entangled qubit pair + Local operations = Nonlocal CNOT gate

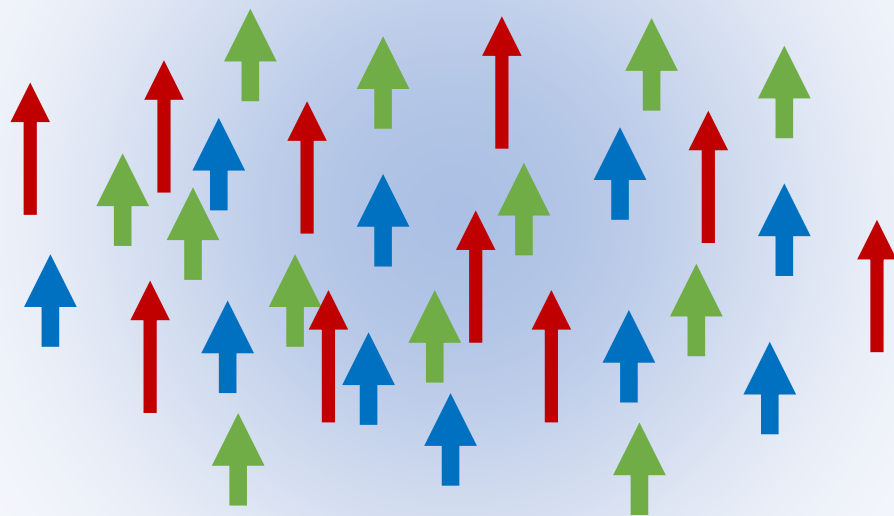


Distributed Quantum Computation Based on Small Quantum Registers. Phys. Rev. A 2007
Optimal Local Implementation of Nonlocal Quantum Gates. Phys. Rev. A 2000

Why one big chip isn't enough

5

Integrated qubits in a quantum node



Local interaction
Local quantum gate

- ❑ I/O fan-out $\uparrow$
- ❑ Crosstalk $\uparrow$
- ❑ Calibration overhead $\uparrow$
- ❑ classical/quantum noise $\uparrow$

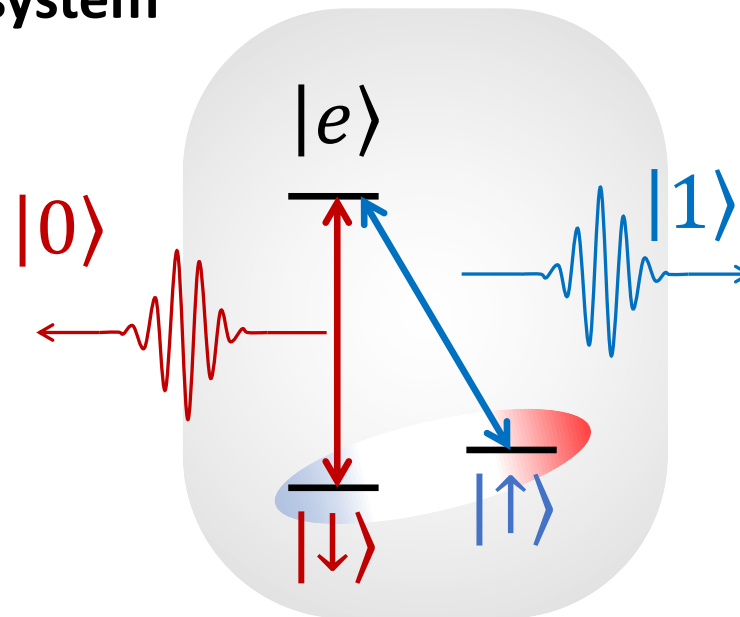
❑ As N grows, control lines $\propto N$, noise/crosstalk $> N$, uniformity $\downarrow$

Colloquium: Quantum Networks with Trapped Ions. Rev. Mod. Phys. (2010)
Scaling the Ion Trap Quantum Processor. Science (2013)

Spin-to-photon interface

17

Atom, or atom-like system



- Single photon source
- Quantum memory
- Correlation between them

$$|\downarrow\rangle + |\uparrow\rangle \rightarrow |\downarrow\rangle|0\rangle + |\uparrow\rangle|1\rangle$$

Spin-to-photon entanglement