

From Bits to Qubits: A Beginner's Journey into Quantum Computing



공경철 / K.C. Kong
University of Kansas



Pusan National University
October 27, 2025



부산대학교
PUSAN NATIONAL UNIVERSITY

- Disclaimer: I am NOT a computer scientist nor an expert on quantum computing

About me

- I am a theorist (not an experimentalist) who works on particle physics
 - U of Florida → Fermilab → SLAC → U of Kansas
 - Work closely with experimentalists
 - Member of Deep Underground Neutrino Experiment (DUNE) and QC4HEP and MLQC4FC working group
- My research focuses on new physics beyond the standard model (extra dimensions, supersymmetry, dark matter, Higgs, new forces, new matter particles, collider physics, cosmology/N-body simulations etc)
- Do a lot of MC simulation and data analysis, in addition to analytical calculations
- Use AI/ML tools and quantum algorithms for applications in particle physics (and other areas)
- Started working on quantum computing, after finding out quantum field theory and group theory are used for error correction and barren plateaus problem
- Anyone can think about application of quantum computers within their own discipline and beyond. Domain expertise is very important. Collaboration between math, physics and engineering is crucial.
- Mentor middle school students in Lego robotics
- Mentor students via google summer of codes program (for quantum machine learning)
- Help develop an App for Monarch Butterflies research (for tagging/recovery)
 - Ask me about this, if interested. I can use your help!



Menu

AIRBUS

Quantum technologies

A potential game-changer in aerospace





Menu

AIRBUS



Quantum technologies

A potential game-changer in aerospace

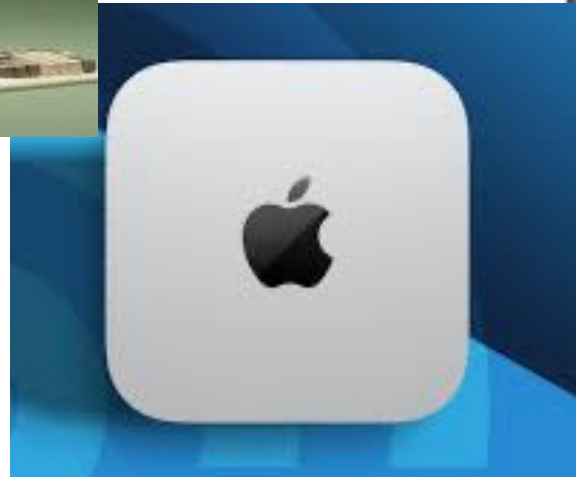
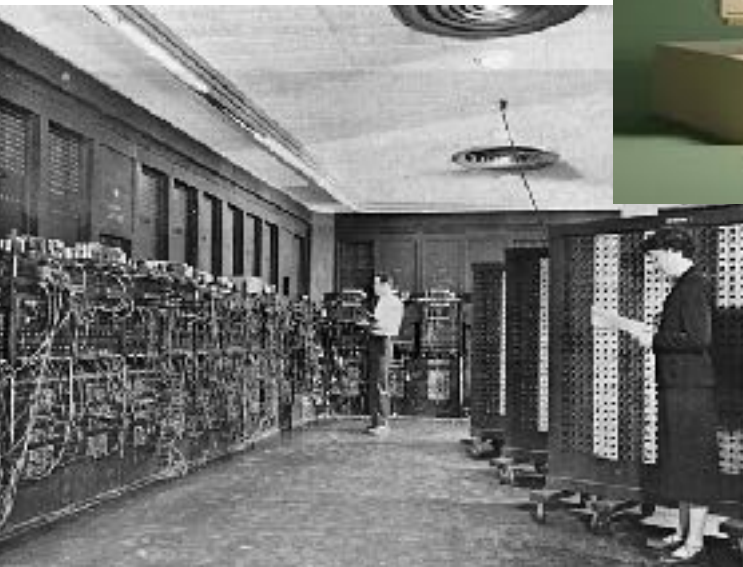
Plan

- Introduction
 - What is a quantum computer?
 - Why quantum computing?
 - Simple examples
- Applications in particle physics
 - Quantum optimization
 - Qubits as sensors for radiative decays of neutrinos and dark matter particles
 - Hamiltonian Simulations: Gross-Neveu model
 - Quantum entanglement and violation of Bell's inequality at collider experiments
- Summary

What is a quantum computer?

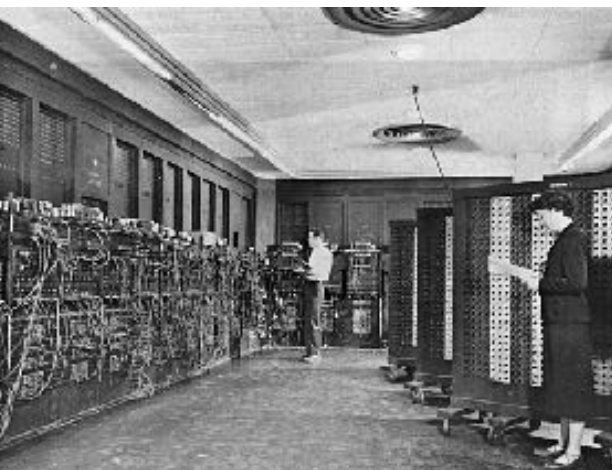
- A quantum computer is a new kind of computer that's based on the laws of **quantum physics**.
- It can do **certain things** faster than normal computers because it follows a different set of rules.

ENIAC (US Army, 1945)



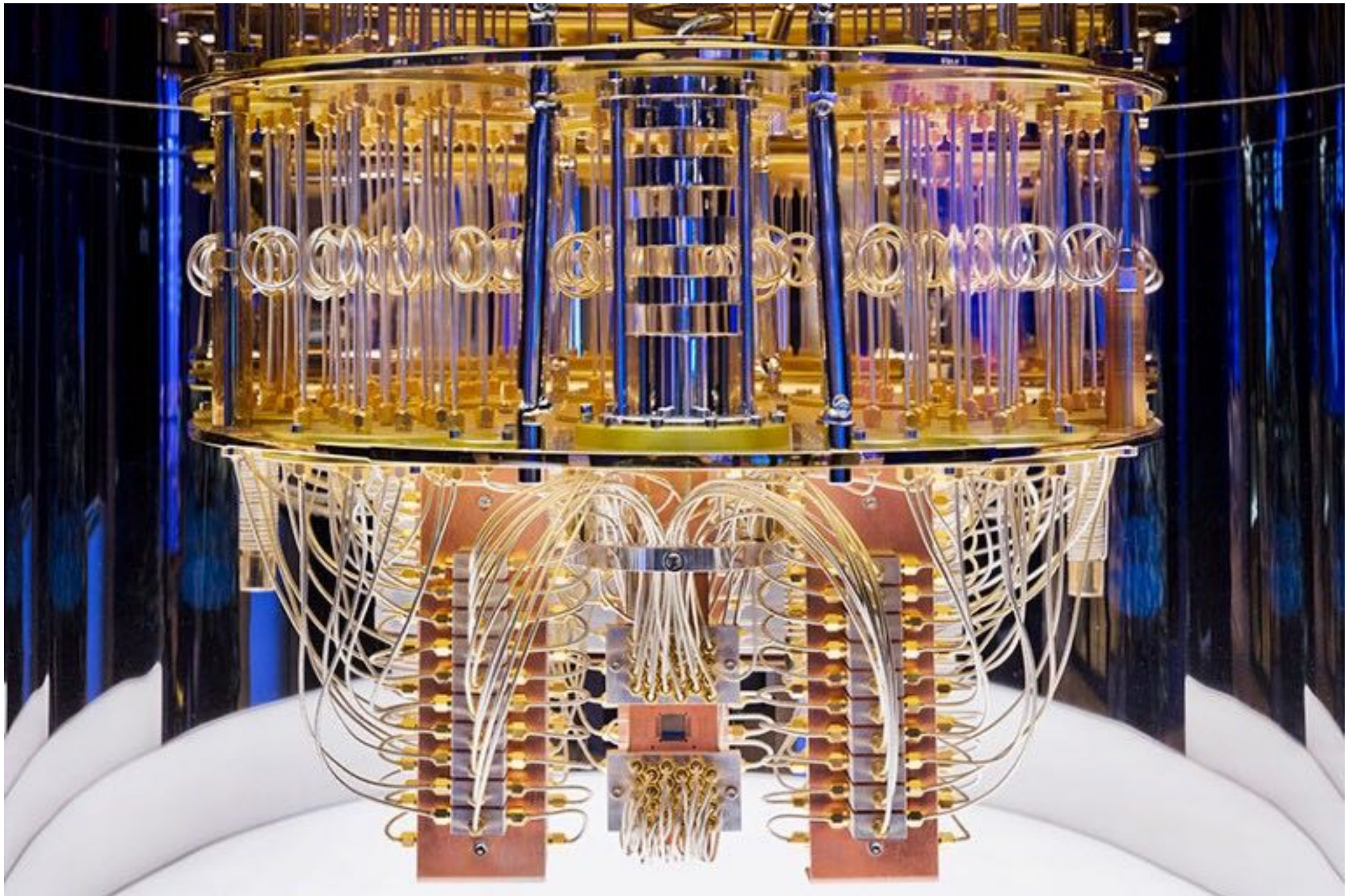


Eagle 127 qubit
IBM Quantum computer
Tens of million dollars
(depending on service contract)



ENIAC (US Army),
**~\$500k (Equivalent
to ~\$7million)**





IBM cloud service charges

Subscription time at National Energy Research Scientific Computing Center (NERSC)
supported by US Department of Energy

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[01 Access plans](#)[02 Compare features](#)[03 Technical services](#)[04 Startup Program](#)[05 Case studies](#)

For getting started

Open Plan

Access quantum computers for up to 10 minutes of runtime per month.

Free

[Sign up](#)[View features](#)

For small-scale work

Pay As You Go Plan

Pay for the quantum computer access you use. Billed per second of usage.

Starts at \$96 USD / minute

[Sign up](#)[View features](#)

For project-based work

Flex Plan

Pre-purchase time for projects that require execution flexibility or bursts of sustained usage.

Starts at \$72 USD / minute

[Contact us](#)[View features](#)

For work at scale

Premium Plan

Subscribe for large-scale capacity with Quantum Allocation Units (QAUs). Use up to 1,600 minutes per QAU every 28 days.

Starts at \$48 USD / minute

[Contact us](#)[View features](#)

For unparallelized access

Dedicated Service

Access to a dedicated on-prem quantum system that is serviced and maintained by IBM Quantum.

Price requires quote

[Contact us](#)[View features](#)



IBM Quantum System Two



부산 세가사미 퀀텀 랜드마크 타워 조감도 및 개요



구분	내용
사업명	부산 우동 1502 복합개발
위치	부산 해운대구 우동 1502번지 일대
대지면적	9,911.2㎡ (2,998평)
연면적	182,372.8㎡ (55,168평)
건폐율/용적률	52.7% / 1,078.9%
규모	지하 6층 - 지상 60층, 1개 동
사업기간	2025~2030년
사업비	약 1조 3500억 원
사업자	파이어아이 하인즈 퀀텀 랜드마크 타워 부산원반사모부동산투자회사

IBM과 한국퀀텀컴퓨팅(KQC) 협력 내용

양자컴퓨터

- 2028년 부산에 IBM 양자컴퓨터 도입 예정
- 여러 양자 모듈 연결해 1000큐비트 이상 성능 예상

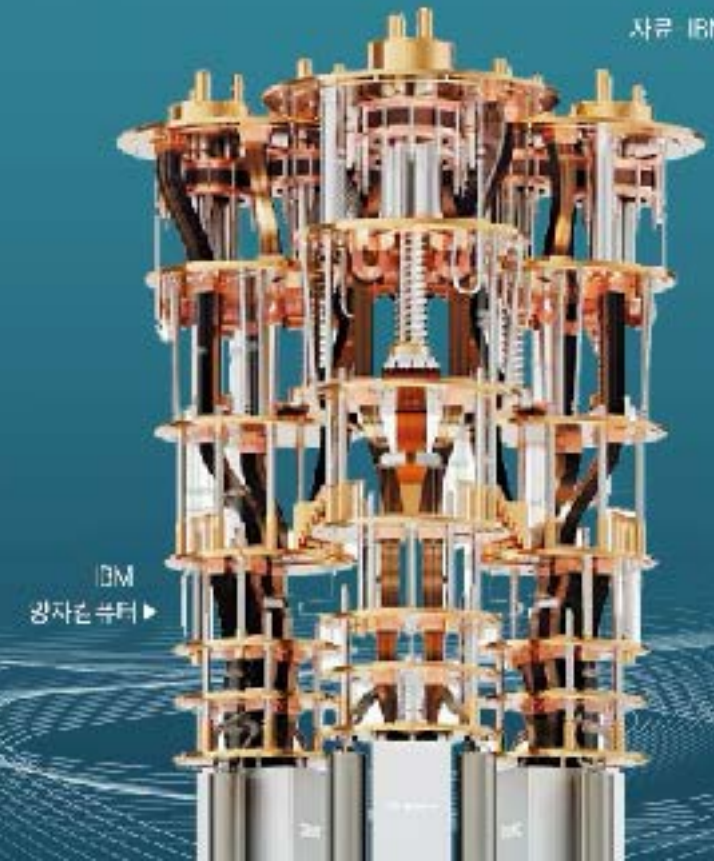
인공지능(AI)

- 올 하반기 GPU와 IBM 자체 칩 수백개 적용한 AI 센터 가동
- IBM의 최신 AI 플랫폼 '왓슨x'제공

기업들 금융, 신약 개발 등에 활용 가능

기업용 생성형 AI 기술 개발

자료 IBM KQC



What is a quantum computer?

- A quantum computer is a new kind of computer that's based on the laws of **quantum physics**.
- It can do **certain things** faster than normal computers because it follows a different set of rules.
- Quantum computers are NOT expected to replace traditional computers for everyday use. Quantum computers excel at specific types of calculations that are intractable for classical computers, but they are not a general-purpose replacement. Classical computers will remain essential for most tasks, while quantum computers will be used for specialized applications.
- Quantum Junction at the International Conference for High Performance Computing (November 16-21, 2025)

Quantum Computing: A Soccer Analogy

By Thomas Wong



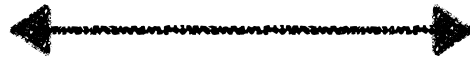
- If the rules were changed so that players could use their hands, soccer would look very different. In some cases, it would be faster or easier for athletes to use their hands to hold, throw, or catch the ball. In other cases, however, it would still be better to kick the ball. For example, most soccer players can kick the ball across the field faster than they can throw the ball or run with it. So, to get the ball across the field as quickly as possible, it may still be better to use one's feet.
- Analogously, the essence of quantum computing is to change the rules so that a computer can now use its “hands.” That is, the rules of the game are changed from the laws of classical physics to the laws of quantum physics. As a result, a quantum computer can solve some problems faster by using its “hands.” For other problems, using one's “feet” is better, so a quantum computer is no faster for these problems.

Digital Computing vs Quantum Computing

Use binaries to perform calculations and solve problems

Use QM / qubits to solve problems

Bit: $x \in \{0,1\}$



Qubit: $|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} \end{pmatrix}$

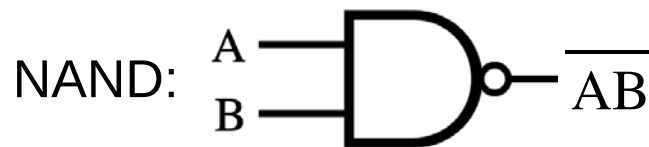
Digital computation with n bits:
 $\{0,1\}^n \longrightarrow \{0,1\}^m, m \leq n$

Quantum computation with n qubits: $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{2^n} \end{pmatrix} \in \mathbb{C}^{2^n}$

Classical gates:



Quantum gates: Unitary transformation
Measurement



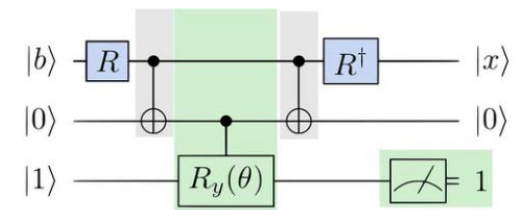
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Rotation around x-axis: $R(\theta) = \exp \left(-i\theta \frac{\sigma_x}{2} \right)$

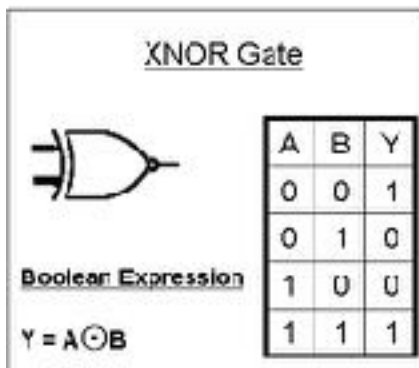
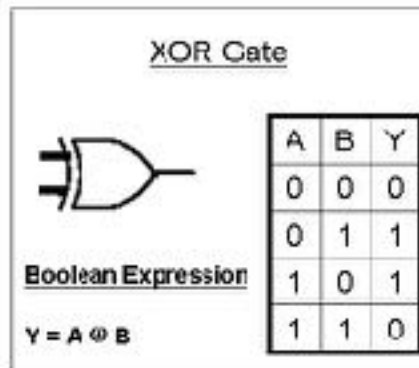
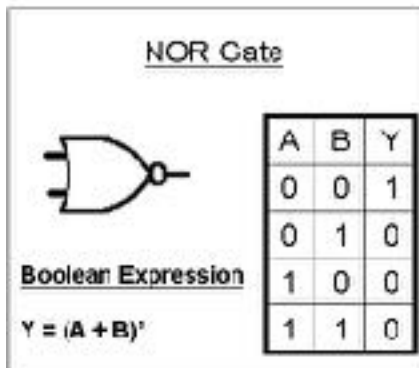
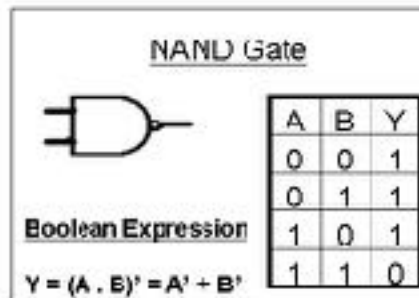
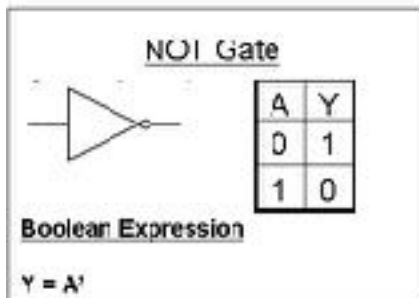
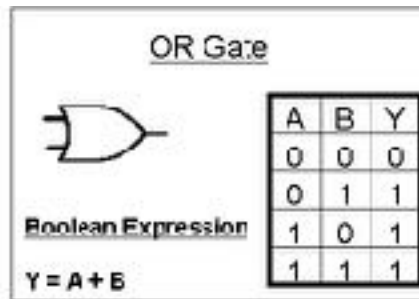
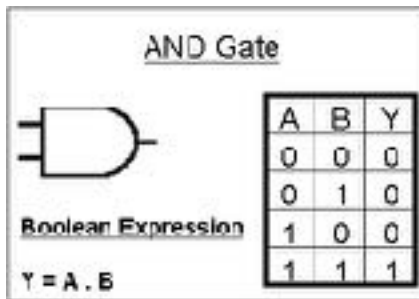
Algorithms

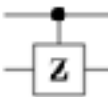


Quantum Algorithms



Classical gates Quantum gates



Operator	Gate(s)		Matrix
Pauli-X (X)			$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)			$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)			$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)			$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)			$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)			$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

These gates are platform-dependent.

Qubit Touchdown

- A quantum computing board game
 - Released Dec. 2019
- Make your own copy – instructions at thomaswong.net
- Buy a professionally produced copy print-on-demand at thegamecrafter.com
- Study involving 107 public high school students in Precalculus, AP Statistics, and/or AP Physics 1
 - Conducted Dec. 2024
 - Submitted Jan. 2025, <https://arxiv.org/abs/2501.10449>

Made by Prof. Tom Wong
Creighton University

Improving Student Self-Efficacy in Quantum Computing with the Qubit Touchdown Board Game

Kristina Armbruster,¹ Gintaras Duda,² and Thomas G. Wong²

¹Bellevue West High School, 1501 Thurston Ave, Bellevue, Nebraska 68123

²Department of Physics, Creighton University, 2500 California Plaza, Omaha, Nebraska 68178



learners to be “provided exposure to QIST via accessible outreach and educational opportunities [including] in informal learning venues such as museums, movies, games, and other media.” Note that games are listed, and the report further recommends that such games and activities begin with K-12 age groups.

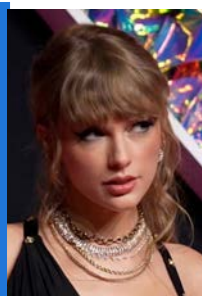
⁴Electronic mail: kristina.armbruster@bpnc.net

⁵Electronic mail: gkduda@creighton.edu

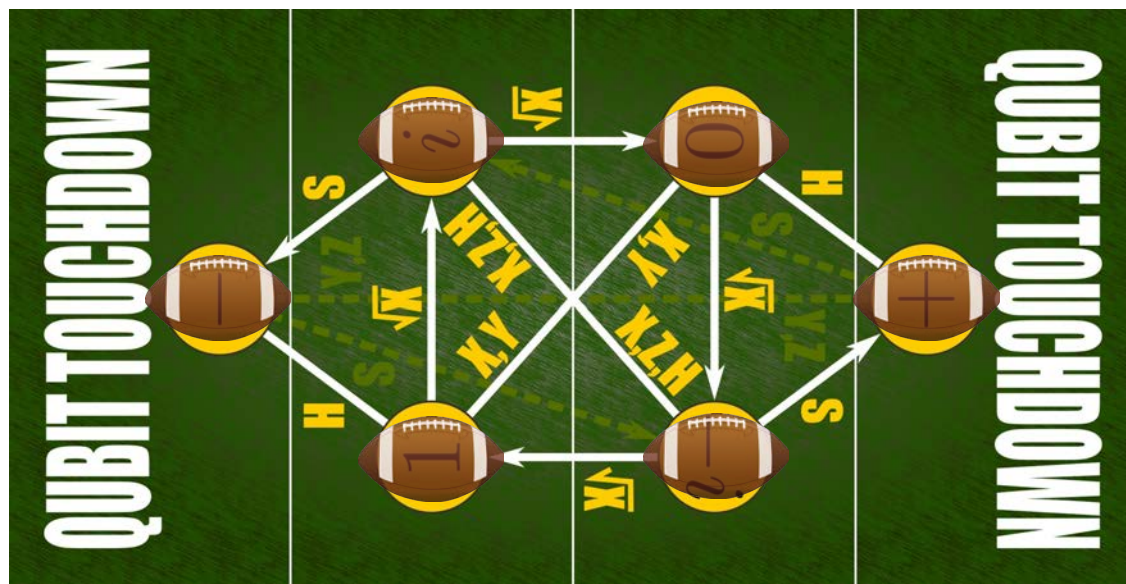
⁶Electronic mail: thomaswong@creighton.edu

II. HOW TO PLAY QUBIT TOUCHDOWN

As a high-level overview, Qubit Touchdown is a two player, competitive game. The game board is shown in Fig. 2 and the football token or gem is placed on the board to indicate the position of the football. The football can only be at the six positions indicated by yellow circles, which are labeled 0, 1, +, -, i , and $-i$. One player is trying to get the football to the + endzone to score touchdowns, and the other player is trying to get the ball to the - endzone to score touchdowns. They take turns playing cards to move the football along the



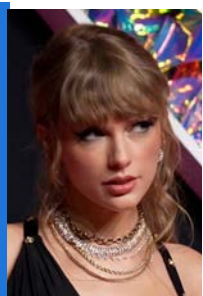
Taylor



Ryan



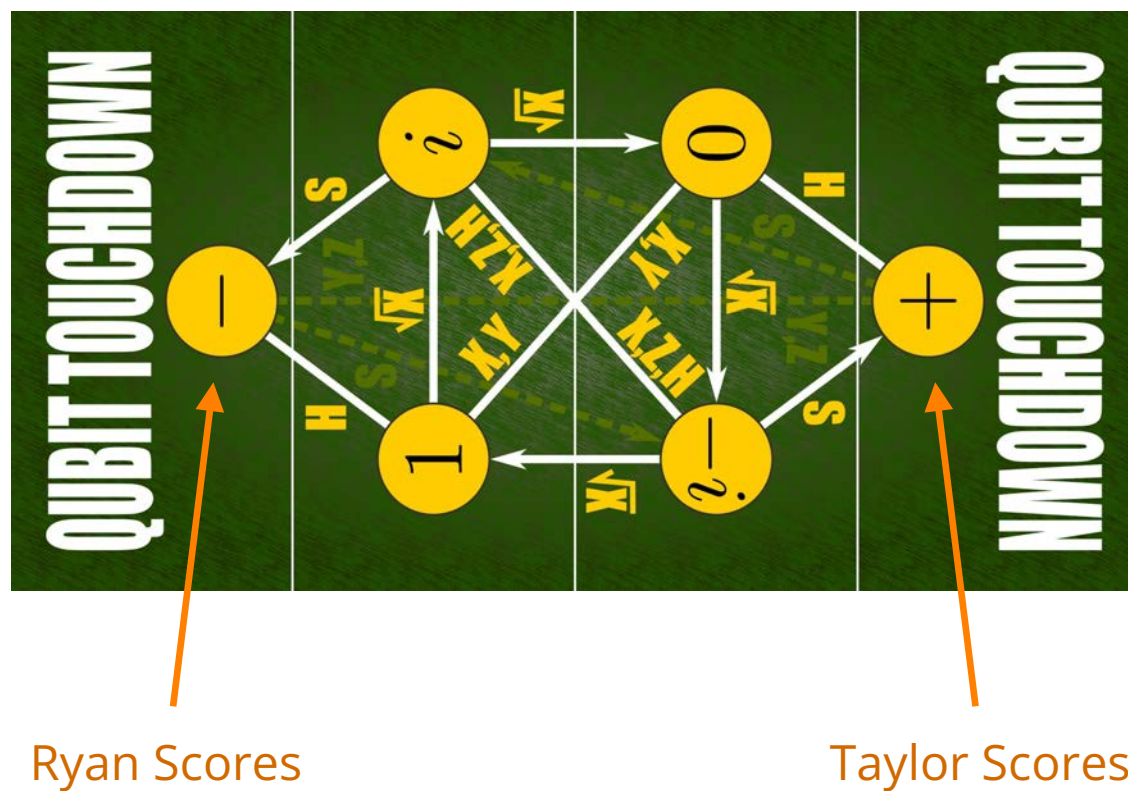
- The football can be in one of six positions: 0 , 1 , $+$, $-$, i , or $-i$.



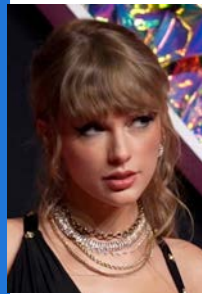
Taylor



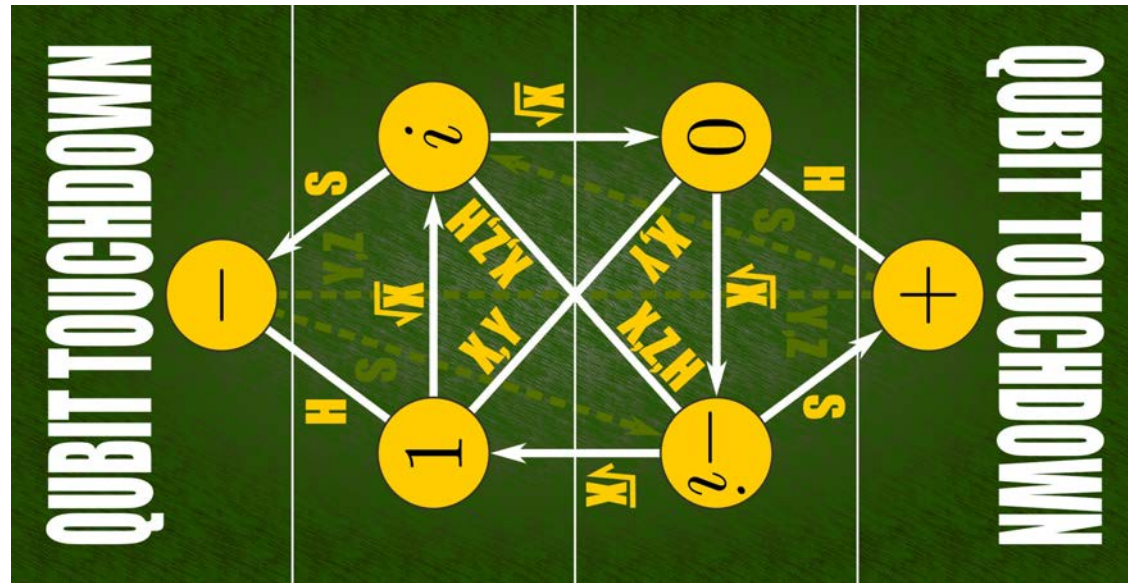
Ryan



- Taylor is trying to get the ball to the “+” endzone, and Ryan is trying to get the ball to the “-” endzone, to score touchdowns.



Taylor

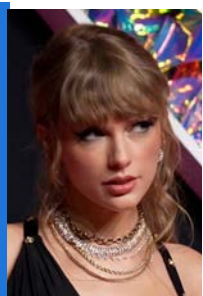


Ryan



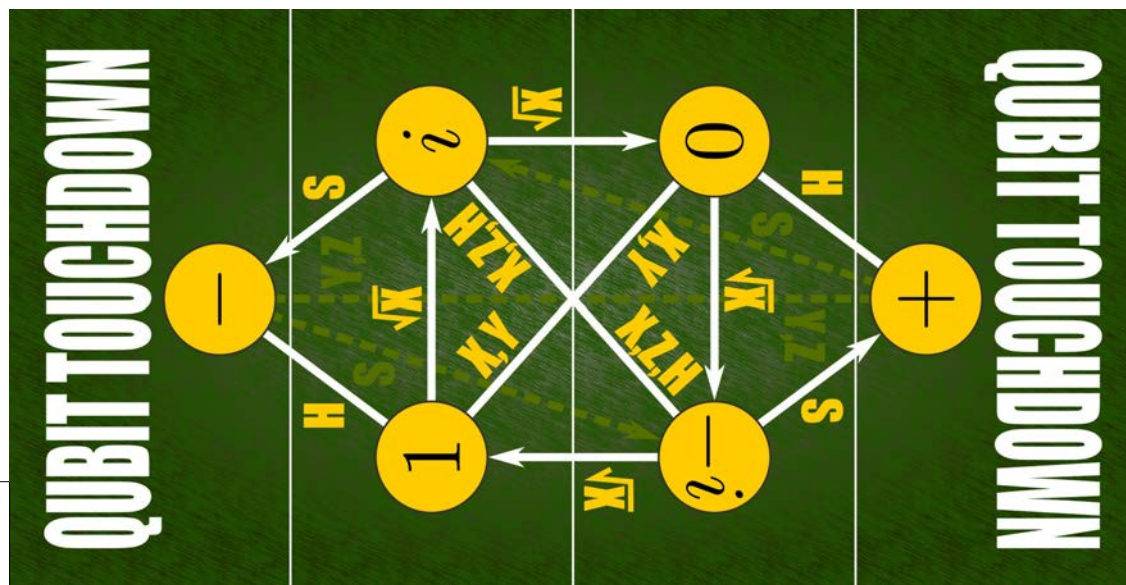
- They take turns playing cards to move the ball along the lines/arrows.
- Whoever scores the most touchdowns by the time all 52 cards are played wins.

Let's play Qubit touchdown!



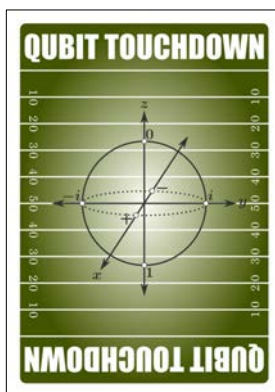
Taylor

Touchdowns: 0



Draw Pile

Discard Pile



44 cards

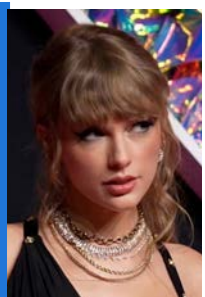
0 cards

Ryan

Touchdowns: 0

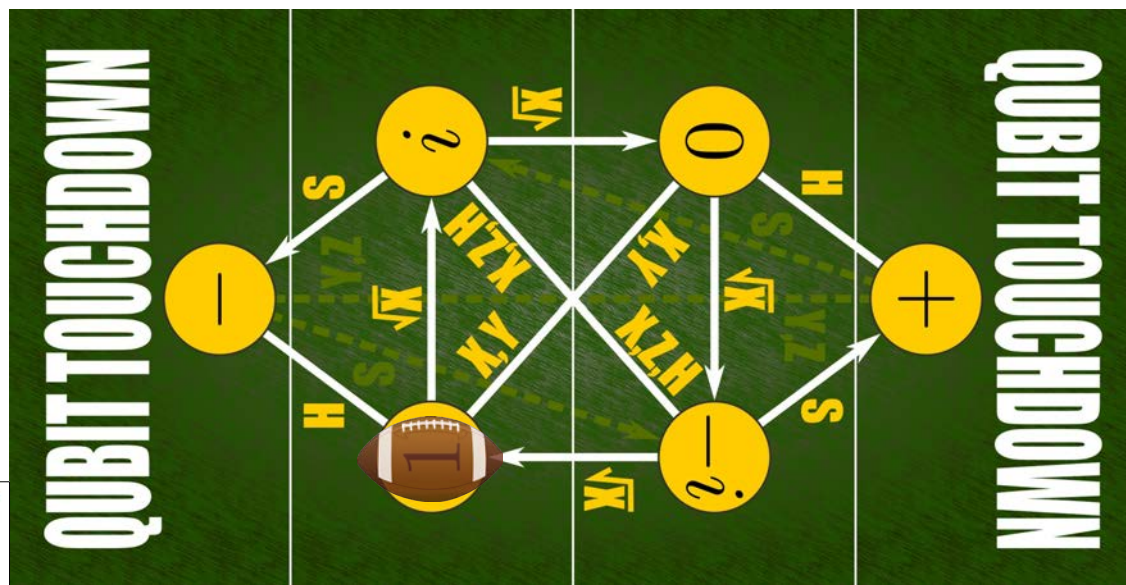


- Shuffle the cards. Deal 4 to each player. Put the rest in the middle as a “draw pile.”



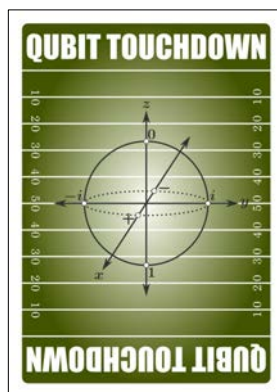
Taylor

Touchdowns: 0



Draw Pile

Discard Pile



44 cards



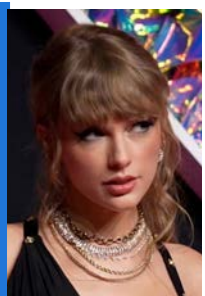
Ryan

Touchdowns: 0



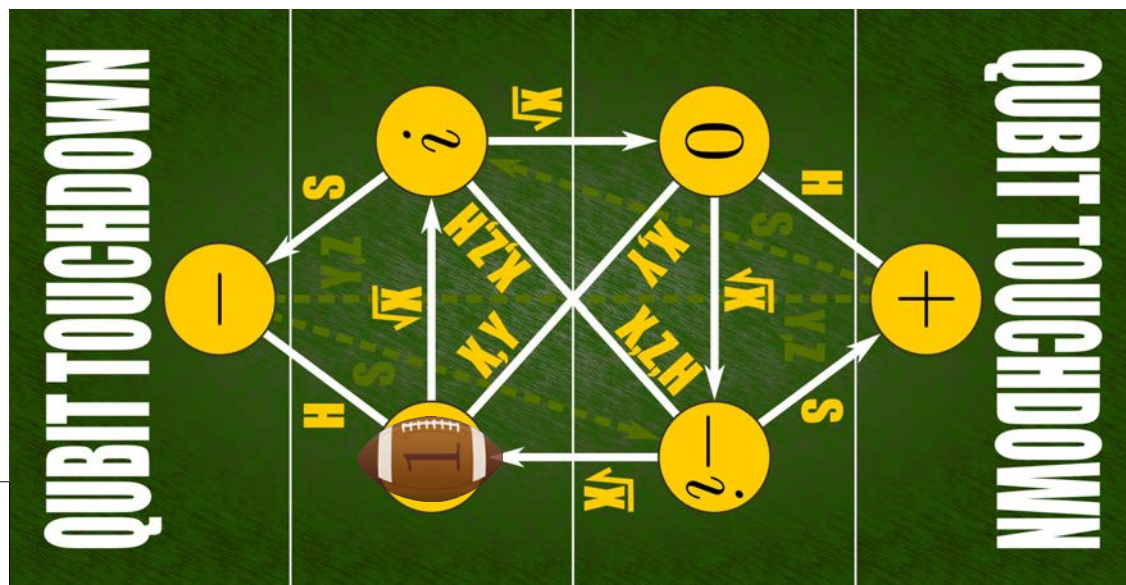
0 cards

- Taylor “kicks off” by rolling the binary die and placing the football at the result.



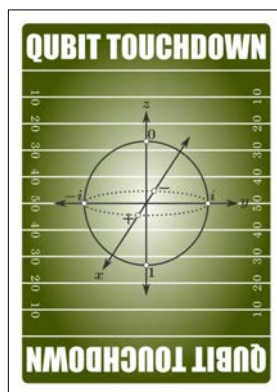
Taylor

Touchdowns: 0



Draw Pile

Discard Pile



44 cards



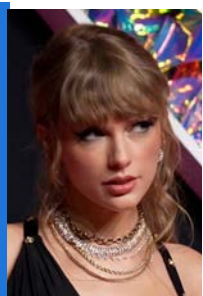
Ryan

Touchdowns: 0



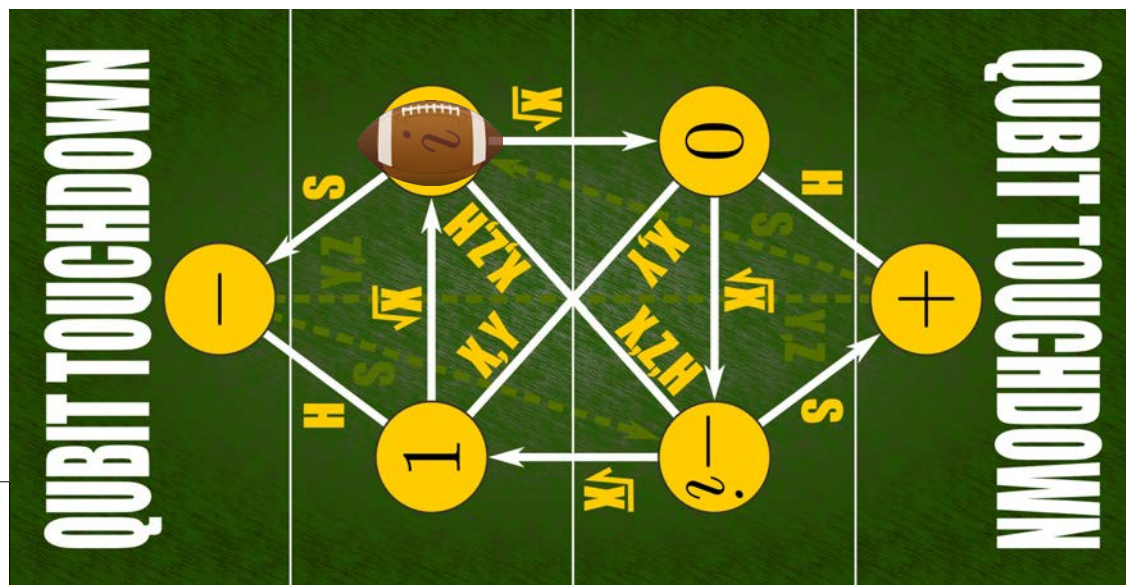
- Ryan's turn. He wants to get the ball to "-" to score, but he doesn't have an "H."

0 cards



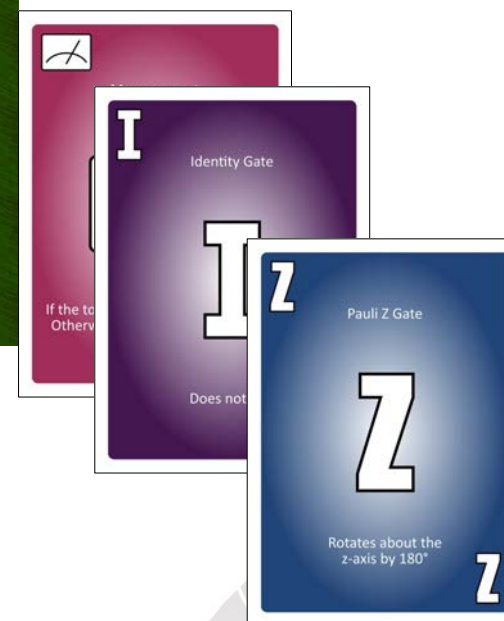
Taylor

Touchdowns: 0

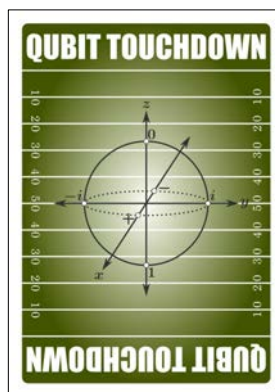


Ryan

Touchdowns: 0

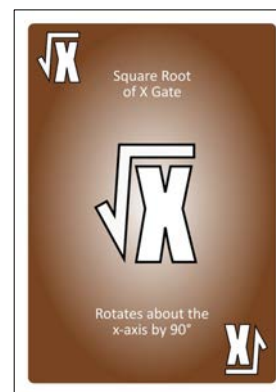


Draw Pile



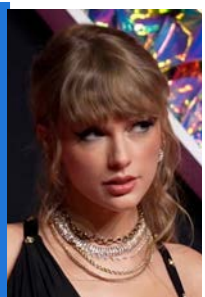
44 cards

Discard Pile



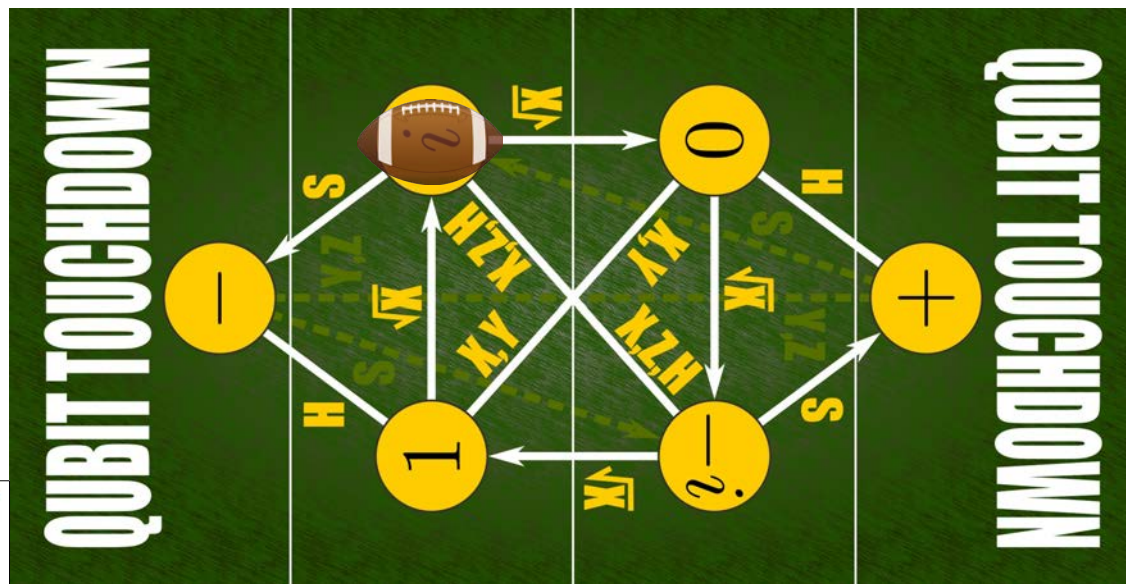
1 card

- He plays " $\sqrt{X}$ " and moves the ball to "i."



Taylor

Touchdowns: 0

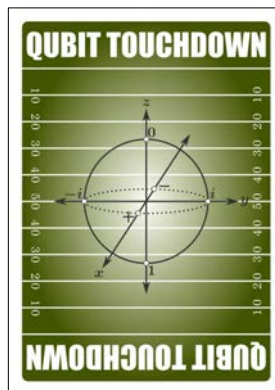


Ryan

Touchdowns: 0

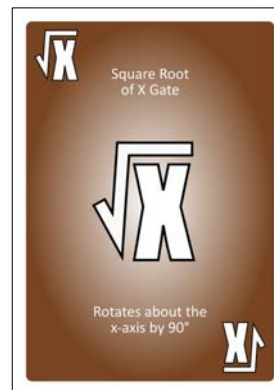


Draw Pile



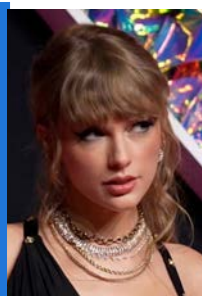
43 cards

Discard Pile



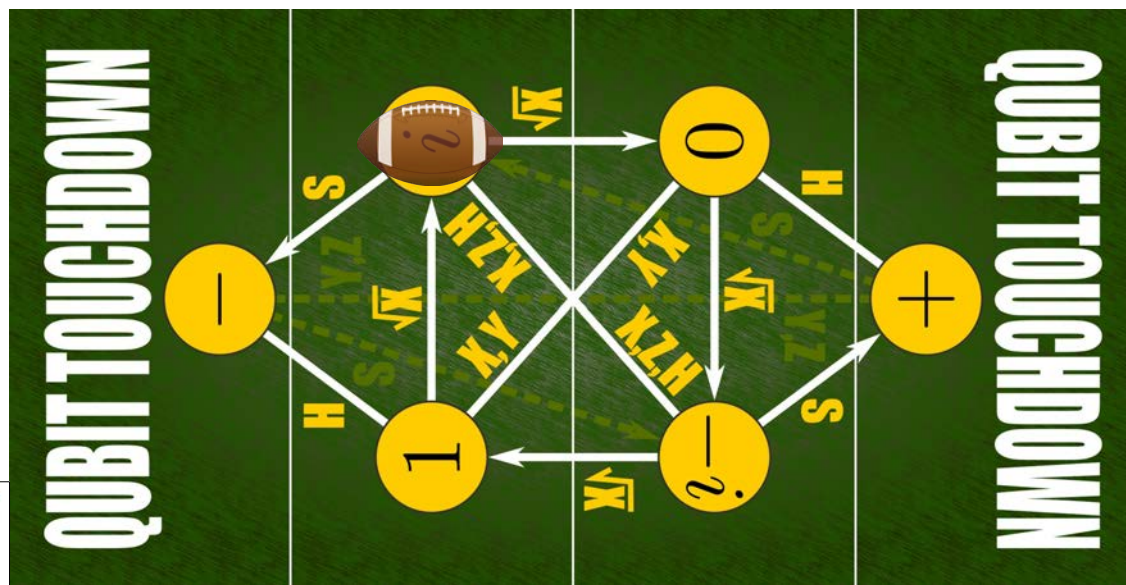
1 card

- He draws a new card.



Taylor

Touchdowns: 0

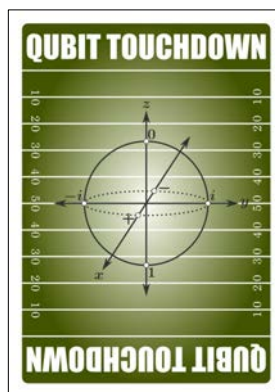


Ryan

Touchdowns: 0

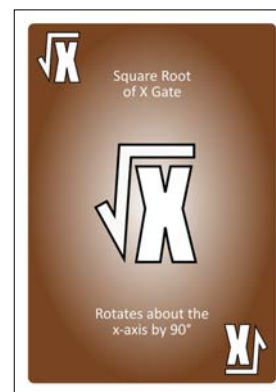


Draw Pile



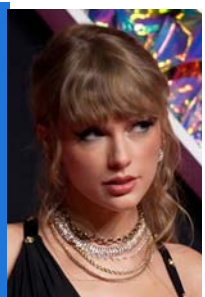
43 cards

Discard Pile



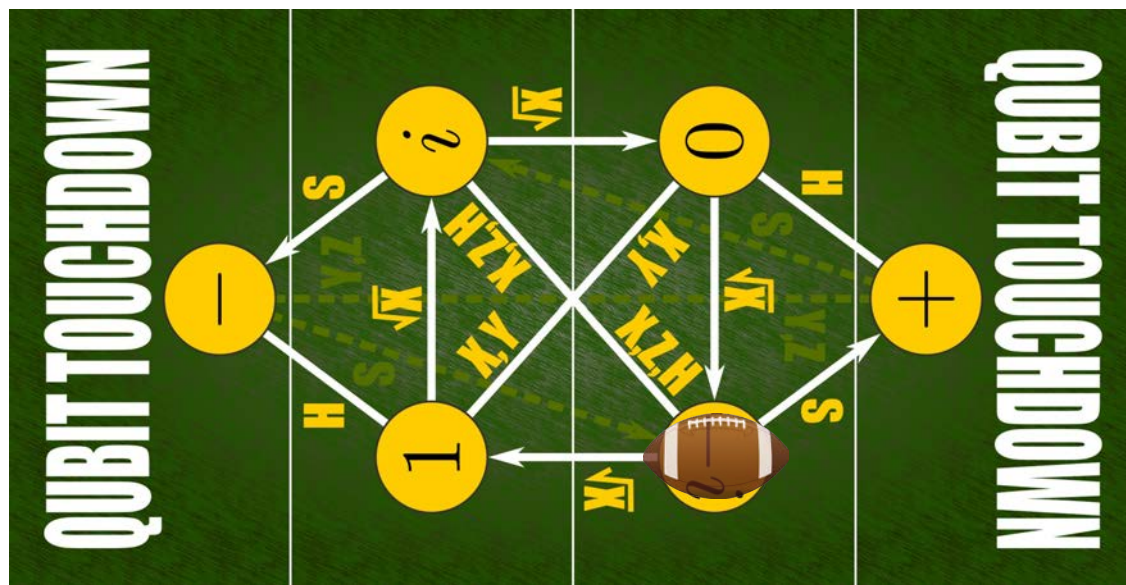
1 card

- Taylor's turn. She wants to move the ball towards "+."



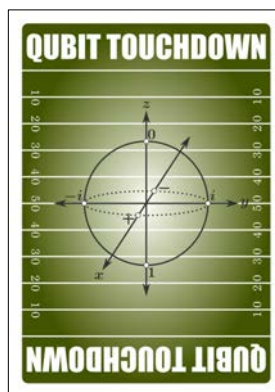
Taylor

Touchdowns: 0

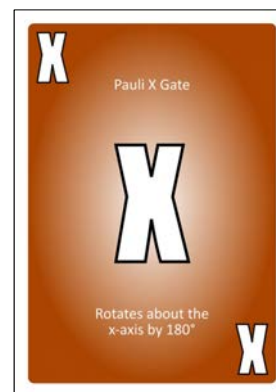


Draw Pile

Discard Pile



43 cards



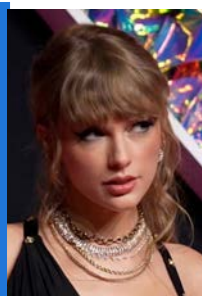
2 cards

Ryan

Touchdowns: 0



- She plays “X” and moves the ball to “-i.”

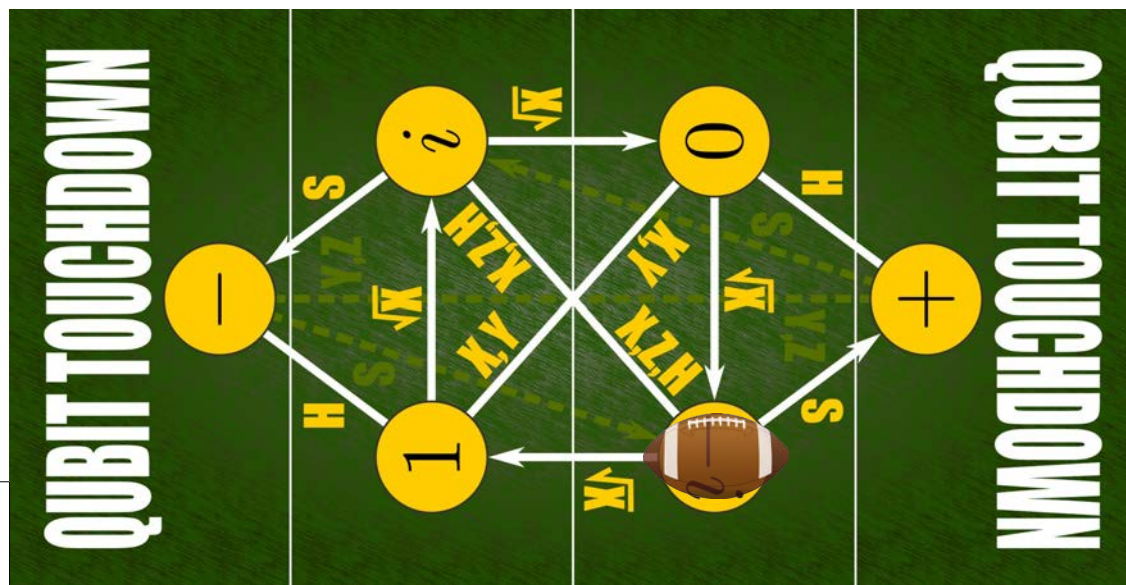


Taylor

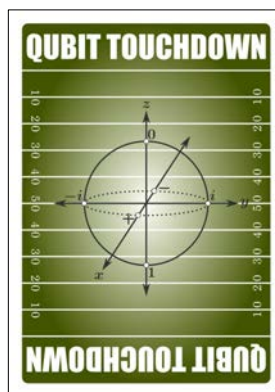
Touchdowns: 0



- She draws a new card.

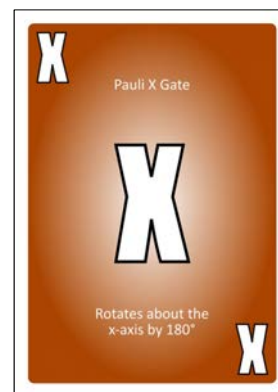


Draw Pile



42 cards

Discard Pile

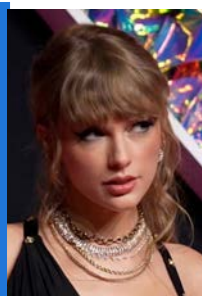


2 cards

Ryan

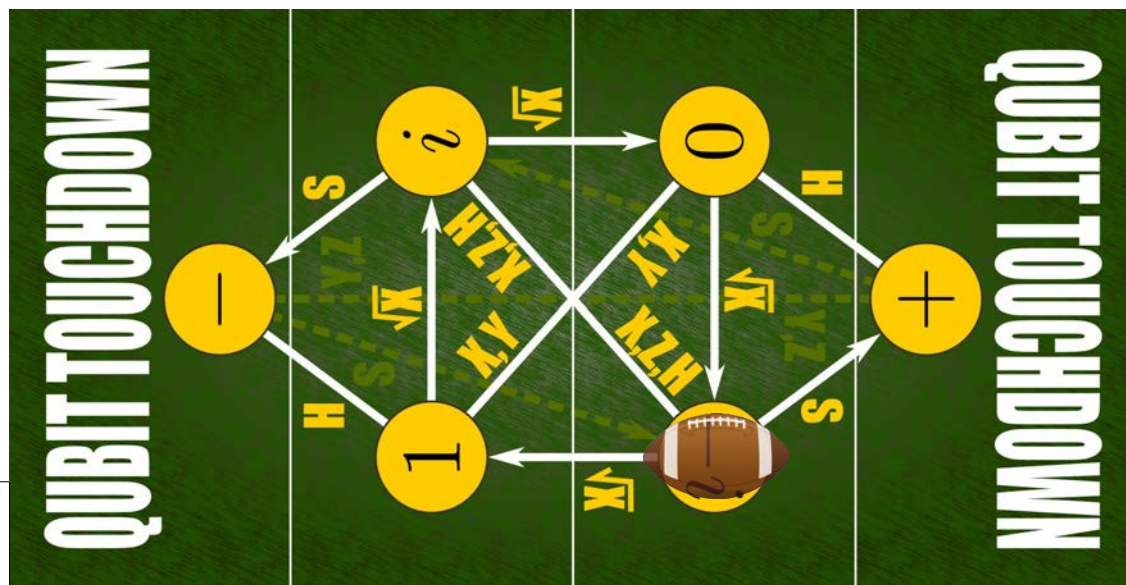
Touchdowns: 0





Taylor

Touchdowns: 0

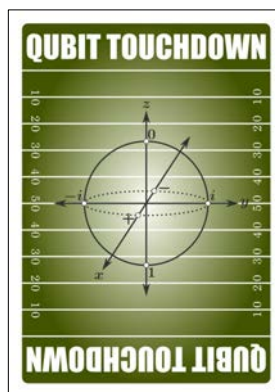


Ryan

Touchdowns: 0

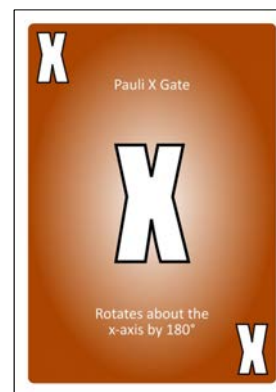


Draw Pile



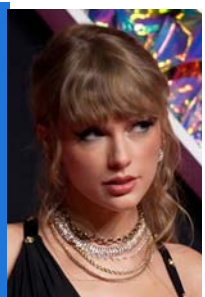
42 cards

Discard Pile



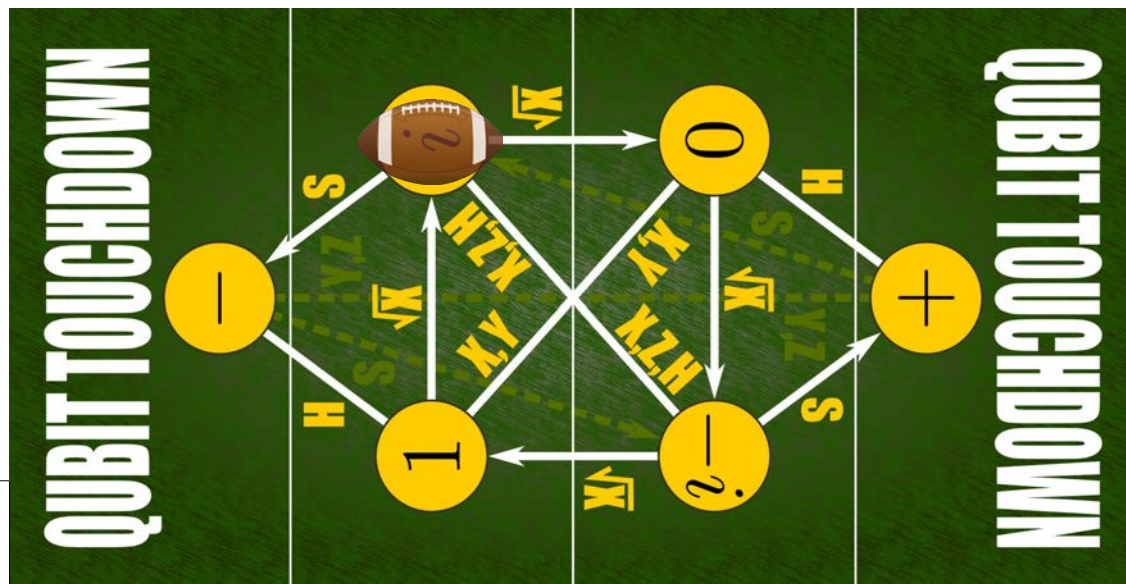
2 cards

- Ryan's turn.



Taylor

Touchdowns: 0



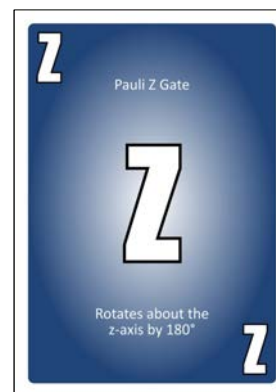
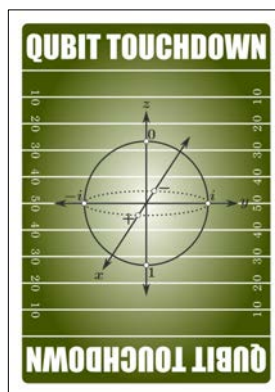
Ryan

Touchdowns: 0



Draw Pile

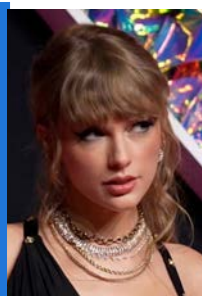
Discard Pile



42 cards

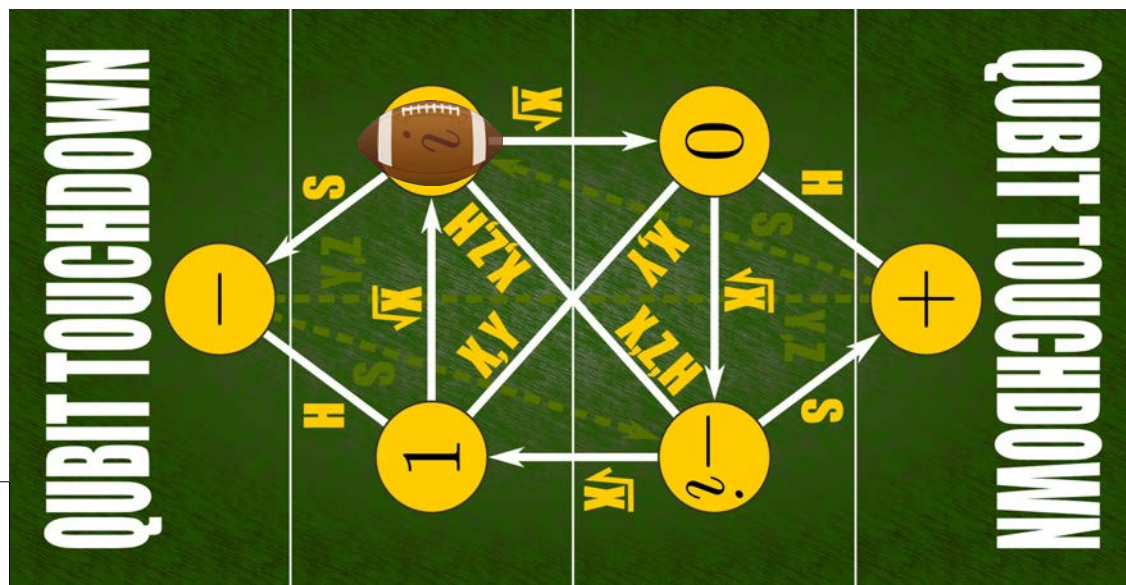
3 cards

- He plays "Z" and moves the ball to "i."



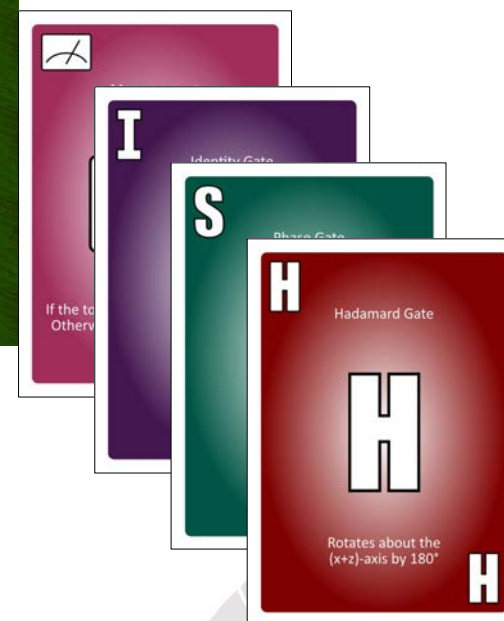
Taylor

Touchdowns: 0



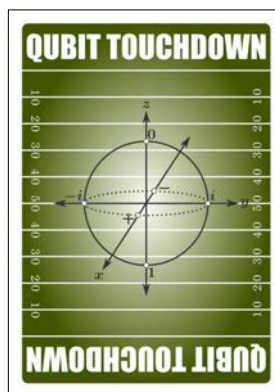
Ryan

Touchdowns: 0

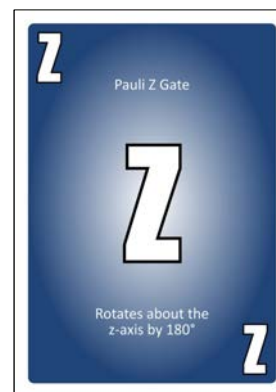


Draw Pile

Discard Pile

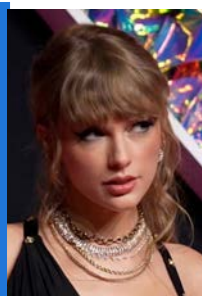


41 cards



3 cards

- He draws another card.

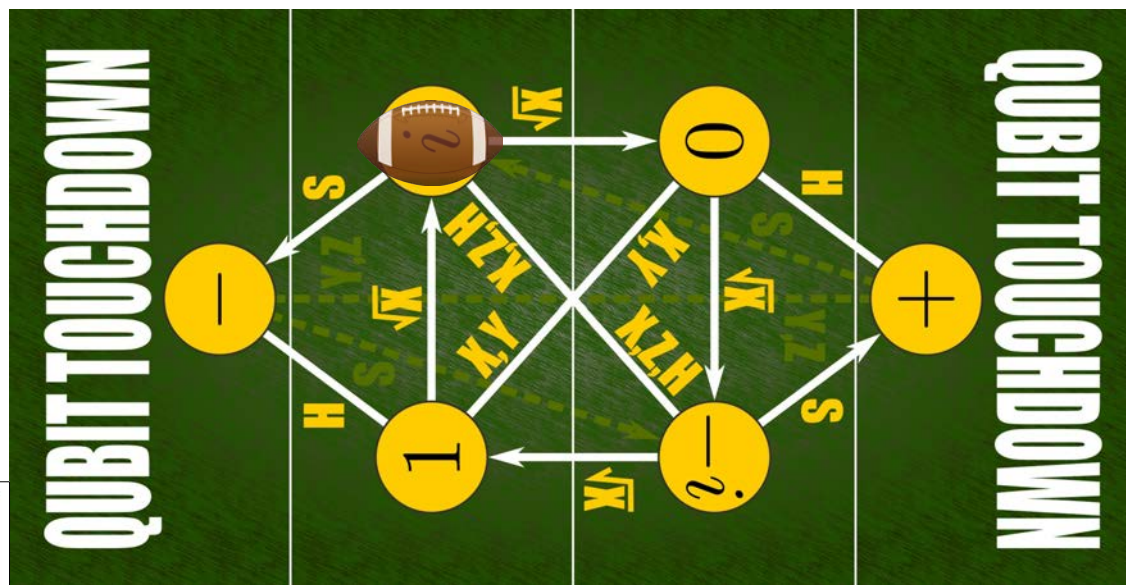


Taylor

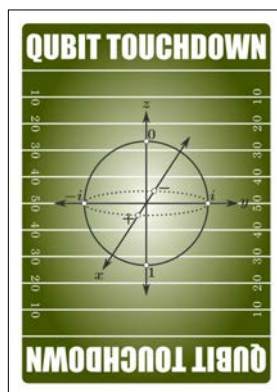
Touchdowns: 0



- Taylor's turn.

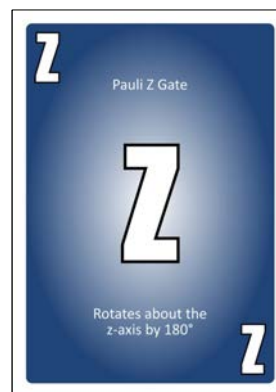


Draw Pile



41 cards

Discard Pile

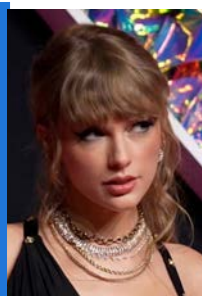


3 cards

Ryan

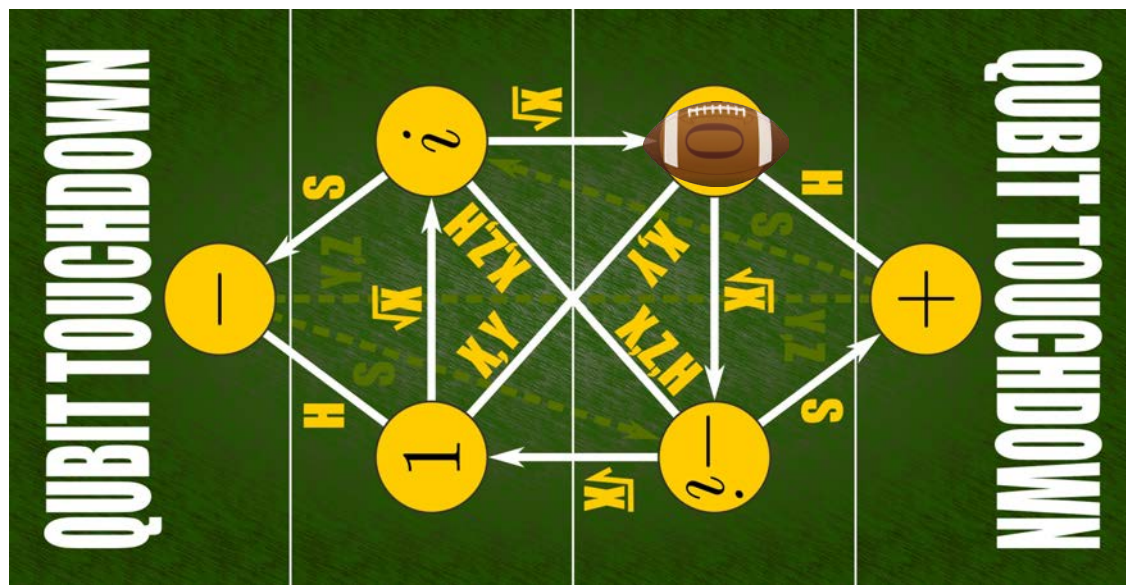
Touchdowns: 0





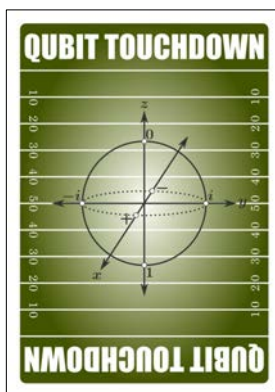
Taylor

Touchdowns: 0

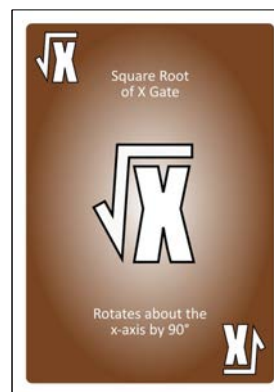


Draw Pile

Discard Pile



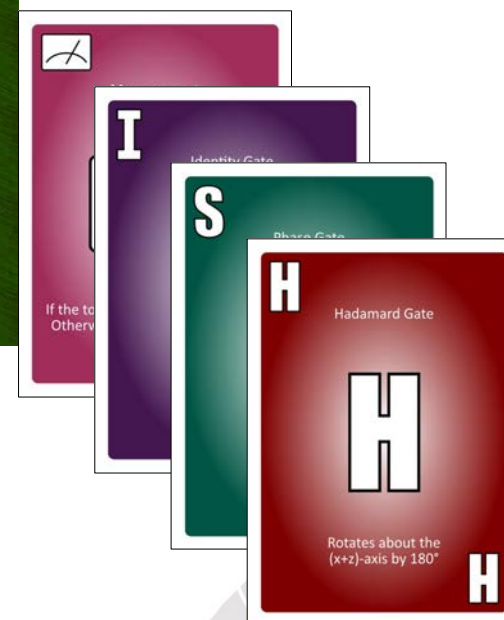
41 cards



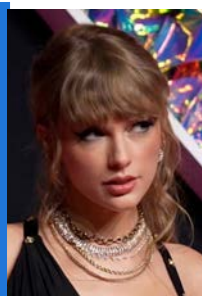
4 cards

Ryan

Touchdowns: 0



- She plays " $\sqrt{X}$ " and moves the ball to "0."

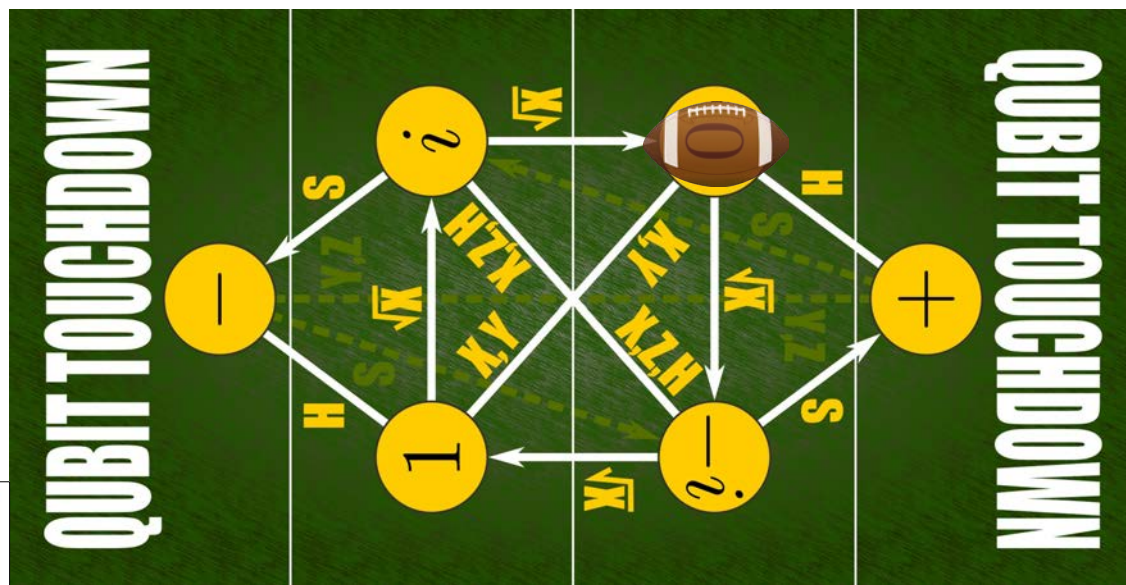


Taylor

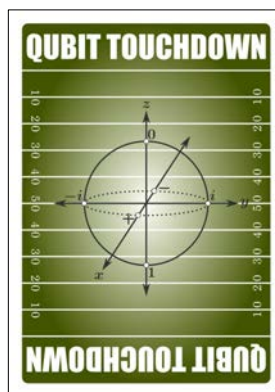
Touchdowns: 0



- She draws another card.

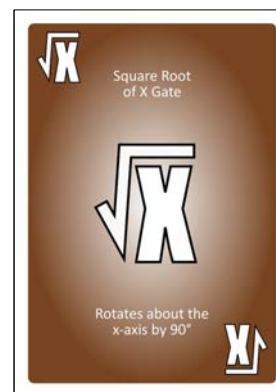


Draw Pile



40 cards

Discard Pile

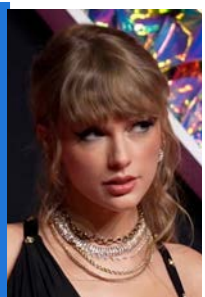


4 cards

Ryan

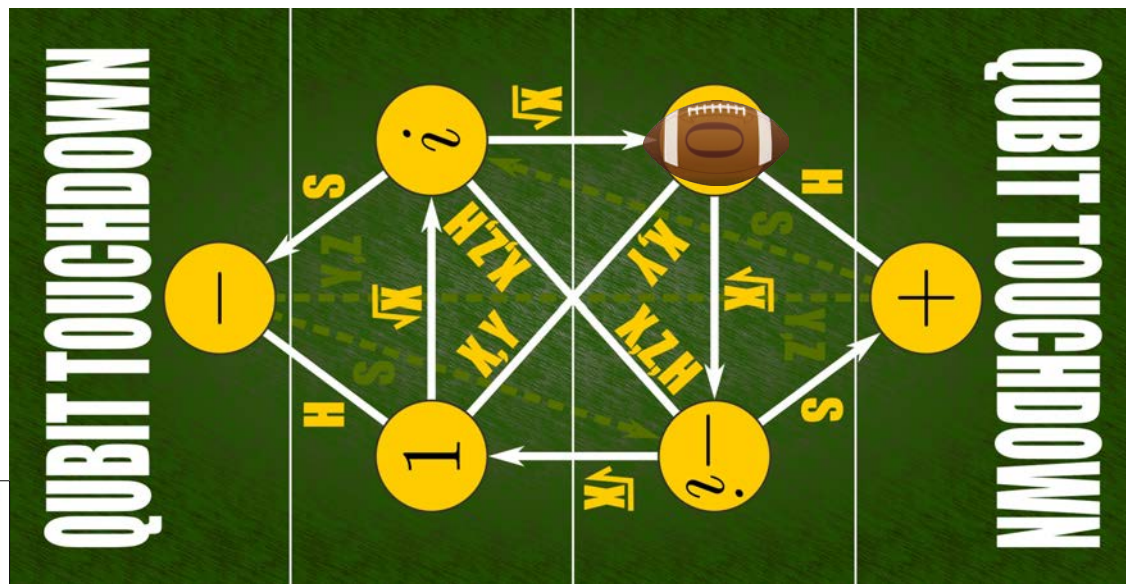
Touchdowns: 0





Taylor

Touchdowns: 0

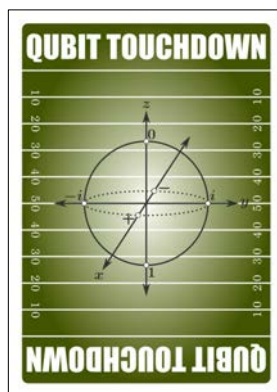


Ryan

Touchdowns: 0

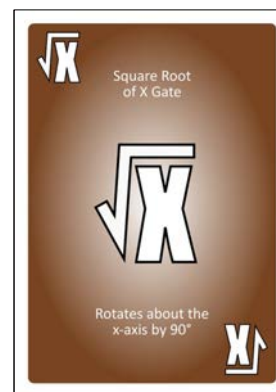


Draw Pile



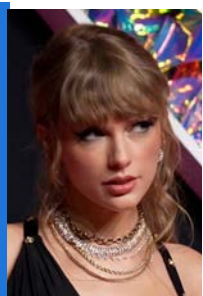
40 cards

Discard Pile



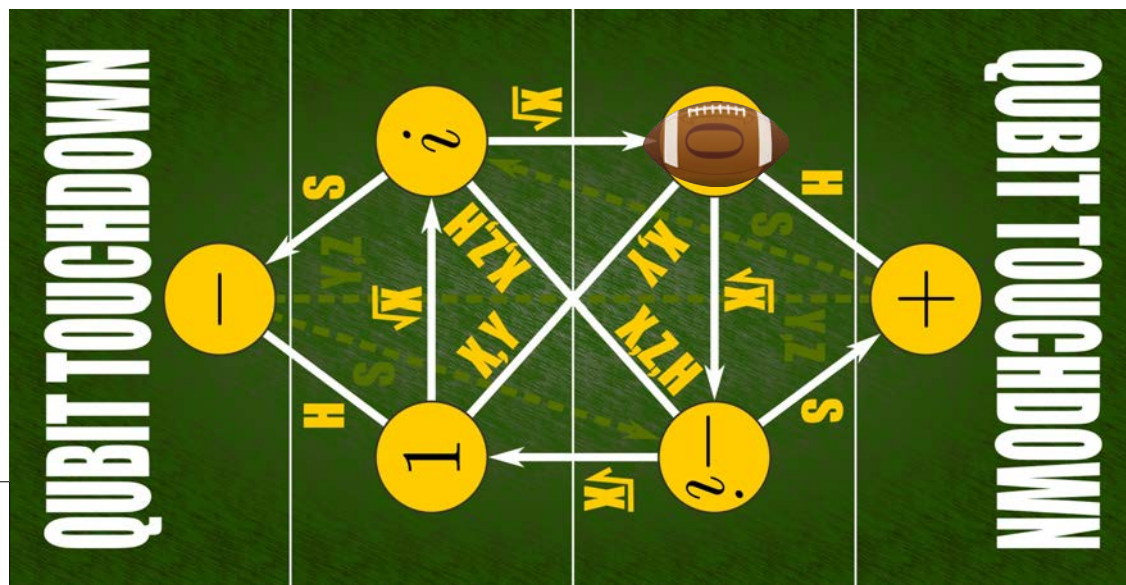
4 cards

- Ryan's turn.

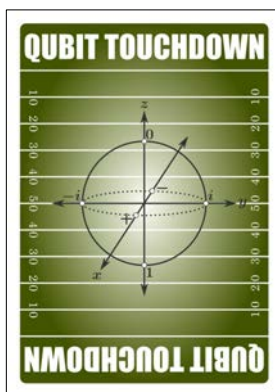


Taylor

Touchdowns: 0

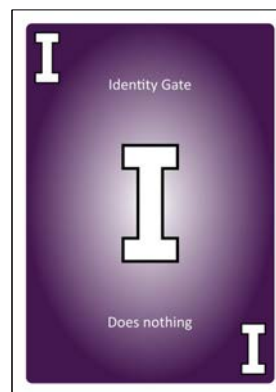


Draw Pile



40 cards

Discard Pile



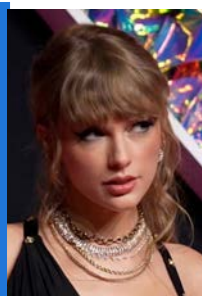
5 cards

Ryan

Touchdowns: 0

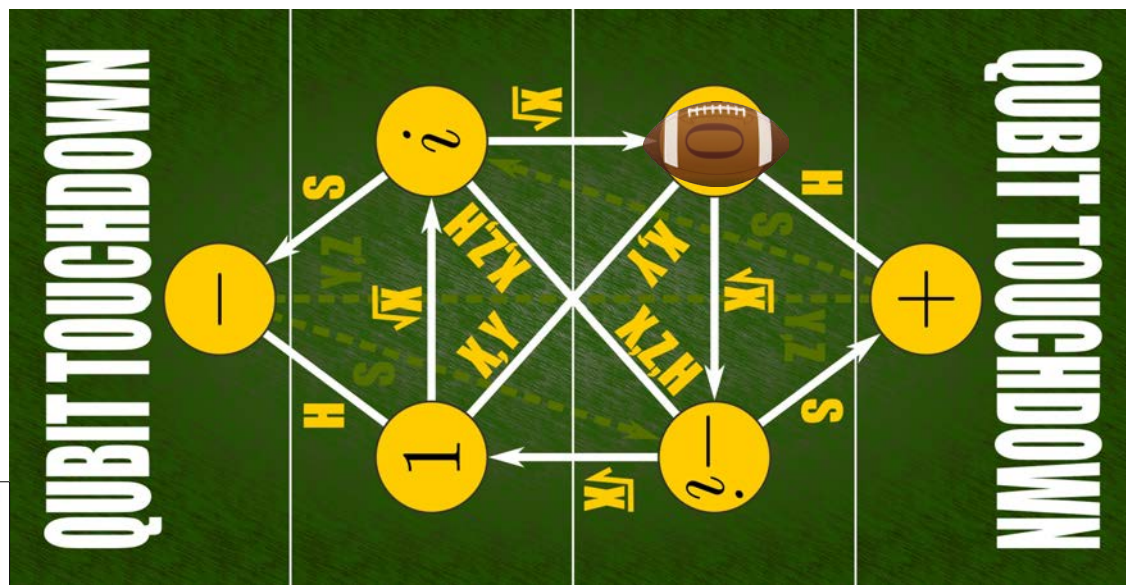


- He plays "I" and leaves the ball unmoved.

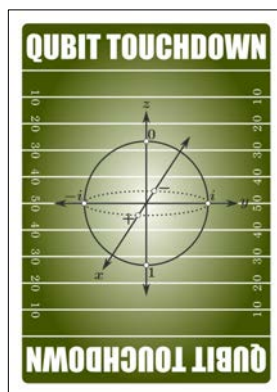


Taylor

Touchdowns: 0

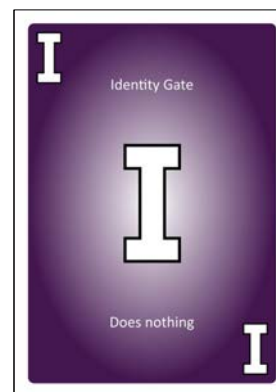


Draw Pile



39 cards

Discard Pile



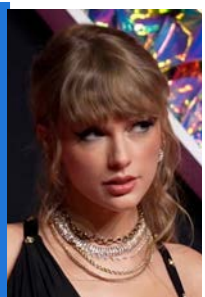
5 cards

Ryan

Touchdowns: 0

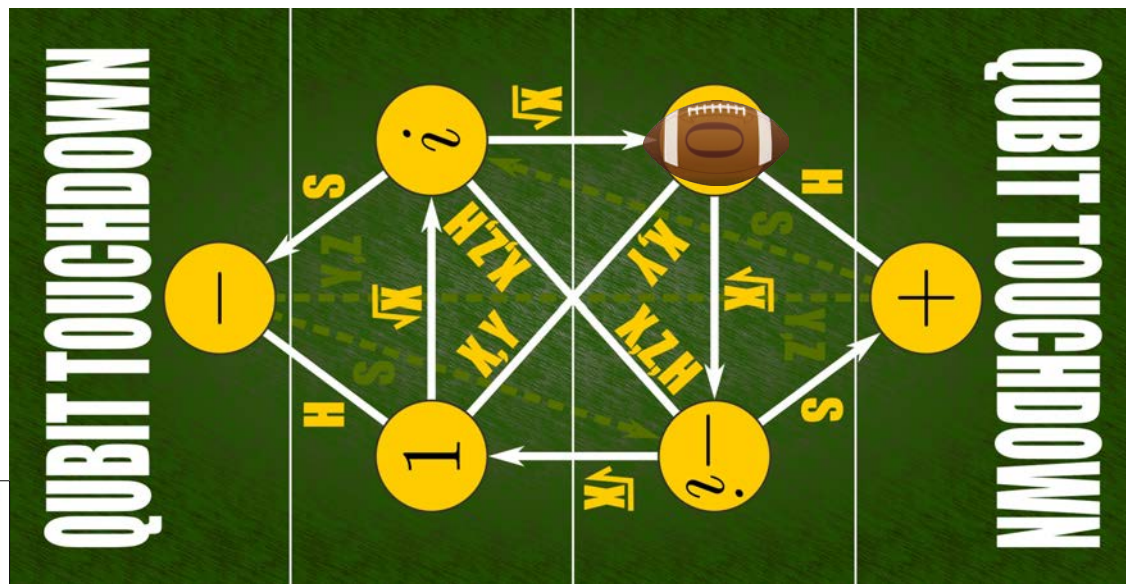


- He draws another card.

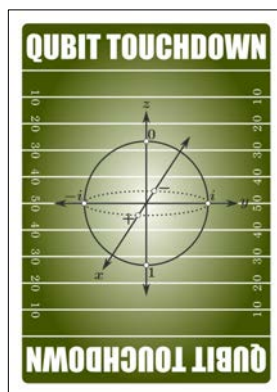


Taylor

Touchdowns: 0

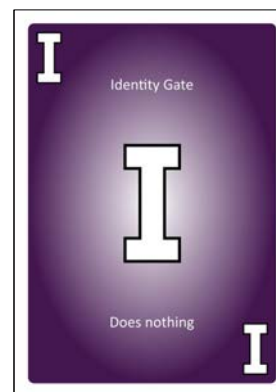


Draw Pile



39 cards

Discard Pile



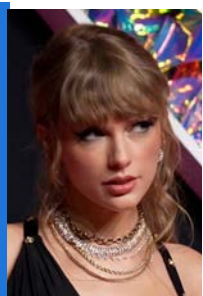
5 cards

Ryan

Touchdowns: 0



- Taylor's turn.

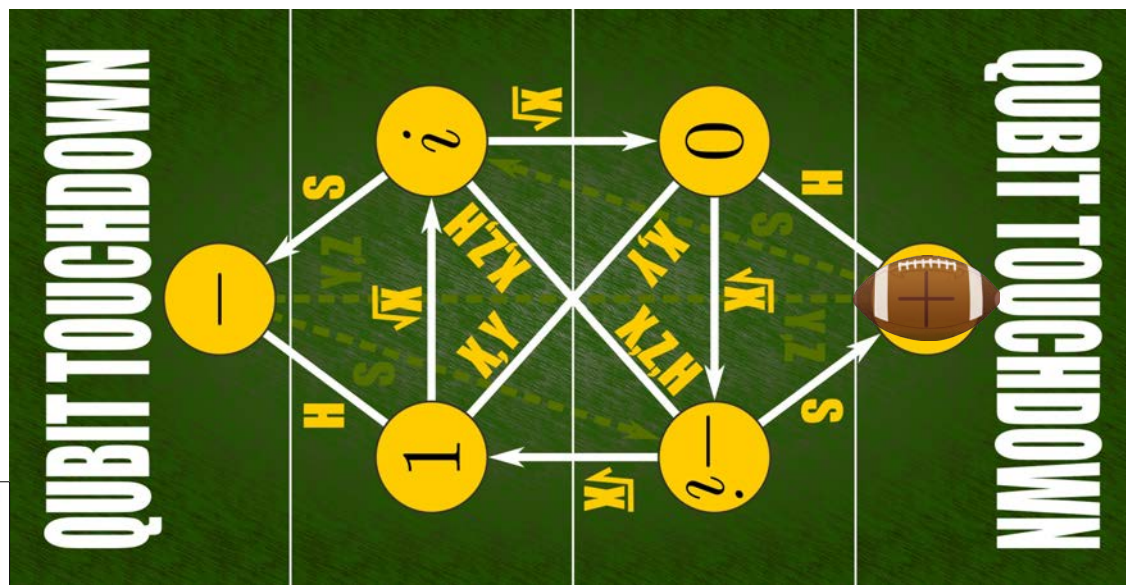


Taylor

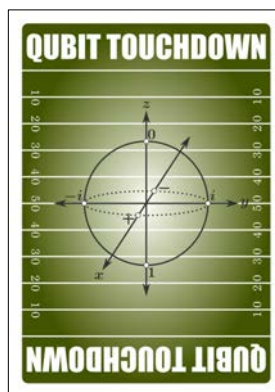
Touchdowns: 1



- She plays “H” and moves the ball to “+.” Touchdown!

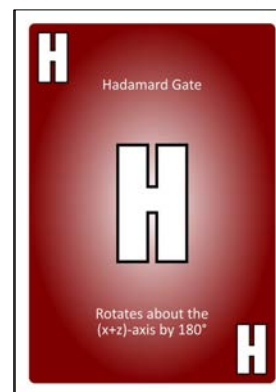


Draw Pile



39 cards

Discard Pile

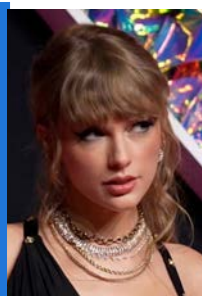


6 cards

Ryan

Touchdowns: 0



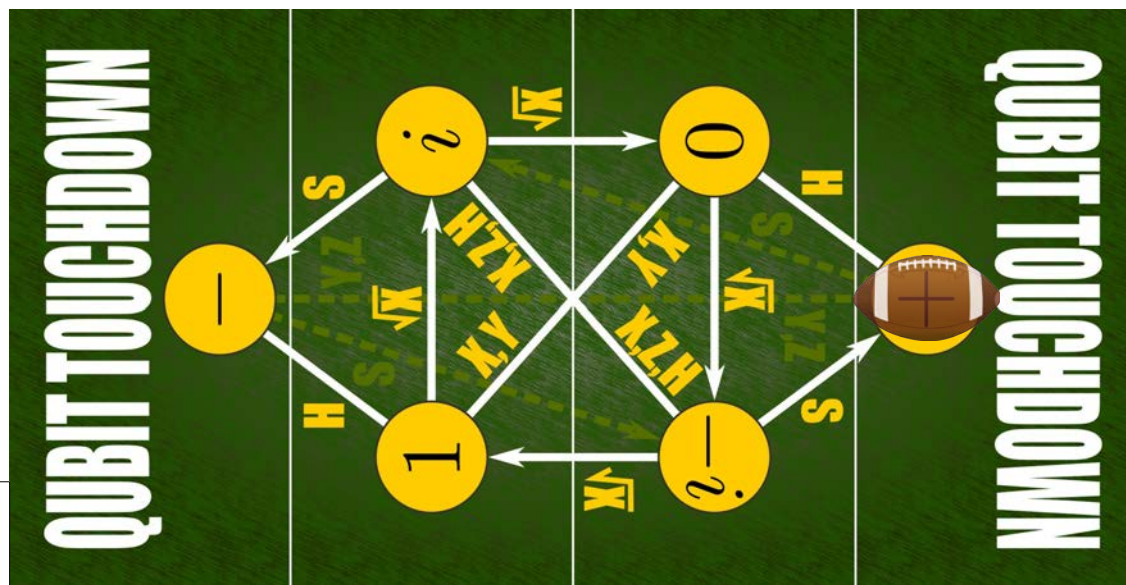


Taylor

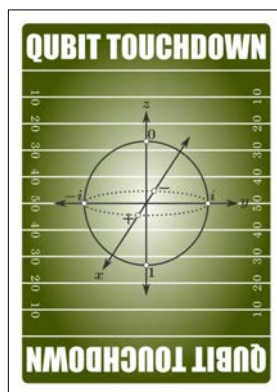
Touchdowns: 1



- She draws another card.

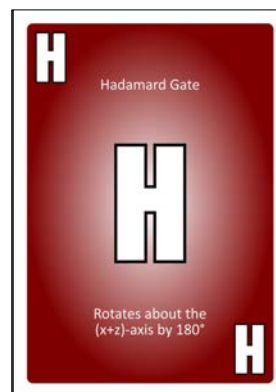


Draw Pile



38 cards

Discard Pile

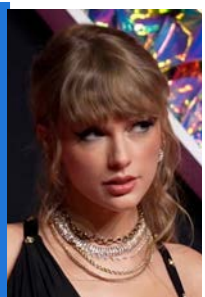


6 cards

Ryan

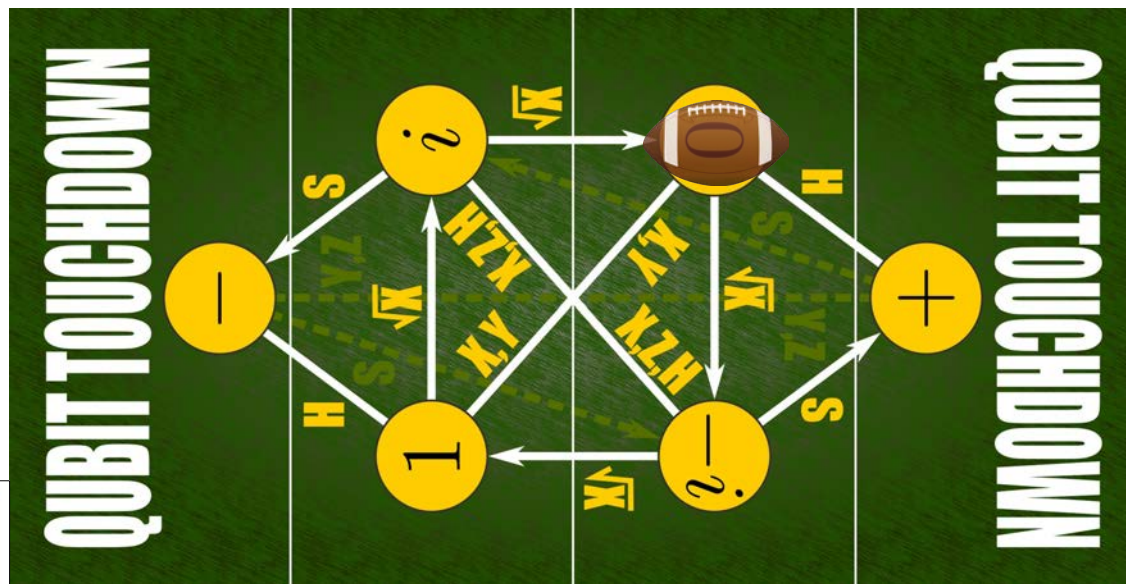
Touchdowns: 0





Taylor

Touchdowns: 1



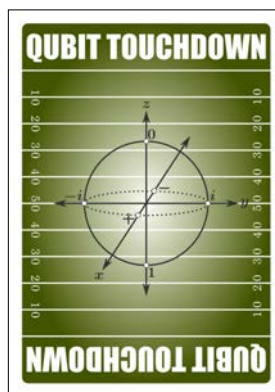
Ryan

Touchdowns: 0

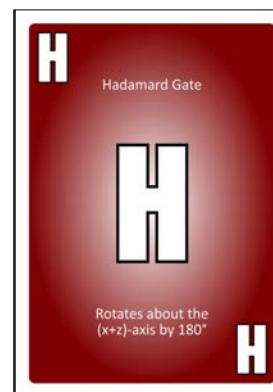


Draw Pile

Discard Pile

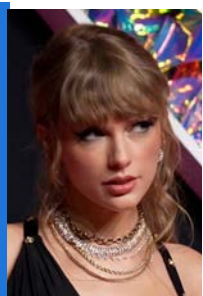


38 cards



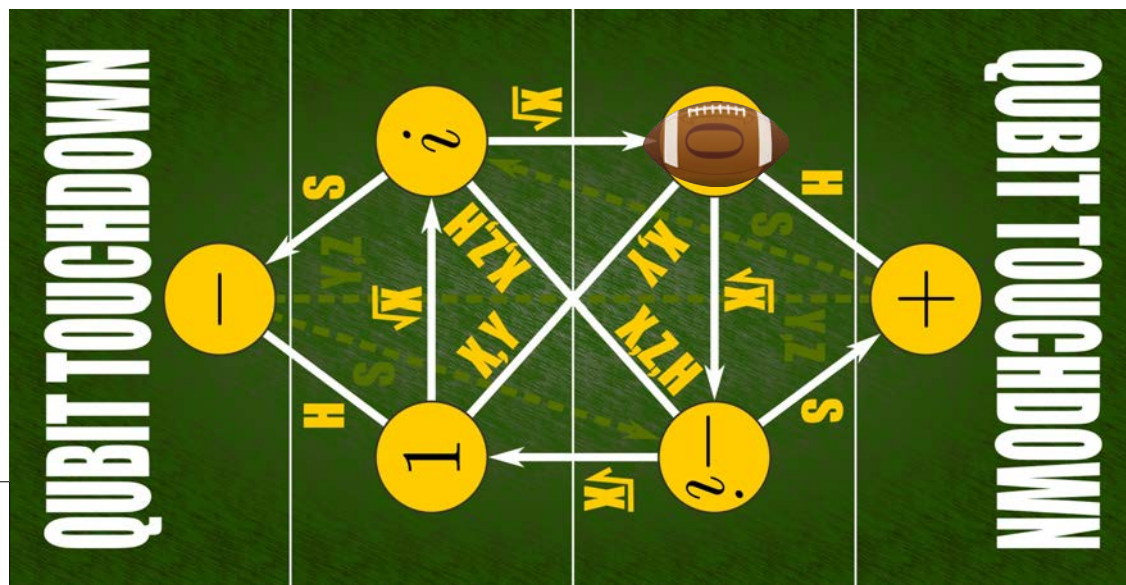
6 cards

- Since she scored a touchdown, she kicks off by rolling the binary die.



Taylor

Touchdowns: 1



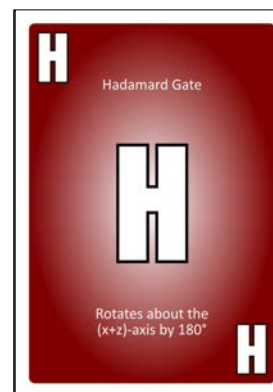
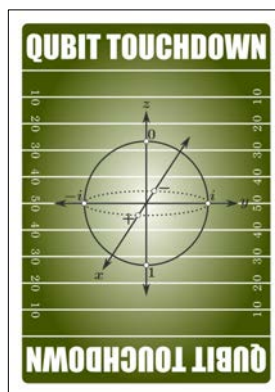
Ryan

Touchdowns: 0



Draw Pile

Discard Pile



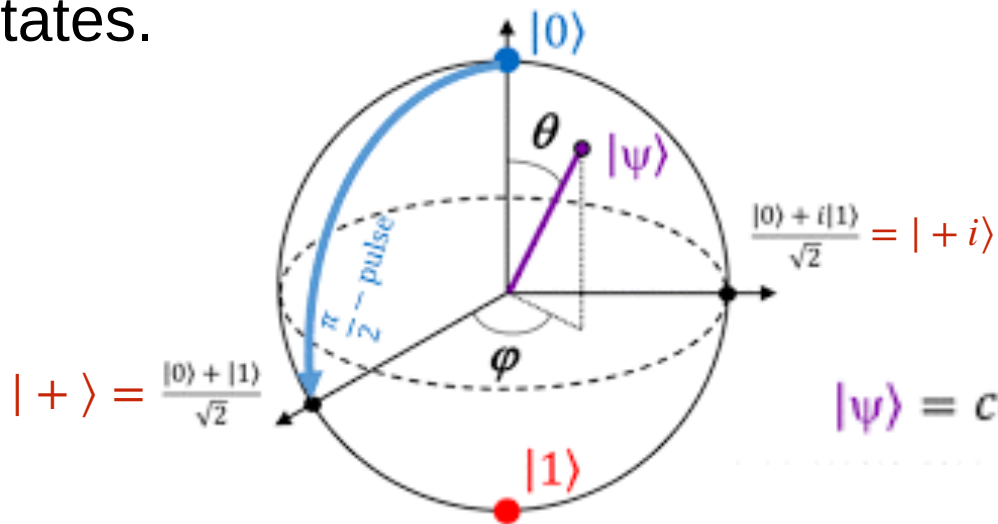
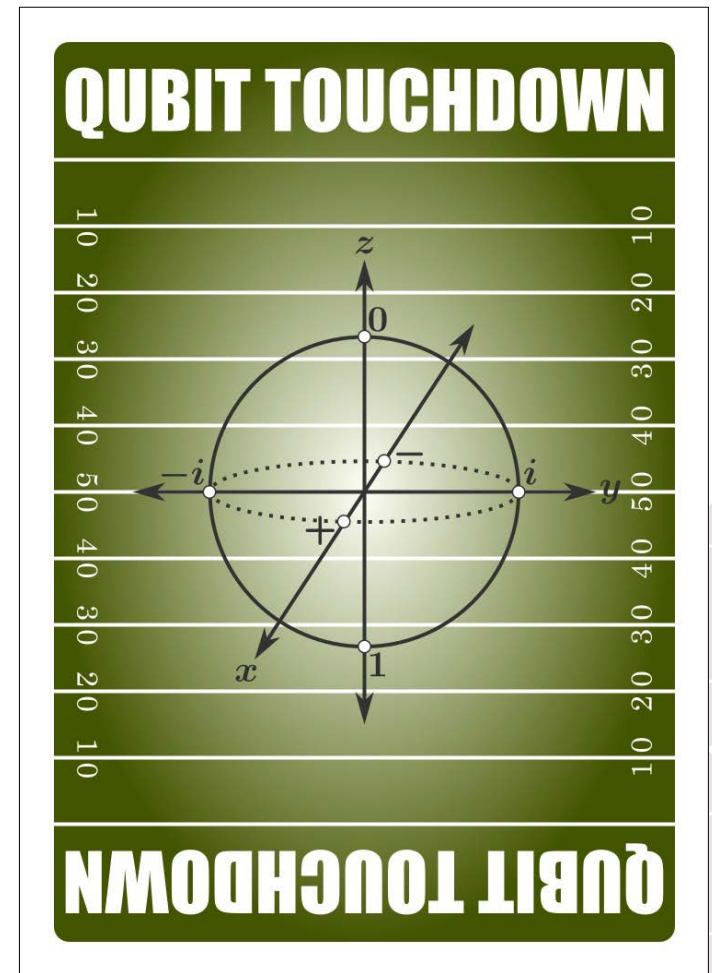
38 cards

6 cards

- Ryan's turn. Play continues until all cards are used up.

The Physics of Qubit Touchdown

- The back of every action card is a **Bloch sphere**.
- Its North and South poles are 0 and 1, the two possible states of a bit.
- A quantum bit, or qubit, can be any point on the Bloch sphere. Qubit Touchdown uses six of them: 0, 1, +, −, i, −i.
- The positions on the game board correspond to these six states, and the football is a qubit moving between these states.



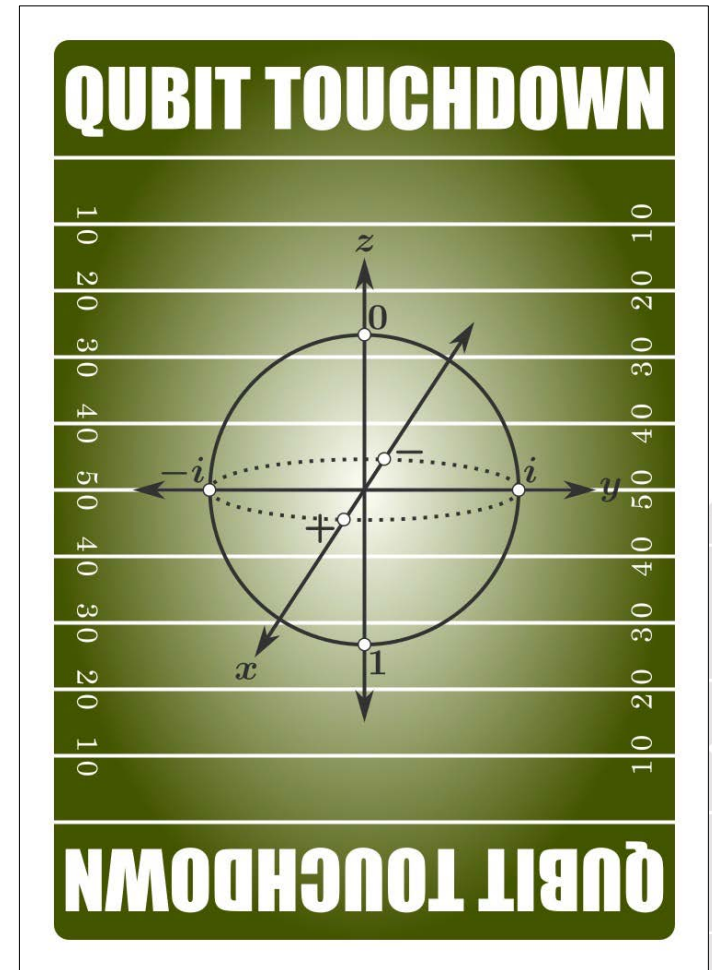
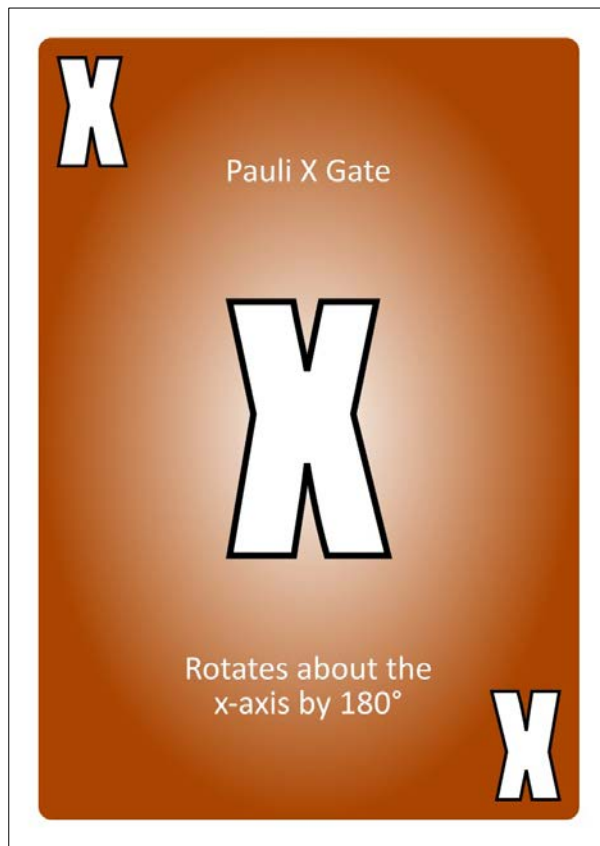
$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

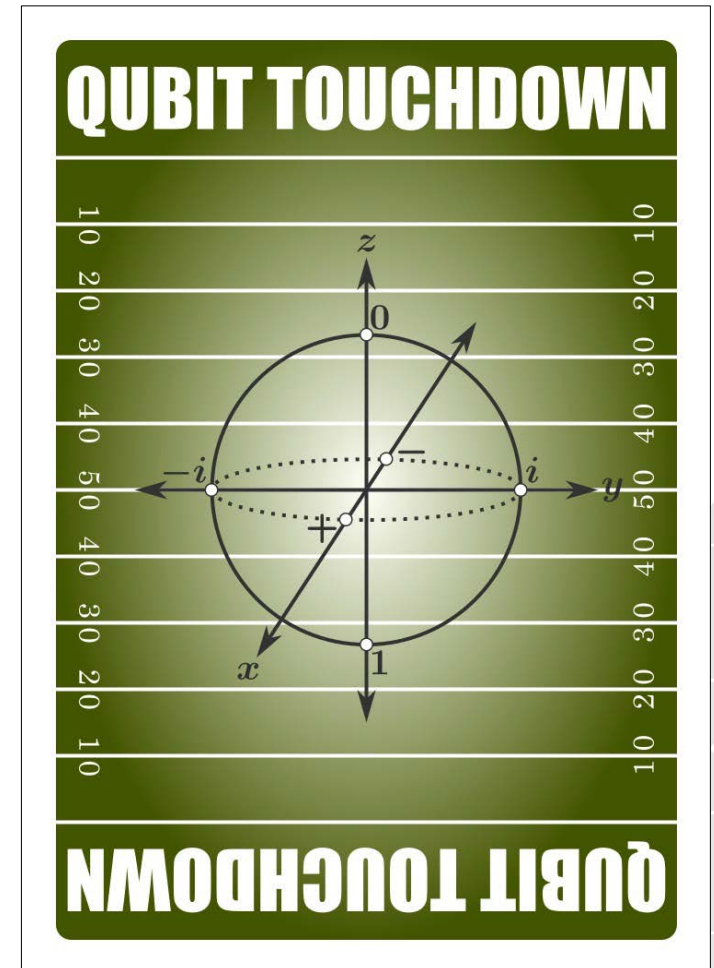
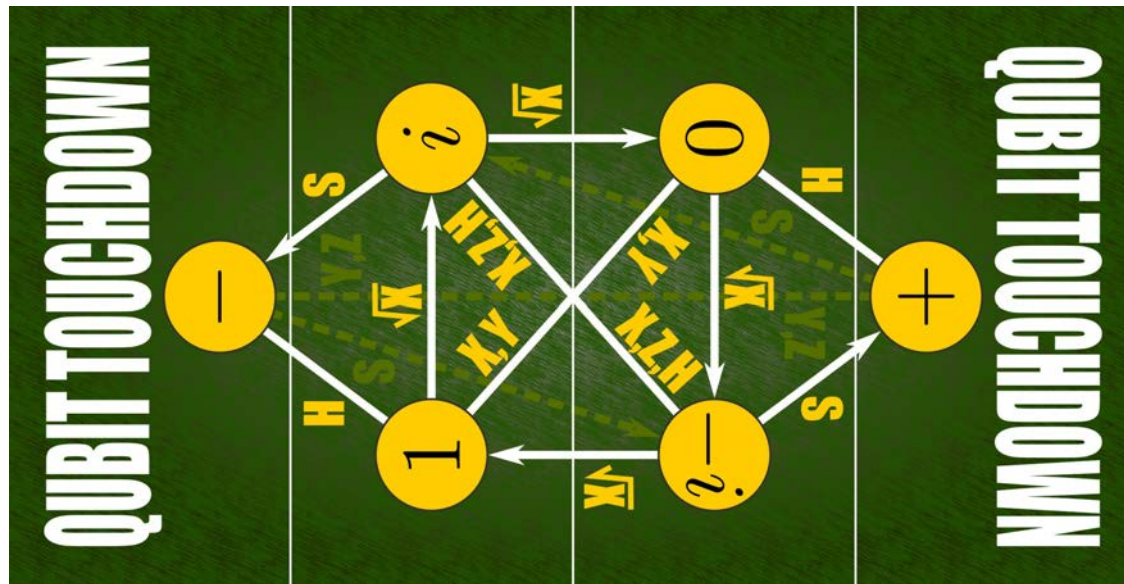
The Physics of Qubit Touchdown

- Quantum gates change a qubit's state



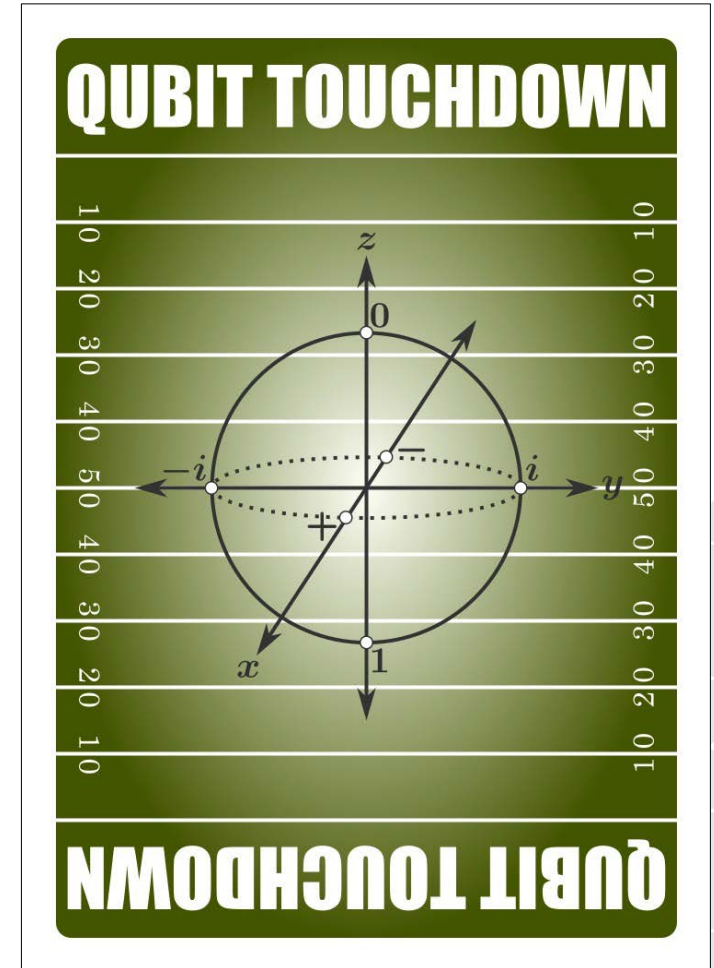
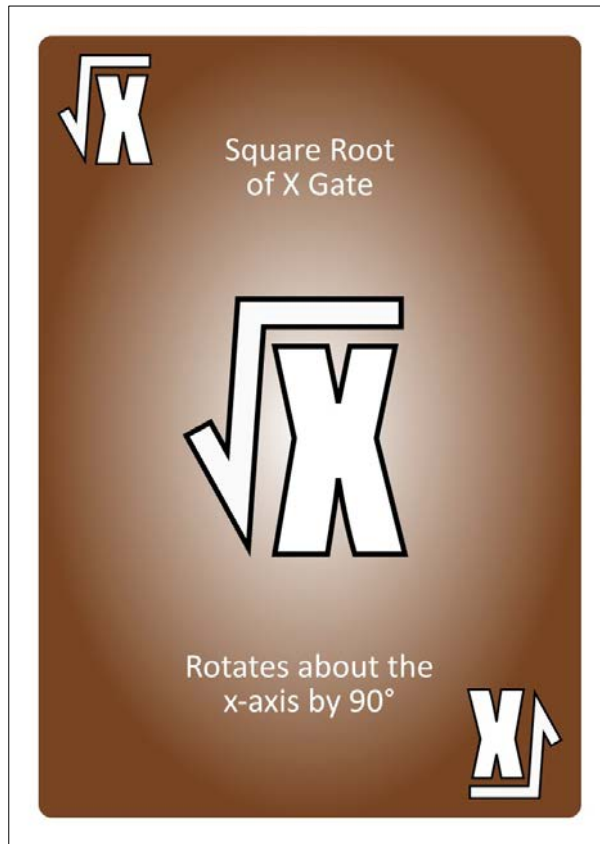
The Physics of Qubit Touchdown

- Quantum gates change a qubit's state



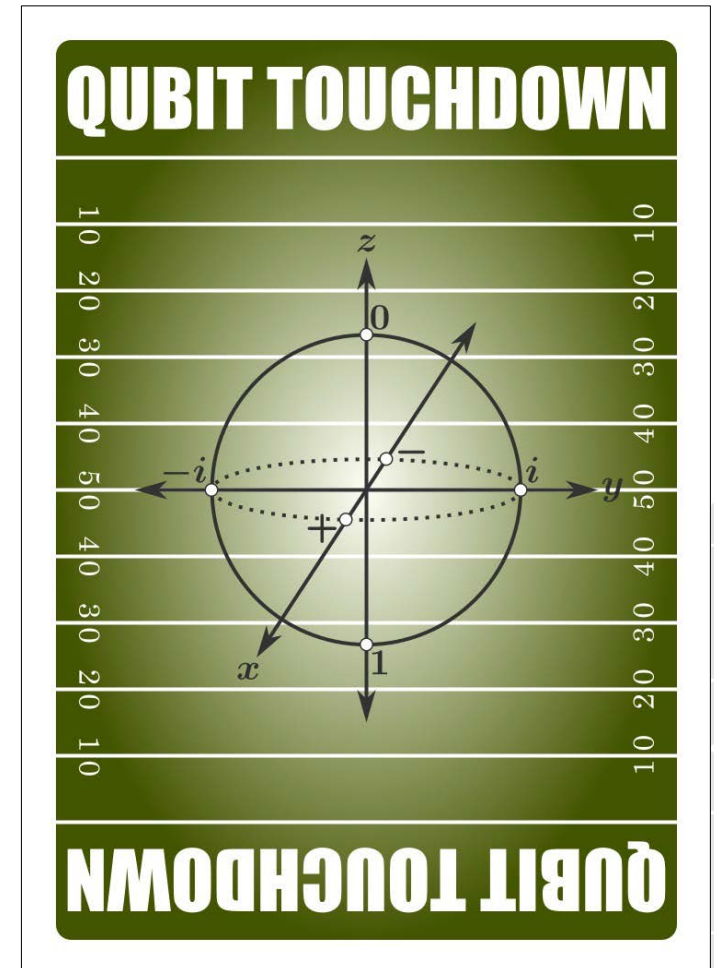
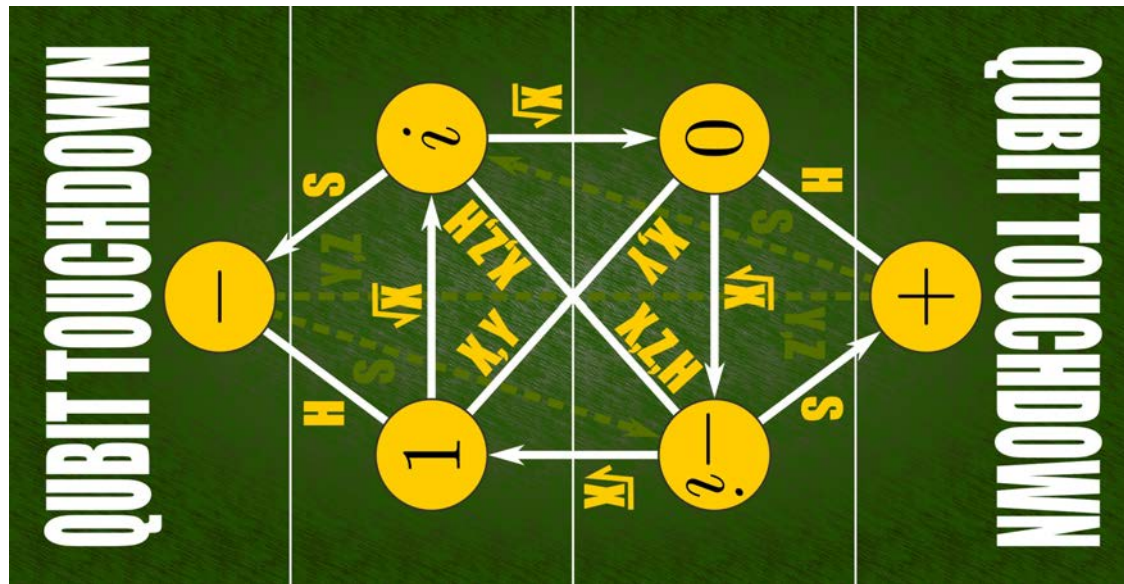
The Physics of Qubit Touchdown

- Quantum gates change a qubit's state



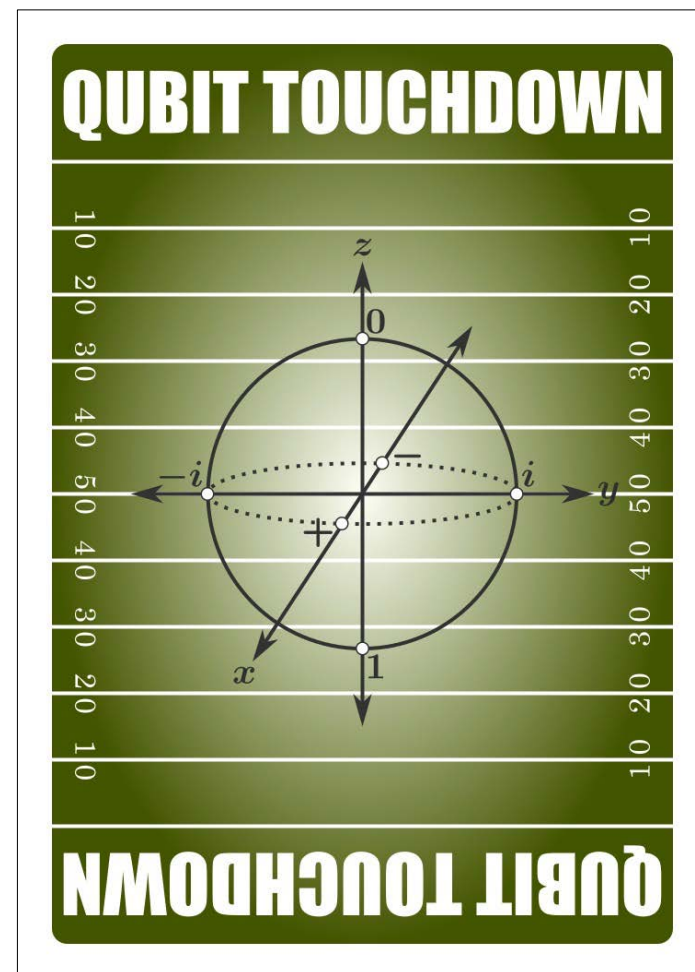
The Physics of Qubit Touchdown

- Quantum gates change a qubit's state



The Physics of Qubit Touchdown

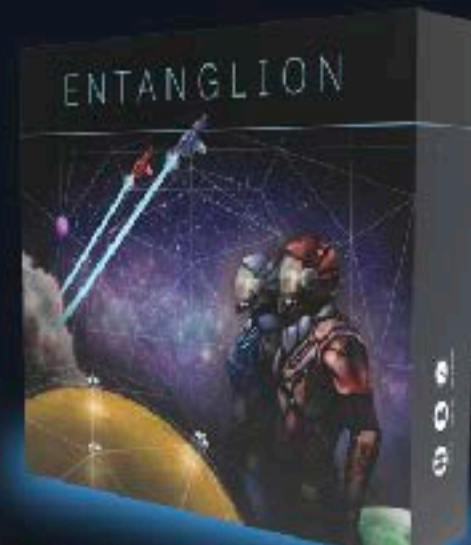
- Although a qubit can be any point on the Bloch sphere, **measuring its value yields 0 or 1, each with some probability.**
- Since $+$, $-$, i , $-i$ lie on the equator, they are **superpositions, or combinations, of half 0 and half 1.** Measuring them yields 0 or 1, each with 50% probability.
- Kicking off after a touchdown corresponds to a measurement.





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THE UNIVERSITY OF ARIZONA

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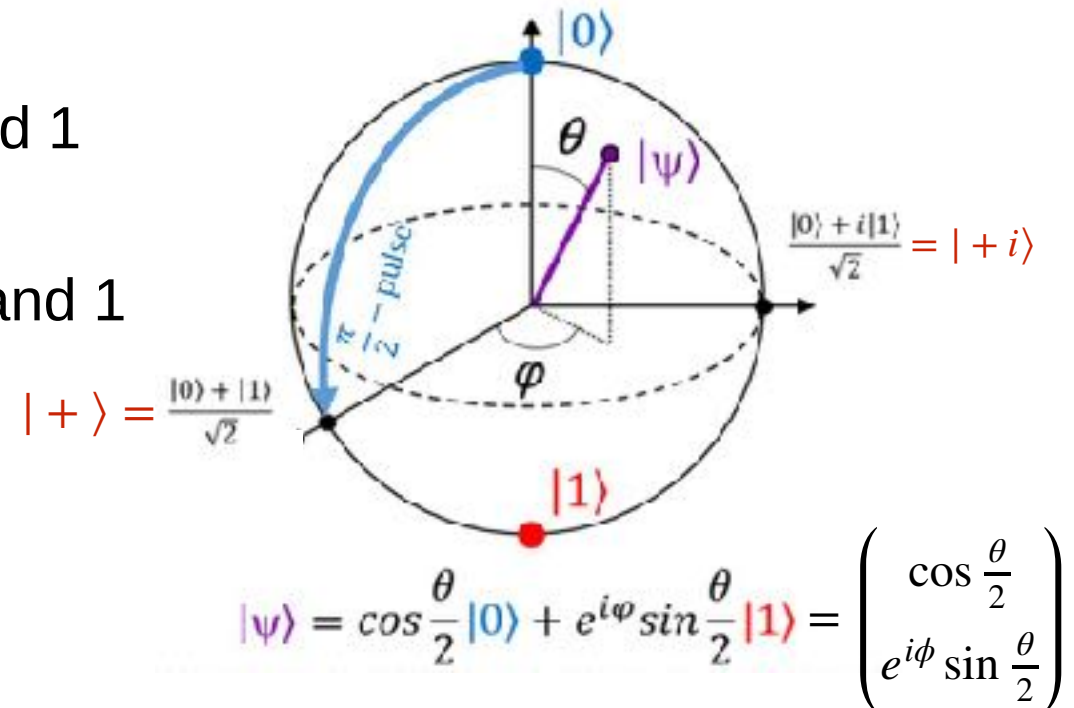
Educational Resources



Center for
Quantum Networks
NSF Engineering Research Center

Quantum Bits (Qubits): Unit-normalized two dimensional complex vector

- **Qubit** - quantum bit
- **Superposition** - combination of 0 and 1 ($|0\rangle$ and $|1\rangle$)
- **Amplitude** - coefficient in front of 0 and 1 ($|0\rangle$ and $|1\rangle$)
- **Measurement** - get $|0\rangle$ or $|1\rangle$ with probability = $|\text{amplitude}|^2$
 - **Qubit collapses** to $|0\rangle$ or $|1\rangle$
- **Normalized** - total probability is 1.
- **Entanglement** - outcome two qubits are intertwined.



$$|\pm\rangle = \frac{1}{\sqrt{2}}|0\rangle \pm \frac{1}{\sqrt{2}}|1\rangle$$

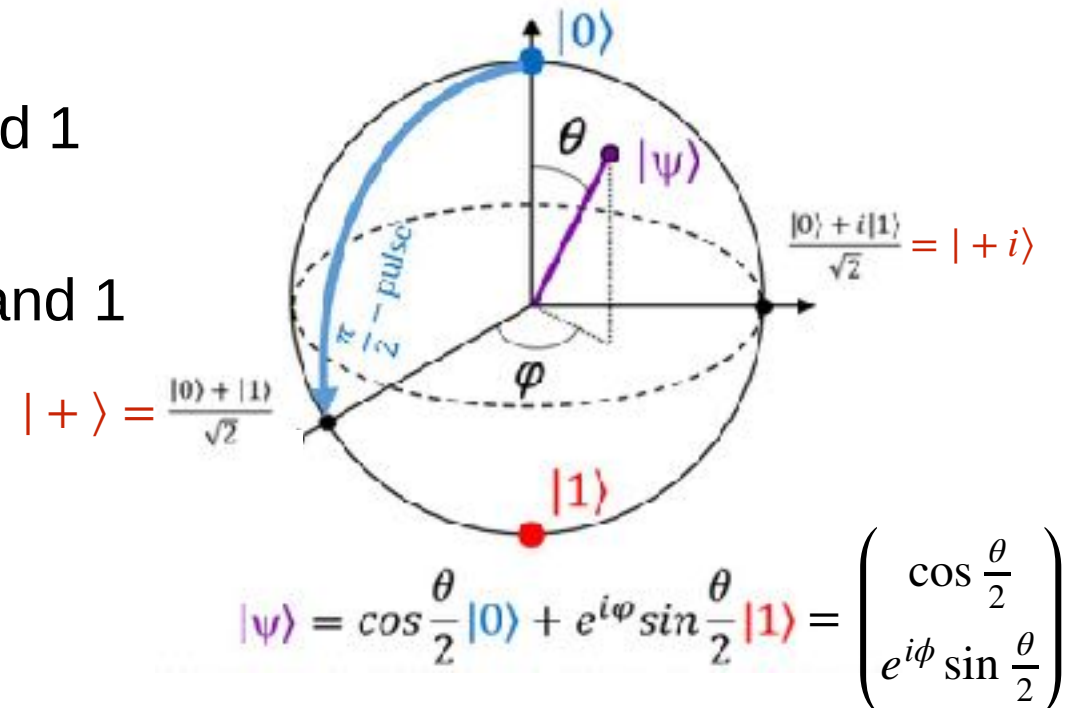
$$|\pm i\rangle = \frac{1}{\sqrt{2}}|0\rangle \pm i\frac{1}{\sqrt{2}}|1\rangle$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \qquad \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$



Quantum Bits (Qubits): Unit-normalized two dimensional complex vector

- **Qubit** - quantum bit
- **Superposition** - combination of 0 and 1 ($|0\rangle$ and $|1\rangle$)
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$$|\pm\rangle = \frac{1}{\sqrt{2}}|0\rangle \pm \frac{1}{\sqrt{2}}|1\rangle$$

$$|\pm i\rangle = \frac{1}{\sqrt{2}}|0\rangle \pm i\frac{1}{\sqrt{2}}|1\rangle$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \qquad \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

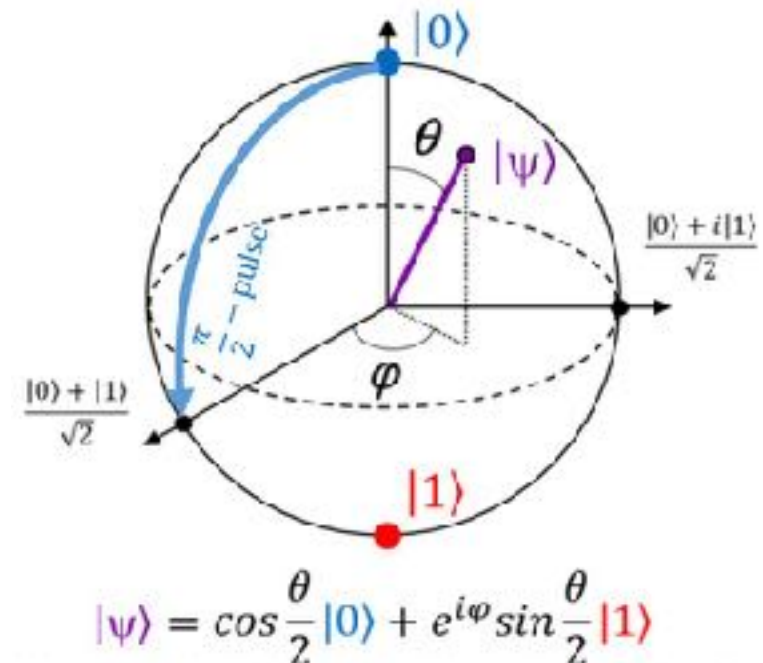


Why Quantum Computing?

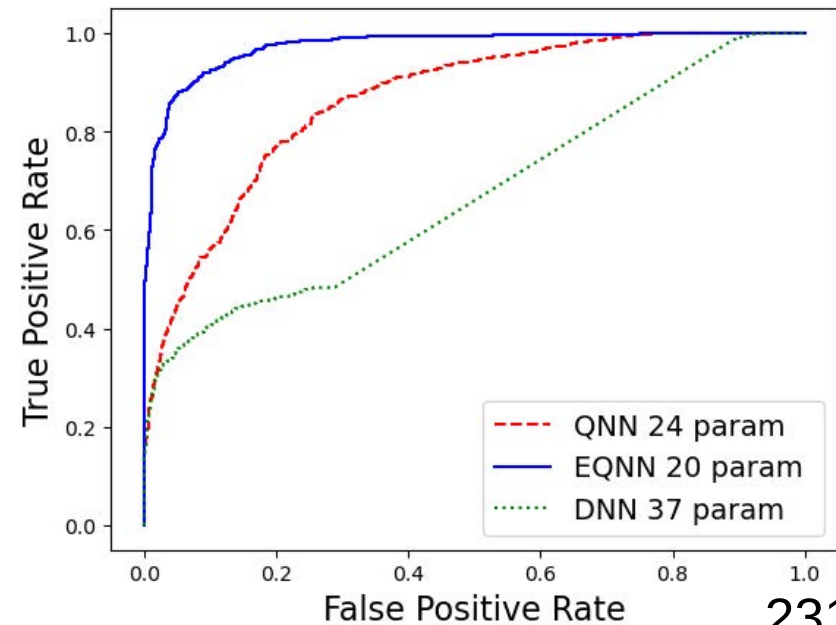
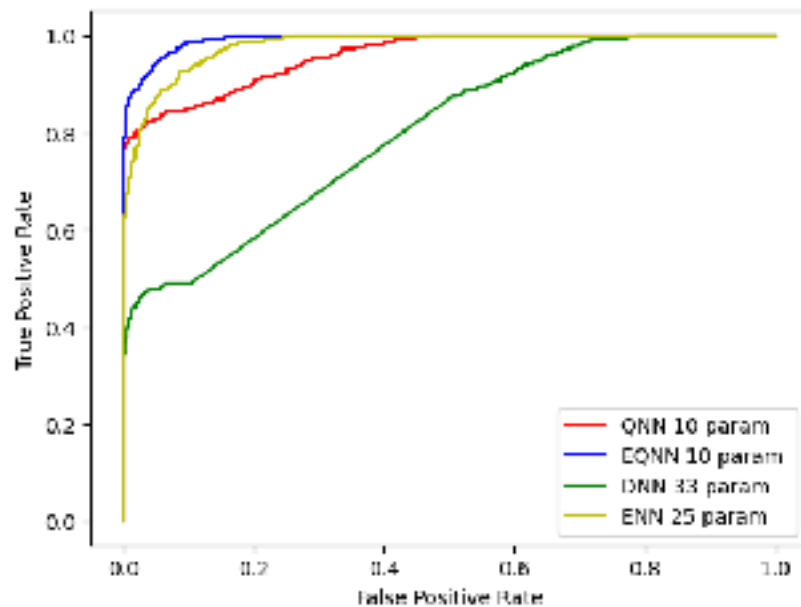
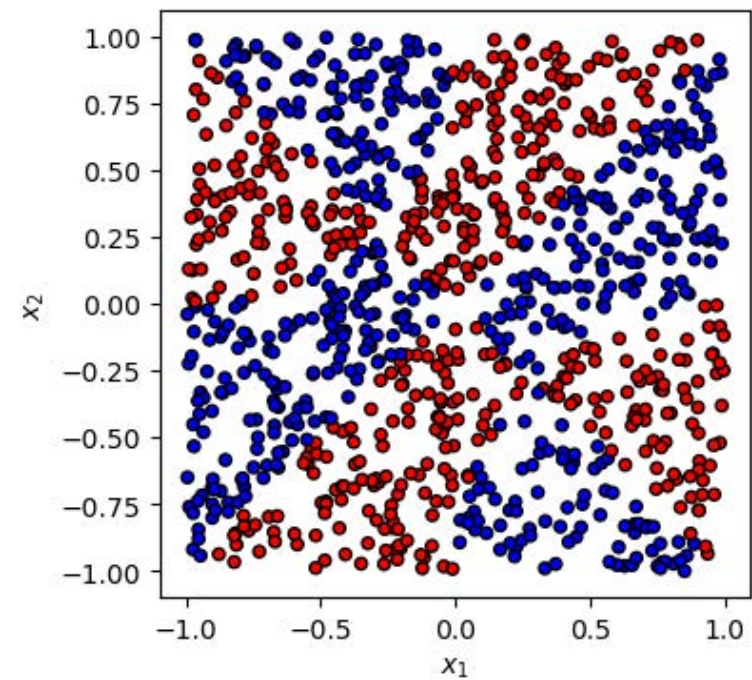
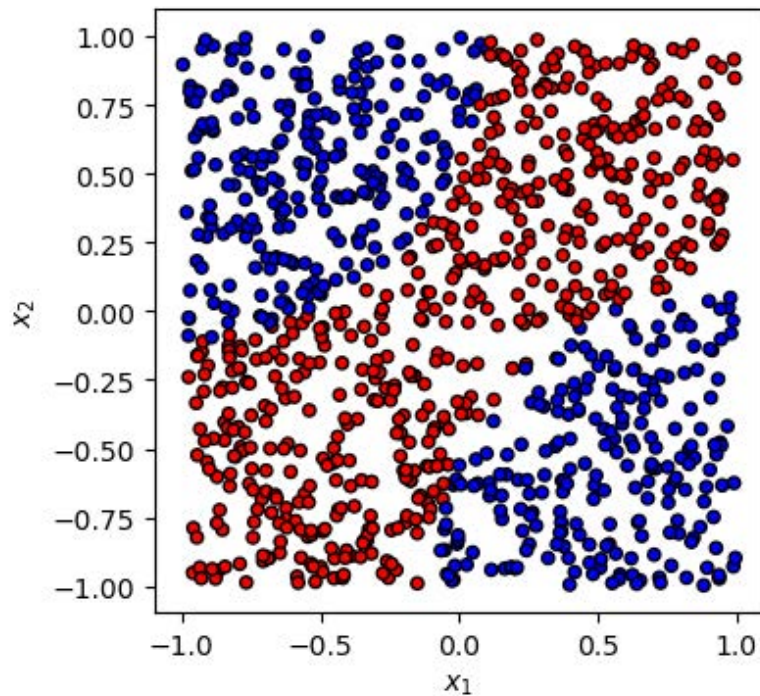
- Cryptography: factoring, **hidden subgroup program**, **discrete logarithm problem**
- **Optimization**
- Search algorithm
- Quantum simulation
- Quantum Machine Learning
 - Quantum Advantages?
 - Learns better with small # of data
 - Faster convergence
 - Less # of parameters
- **What are the interesting problems?**

Find the generators of the subgroup $H < G$, that is implicitly defined by a function f on G such that f is constant and takes distinct values on each coset of H .

Find the discrete logarithm of $y \in \mathbb{Z}_p^*$ with respect to base b such that $b^x = y \pmod{p}$



Equivariant Quantum Neural Networks

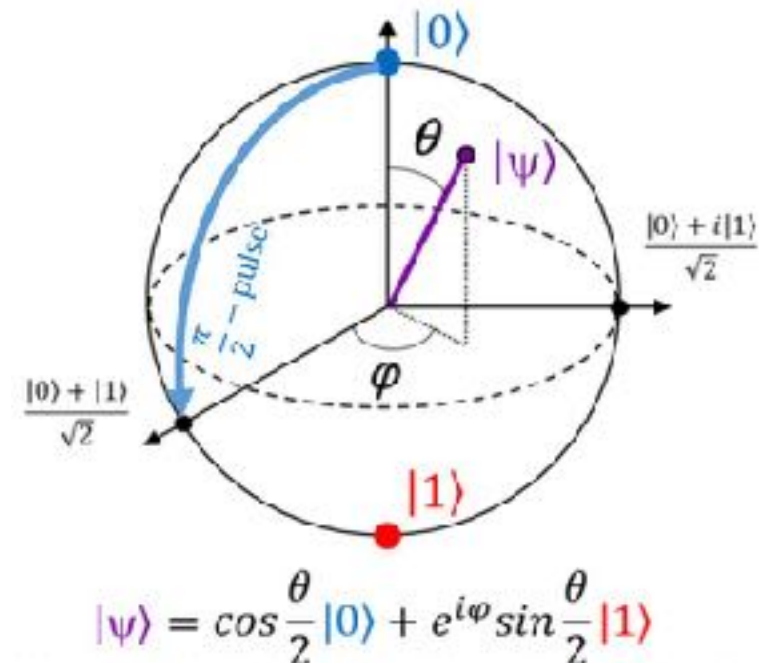


Why Quantum Computing?

- Cryptography: factoring, **hidden subgroup program**, **discrete logarithm problem**
- **Optimization**
- Search algorithm
- Quantum simulation
- Quantum Machine Learning
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Example: Inner product

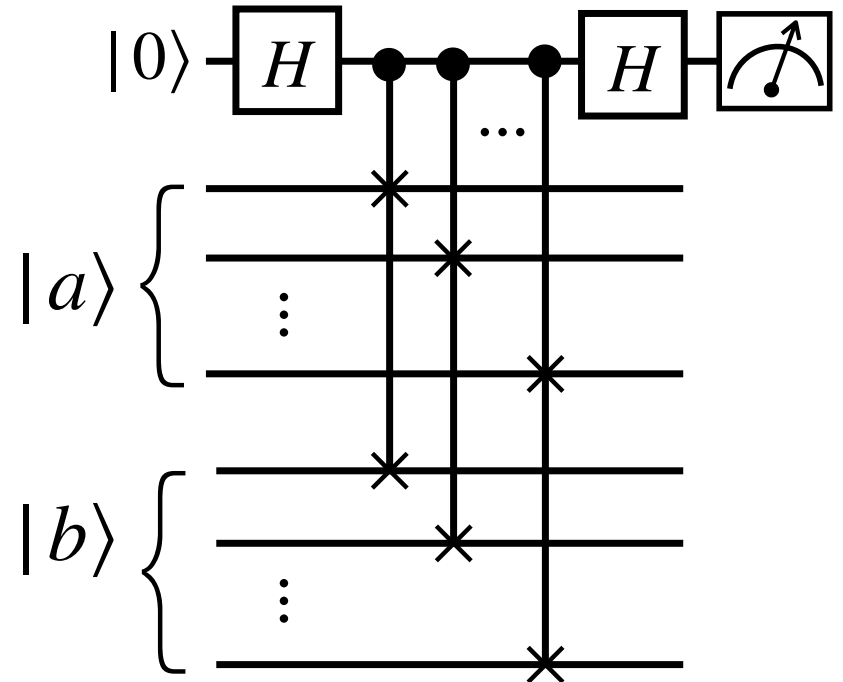
$$\vec{a} = (a_1, a_2, \dots, a_{1024}) = |a\rangle$$

$$\vec{b} = (b_1, b_2, \dots, b_{1024}) = |b\rangle$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \dots + a_{1024} b_{1024}$$

How many operations

= 1024 multiplications + 1023 additions



$|a\rangle$: 10 qubits are needed to embed $\vec{a} = (a_1, a_2, \dots, a_{1024})$

$|b\rangle$: 10 qubits are needed to embed $\vec{b} = (b_1, b_2, \dots, b_{1024})$

$|0\rangle$: 1 additional qubit as a registry

$$2^{10} = 1024$$

12 operations are needed.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

Example: Inner product

- Let $|\psi\rangle, |\phi\rangle \in \mathbb{C}^{2^n}$ be two $N = 2^n$ dimensional vectors. How to compute the magnitude of the inner product $|\langle\phi|\psi\rangle|^2$?

- Digital

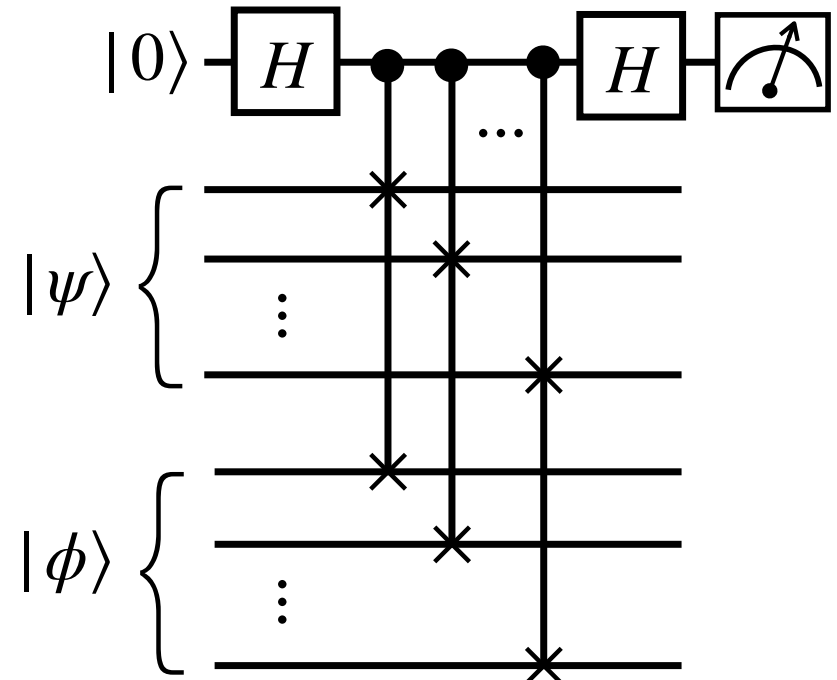
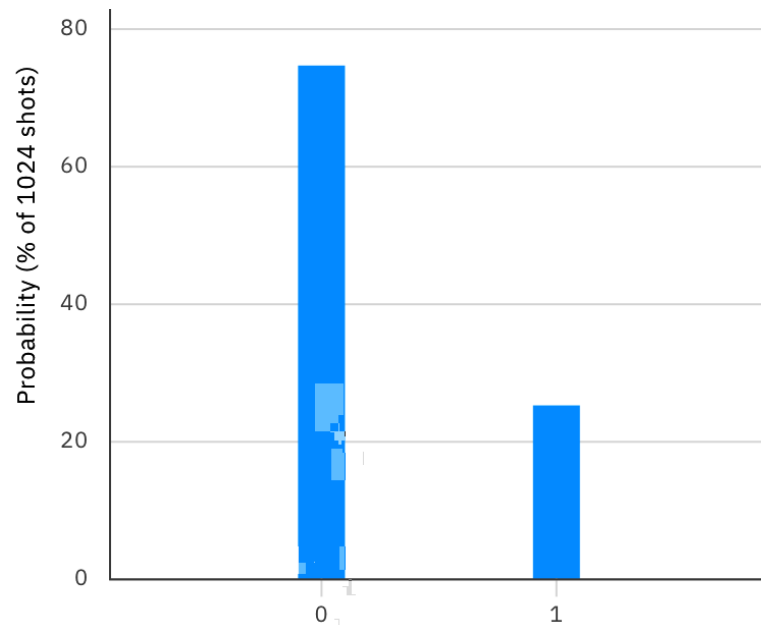
- $N = 2^n$ multiplication and additions
- Decompose multiplications and additions as NAND gates

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Quantum

- Run the following circuits with $2n + 1$ qubits and $n + 2$ gates
- $\text{Prob}(0) - \text{Prob}(1) = |\langle\phi|\psi\rangle|^2$



Proof: Inner product

$$|a\rangle \otimes |b\rangle = |a\rangle |b\rangle$$

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

$$(1) \quad H(|0\rangle |a\rangle |b\rangle) = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |a\rangle |b\rangle$$

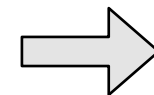
$$(2) \quad SWAP H(|0\rangle |a\rangle |b\rangle) = \frac{1}{\sqrt{2}} SWAP(|0\rangle + |1\rangle) |a\rangle |b\rangle = \frac{1}{\sqrt{2}} (|0\rangle |a\rangle |b\rangle + |1\rangle |b\rangle |a\rangle)$$

$$(3) \quad \begin{aligned} |\psi\rangle &= H SWAP H(|0\rangle |a\rangle |b\rangle) = \frac{1}{\sqrt{2}} H(|0\rangle |a\rangle |b\rangle + |1\rangle |b\rangle |a\rangle) \\ &= \frac{1}{2} |0\rangle (|a\rangle |b\rangle + |b\rangle |a\rangle) + \frac{1}{2} |1\rangle (|a\rangle |b\rangle - |b\rangle |a\rangle) \end{aligned}$$

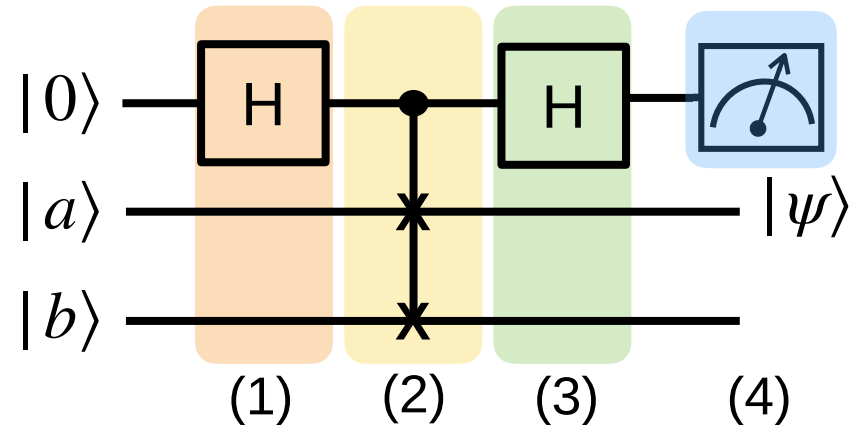
sum of the “unswapped” and the “swapped”

difference

$$(4) \quad \begin{aligned} P(0) &= |(\langle 0 \otimes I) |\psi\rangle|^2 = \frac{1}{2} - \frac{1}{2} |\langle a|b\rangle|^2 \\ P(1) &= |(\langle 1 \otimes I) |\psi\rangle|^2 = \frac{1}{2} + \frac{1}{2} |\langle a|b\rangle|^2 \end{aligned}$$



$$P(1) - P(0) = |\langle a|b\rangle|^2$$



Efficient vs inefficient algorithms

- Efficient: computation time scales polynomially with problem's size.

– (Ex) n^4 (tensor operations, i.e., $\langle ij | k\ell \rangle$):

- $n=1000$: $t = 1$ sec

- $n=1050$: $t = 1.2$ sec

$$\left(\frac{1050}{1000}\right)^4 \approx 1.2$$

- Inefficient: computation time scales exponentially.

– (Ex) 2^n (factoring)

- $n=1000$: $t = 1$ sec

- $n=1001$: $t = 2$ sec

- $n=1006$: $t = 1$ min

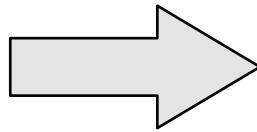
- $n=1012$: $t = 1$ hour

- $n=1050$: $t \sim 3.3$ million years

$$\left(\frac{2^{1006}}{2^{1000}}\right) = 2^6 = 64$$

$$\left(\frac{2^{1050}}{2^{1000}}\right) = 2^{50}$$

Using vector notation



Using Dirac bracket notation

$$\vec{a} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

$$\vec{b} = (1, 0)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \frac{1}{\sqrt{2}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 \\ &= \frac{1}{\sqrt{2}} \approx 0.707 \end{aligned}$$

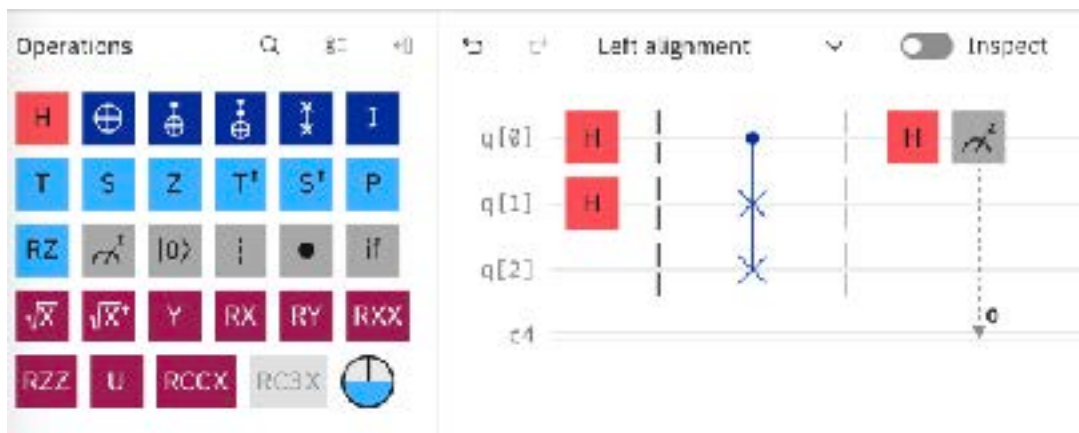
$$|\vec{a} \cdot \vec{b}|^2 = 0.5$$

$$|a\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = H|0\rangle$$

$$|b\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{aligned} \langle a|b\rangle &= \frac{1}{\sqrt{2}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 \\ &= \frac{1}{\sqrt{2}} \approx 0.707 \end{aligned}$$

$$|\langle a|b\rangle|^2 = P(1) - P(0) = 0.5$$



IBM Quantum
Composer

Example: MathMagic

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

2	3	6	7	10	11	14	15
18	19	22	23	26	27	30	31
34	35	38	39	42	43	46	47
50	51	54	55	58	59	62	63

4	5	6	7	12	13	14	15
20	21	22	23	28	29	30	31
36	37	38	39	44	45	46	47
52	53	54	55	60	61	62	63

8	9	10	11	12	13	14	15
24	25	26	27	28	29	30	31
40	41	42	43	44	45	46	47
56	57	58	59	60	61	62	63

16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

XXXXXX1

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

XXXXX1X

2	3	6	7	10	11	14	15
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XXX1XX

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48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

XX1XXX

X1XXXX

1XXXXXX

Suppose your number was 43

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

2	3	6	7	10	11	14	15
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40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

In base 2: 101011

Suppose your number was 43

1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

2	3	6	7	10	11	14	15
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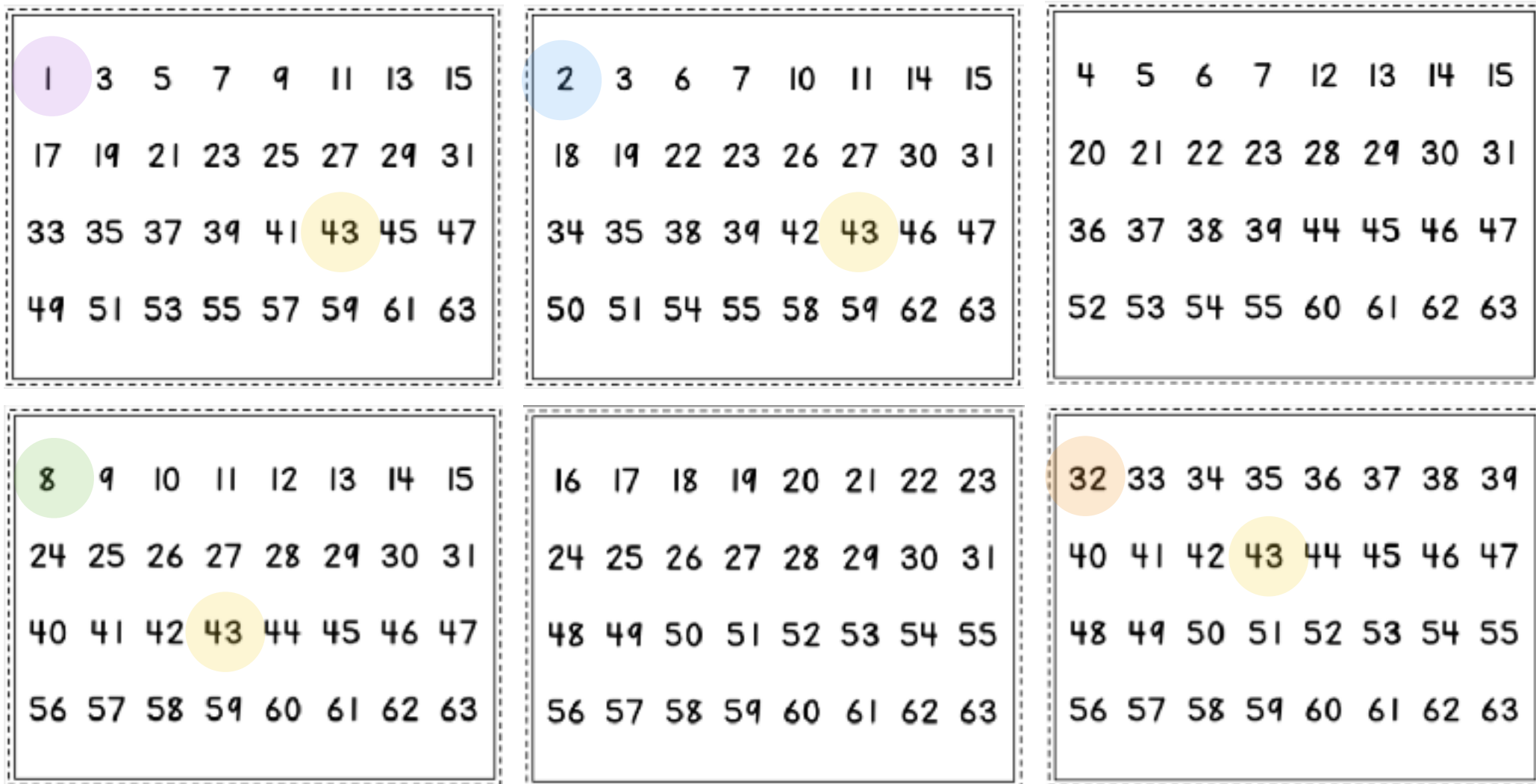
8	9	10	11	12	13	14	15
24	25	26	27	28	29	30	31
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40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

In base 2: $101011 = 100000 + 001000 + 000010 + 000001$

Suppose your number was 43



In base 2: $101011 = 100000 + 001000 + 000010 + 000001$

In base 10:

= 32 + 8 + 2 + 1

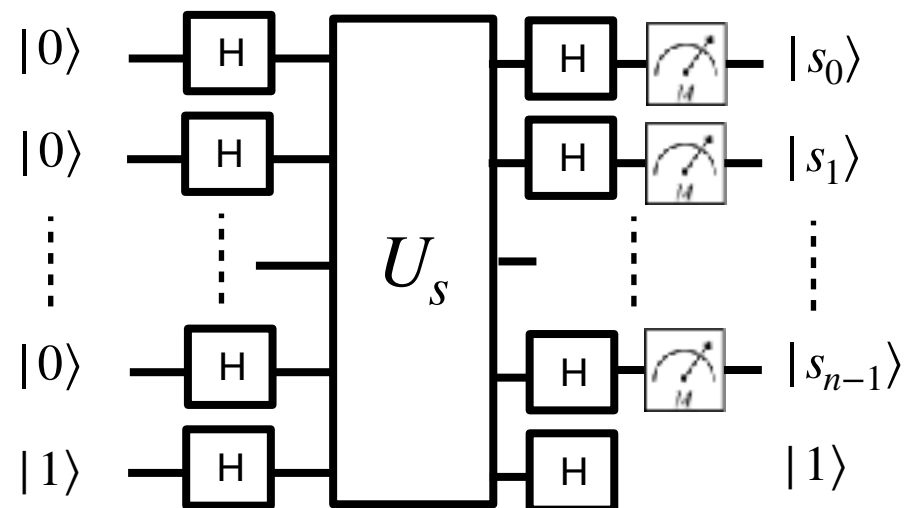
= 43

Is there a way to find out the answer by asking less than 6 questions?

- Core components of Quantum Information Science and Technology
 - Need different set of rules and a novel algorithm must be developed (quantum algorithms)
 - Need a device that can execute the algorithm (quantum devices)
 - Suitable materials are required to build such a device (quantum materials)

Bernstein-Vazirani Algorithm

- A n -bit function $f: \{0,1\}^{\otimes n} \longrightarrow \{0,1\}$, which outputs a single bit, is guaranteed to be of the form $f_s(x) = x \cdot s$, where s is an unknown n -bit string and $x \cdot s = x_0s_0 + \dots + x_{n-1}s_{n-1} = \sum_{i=0}^{n-1} x_i s_i \pmod{2}$. Find the unknown string $s = (s_0s_1 \dots s_{n-1})$.
- Best classical algorithm uses $\mathcal{O}(n)$ calls to $f_s(x) = x \cdot s \pmod{2}$. Each query gives one bit of information of s (because f outputs one bit).
- How do we find s with less than n queries? $\rightarrow$ Use superposition (over all possible input bit strings)



\$ vs Tiger

- Teaching quantum information science to high-school and early undergraduate students by Sophia Economou, Terry Rudolph, Edwin Barnes, 2005.07874
- You encounter two doors:
Money behind at least one door

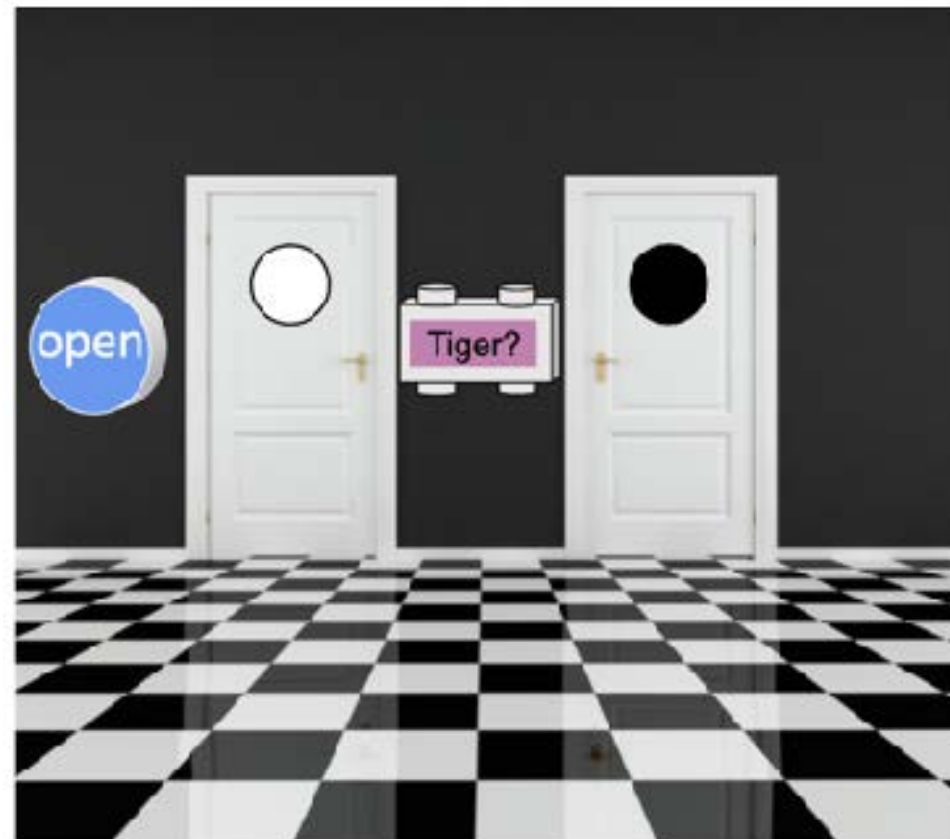


- Tiger might be lurking behind one door



\$ vs Tiger

- The button on the left opens both doors
- YOU WANT TO BE SURE THERE'S NO TIGER BEHIND EITHER DOOR BEFORE YOU PUSH THE "OPEN" BUTTON
- The device in the middle will tell you **if there is a tiger behind the door that you ask about** – but **you only get to use it once**



Deutsch Algorithm

1985

- We want to find out whether a particular function, with one input bit and one output bit is constant or balanced. **Classically, we need to evaluate the function twice** (i.e., for input = 0 and input = 1), but remarkably, **we only need to evaluate the function once using quantum algorithm**, by using Deutsch's algorithm.

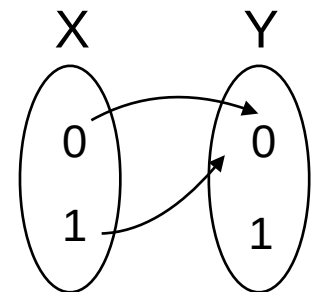
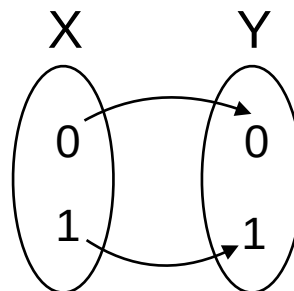
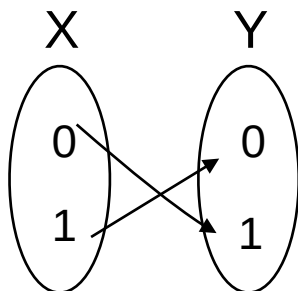
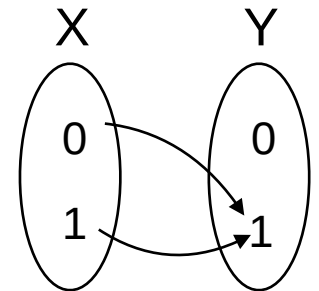
- Consider a simple function, $f(x) : \{0,1\} \longrightarrow \{0,1\}$

- For possible functions

– Identity: $f(0) = 0$ and $f(1) = 1$

– Bit-flip function: $f(0) = 1$ and $f(1) = 0$

– Constant function: $f(x) = 0$ or $f(x) = 1$



The First Wave of Quantum Machine Learning?

PRL 103, 150502 (2009)

PHYSICAL REVIEW LETTERS

week ending
9 OCTOBER 2009



Quantum Algorithm for Linear Systems of Equations

Aram W. Harrow,¹ Avinatan Hassidim,² and Seth Lloyd³

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(Received 5 July 2009; published 7 October 2009)

Solving linear systems of equations is a common problem that arises both on its own and as a subroutine in more complex problems: given a matrix A and a vector $\vec{b}$, find a vector $\vec{x}$ such that $A\vec{x} = \vec{b}$. We consider the case where one does not need to know the solution $\vec{x}$ itself, but rather an approximation of the expectation value of some operator associated with $\vec{x}$, e.g., $\vec{x}^\dagger M \vec{x}$ for some matrix M . In this case, when A is sparse, $N \times N$ and has condition number κ , the fastest known classical algorithms can find $\vec{x}$ and estimate $\vec{x}^\dagger M \vec{x}$ in time scaling roughly as $N\sqrt{\kappa}$. Here, we exhibit a quantum algorithm for estimating $\vec{x}^\dagger M \vec{x}$ whose runtime is a polynomial of $\log(N)$ and κ . Indeed, for small values of κ [i.e., $\text{poly} \log(N)$], we prove (using some common complexity-theoretic assumptions) that any classical algorithm for this problem generically requires exponentially more time than our quantum algorithm.

$$Ax = b$$

Complexity of inversion of a regular matrix= $O(N^3)$

Complexity of inversion of a sparse matrix= $O(N)$

The First Wave of Quantum Machine Learning?

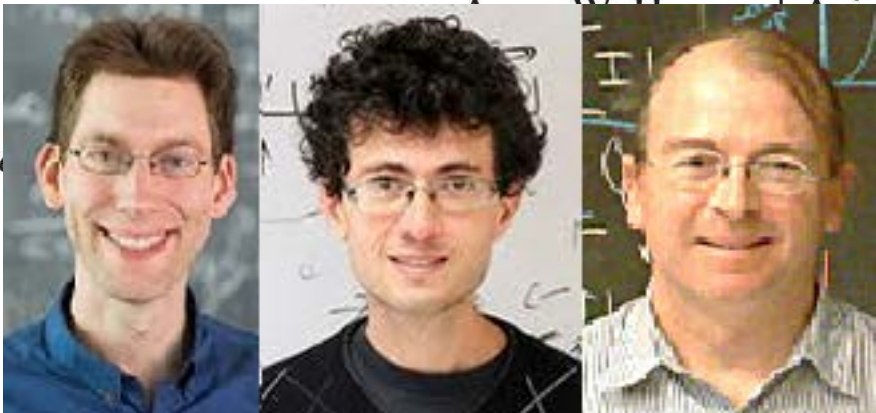
PRL 103, 150502 (2009)

PHYSICAL REVIEW LETTERS

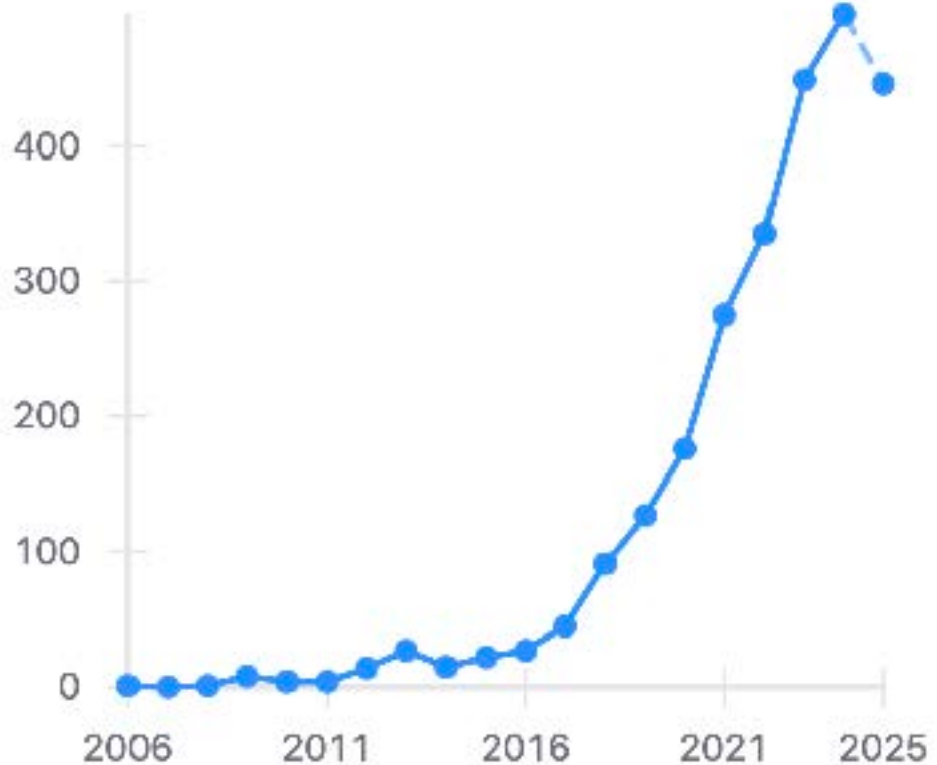
week ending
9 OCTOBER 2009

Quantum Algorithm for Linear Algebra

Citations per year



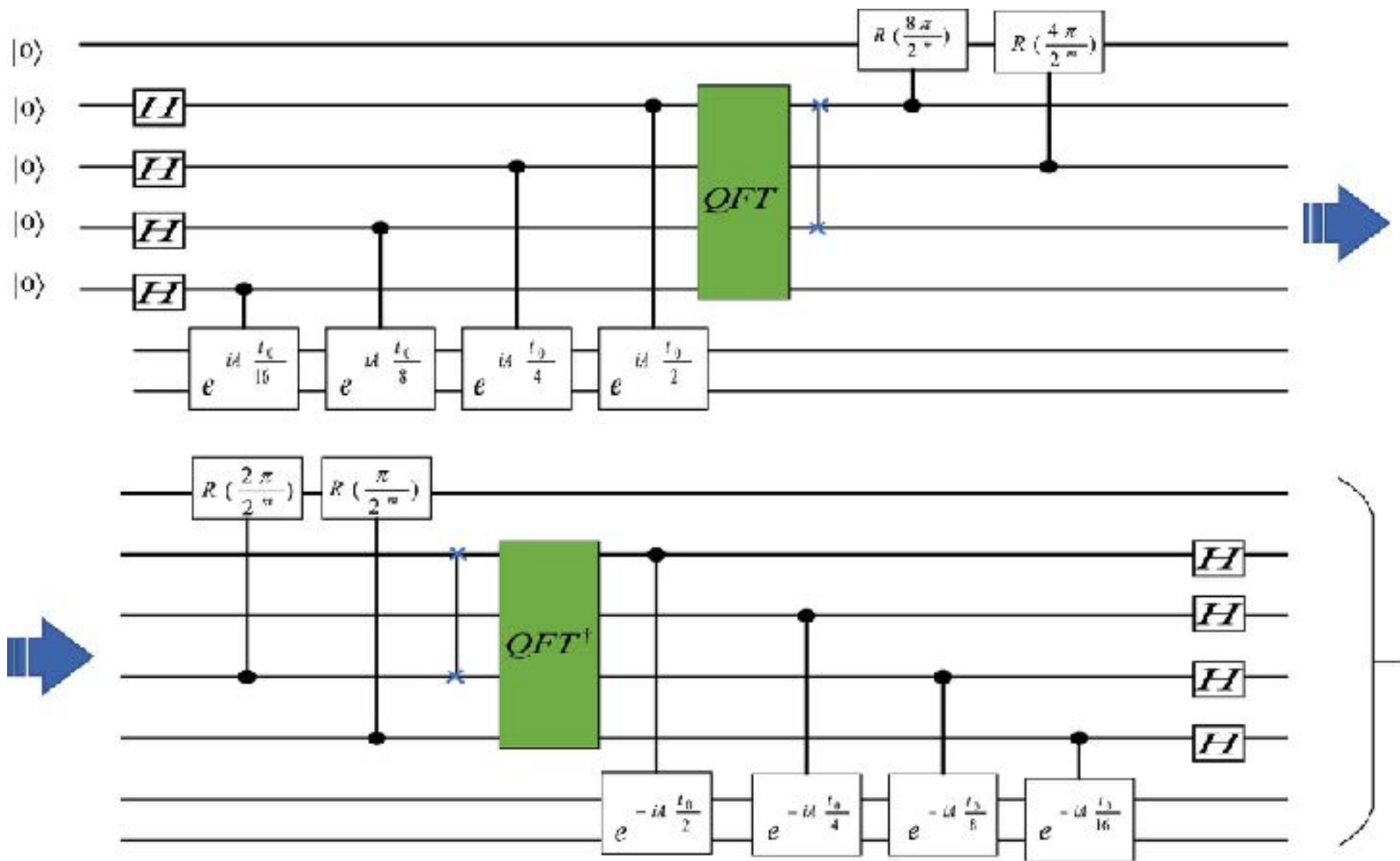
expectation value of some operator associated with is sparse, $N \times N$ and has condition number κ , to estimate $\tilde{x}^\dagger M \tilde{x}$ in time scaling roughly as $N\sqrt{\kappa}$. $\tilde{x}^\dagger M \tilde{x}$ whose runtime is a polynomial of $\log(N)$ and prove (using some common complexity-theoretic problem generically requires exponentially more t



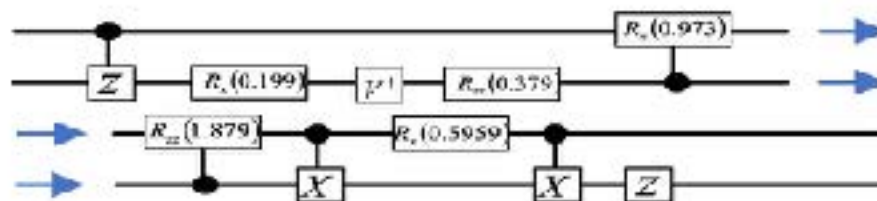
$$Ax = b$$

Complexity of inversion of a regular matrix = $O(N^3)$

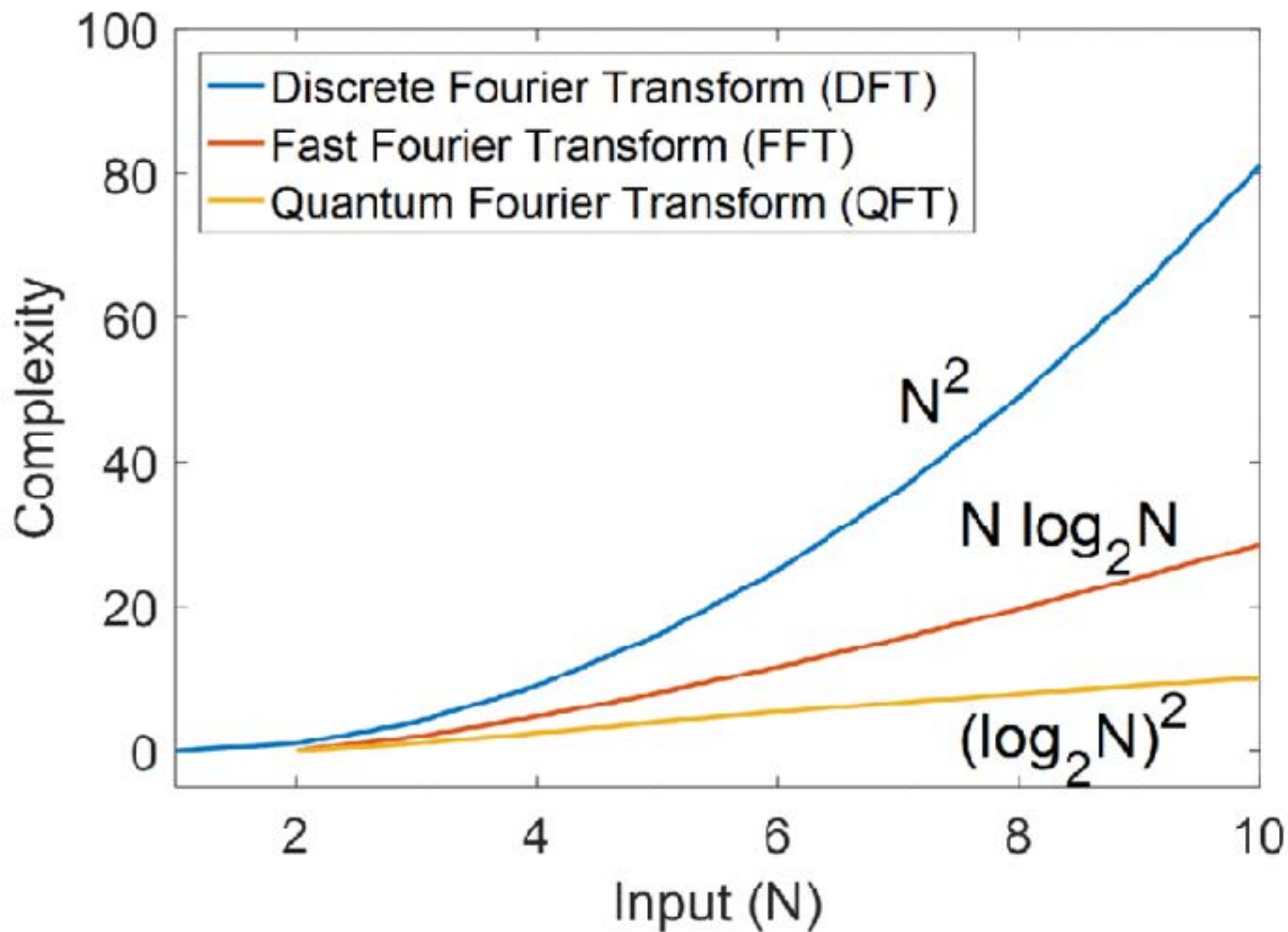
Complexity of inversion of a sparse matrix = $O(N)$



$$\left[\begin{array}{c} \text{---} \\ \text{---} \end{array} \right] e^{-i\theta_1/16} \left[\begin{array}{c} \text{---} \\ \text{---} \end{array} \right] =$$



DFT: N^2
 FFT: $N \log_2 N$
 QFT: $(\log_2 N)^2$



Experimental realization

PRL 110, 230501 (2013)

PHYSICAL REVIEW LETTERS

week ending
7 JUNE 2013



Experimental Quantum Computing to Solve Systems of Linear Equations

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(Received 6 March 2013; published 6 June 2013)

Solving linear systems of equations is ubiquitous in all areas of science and engineering. With rapidly growing data sets, such a task can be intractable for classical computers, as the best known classical algorithms require a time proportional to the number of variables N . A recently proposed quantum algorithm shows that quantum computers could solve linear systems in a time scale of order $\log(N)$, giving an exponential speedup over classical computers. Here we realize the simplest instance of this algorithm, solving 2×2 linear equations for various input vectors on a quantum computer. We use four quantum bits and four controlled logic gates to implement every subroutine required, demonstrating the working principle of this algorithm.

["A two-qubit photonic quantum processor and its application to solving systems of linear equations". Scientific Reports. 4: 6115.](#)
["Experimental realization of quantum algorithm for solving linear systems of equations". Physical Review A. 89 \(2\): 022313](#)

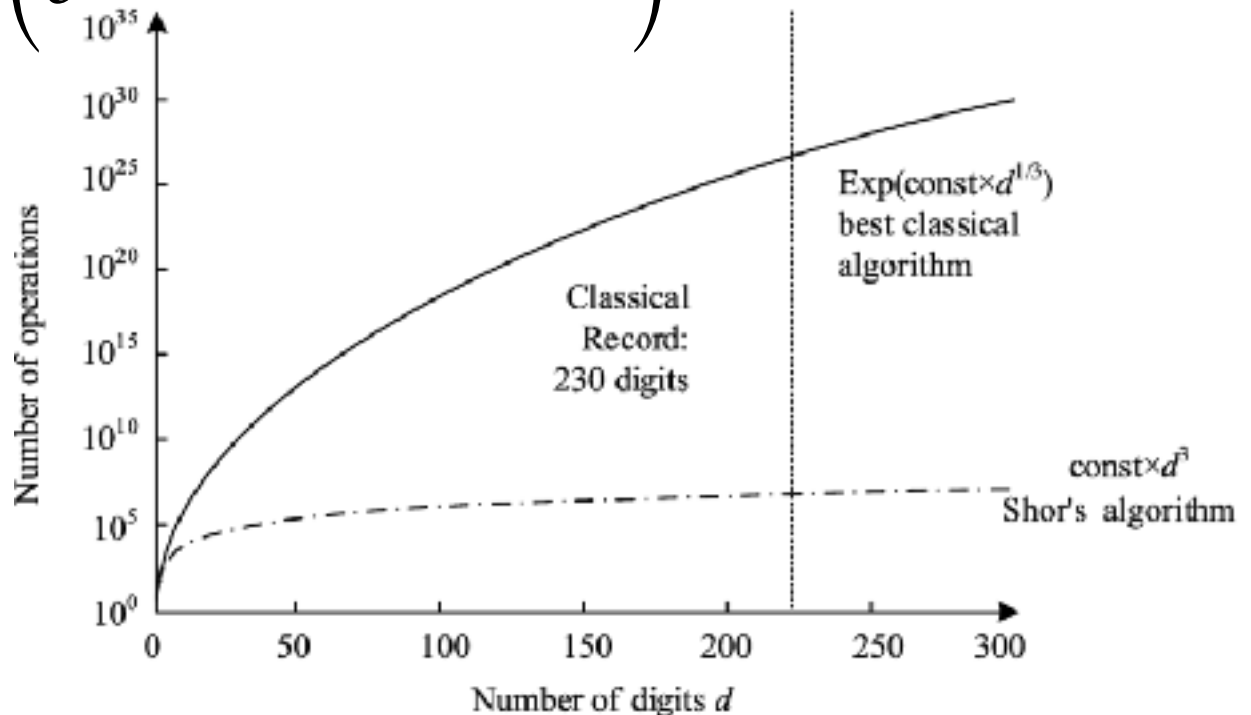
[Quantum algorithms for systems of linear equations inspired by adiabatic quantum computing, Phys.Rev.Lett. 122 \(2019\) 6, 060504](#)
==> ["Experimental realization of quantum algorithms for a linear system inspired by adiabatic quantum computing". Phys. Rev. A 99, 012320.](#) 8 dimensional linear equation.

Shor's factoring algorithm

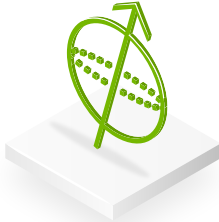
- To factor an integer N , Shor's algorithm runs in polynomial time, meaning the time taken is polynomial in $\log N$, **the size of the integer** given as input. Specifically, it takes quantum gates of order $O((\log N)^2(\log \log N)(\log \log \log N))$.
- This is significantly faster than the most efficient known classical factoring algorithm, the general number field sieve, which works in sub-exponential time: $O(e^{1.9(\log N)^{1/3}(\log \log N)^{2/3}})$

RSA-250 =
641352894770715802787901901705773
890848250147429434472081168596320
245323446302386235987526683477087
37661925585694639798853367
 ×
 333720275949781565562260106053551
 142279407603447675546667845209870
 238417292100370802574486732968818
 77565718986258036932062711

RSA factoring challenge
 (Product of exactly two primes)



A few popular tools for quantum simulation (Classical Quantum Computers)

- Qiskit (IBM)  **Qiskit**
 - IBM Quantum Composer
 - IBM Quantum Platform
 - PennyLane and Strawberry Fields (Xanadu)  PENNYLANE
 - TensorFlowQuantum (Google)  TensorFlow Quantum
 - CUDA Quantum (NVIDIA) 
 - TensorCircuit 
- 

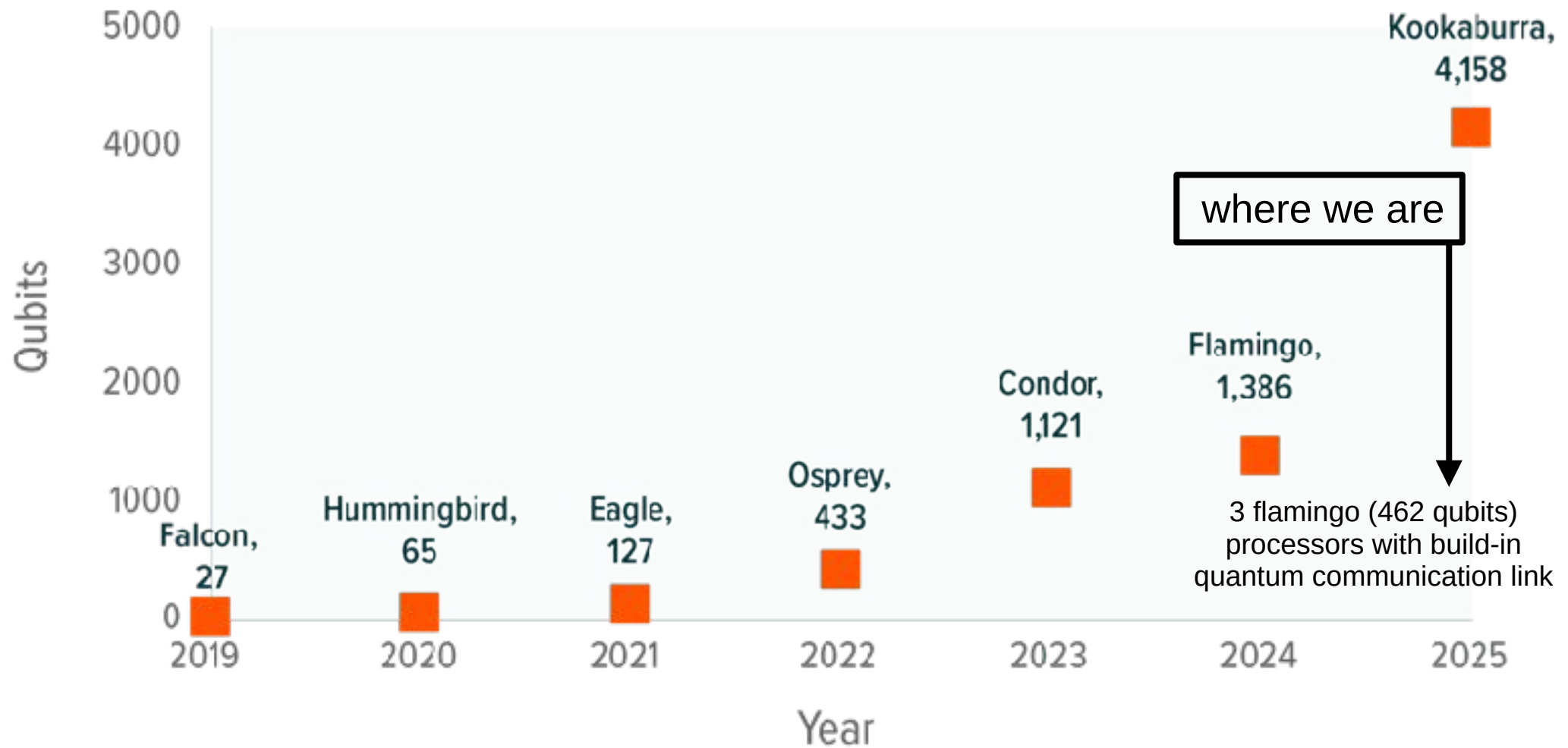
NetKet



Tensor Network

IBM QUANTUM PROCESSORS ROADMAP

Sources: Global X analysis of information derived from: IBM. (2022). Our new 2022 development roadmap.



Note: 2022 onwards includes planned processor launches.

IBM has been following a quantum-computing road map that roughly doubles the number of qubits every year.

Quantum computers are hard to build

- Qubits, unlike classical bits, need to interact strongly among themselves to form entangled states, which in turn form the basis for computation in quantum computers. But to achieve this experimentally is incredibly hard.

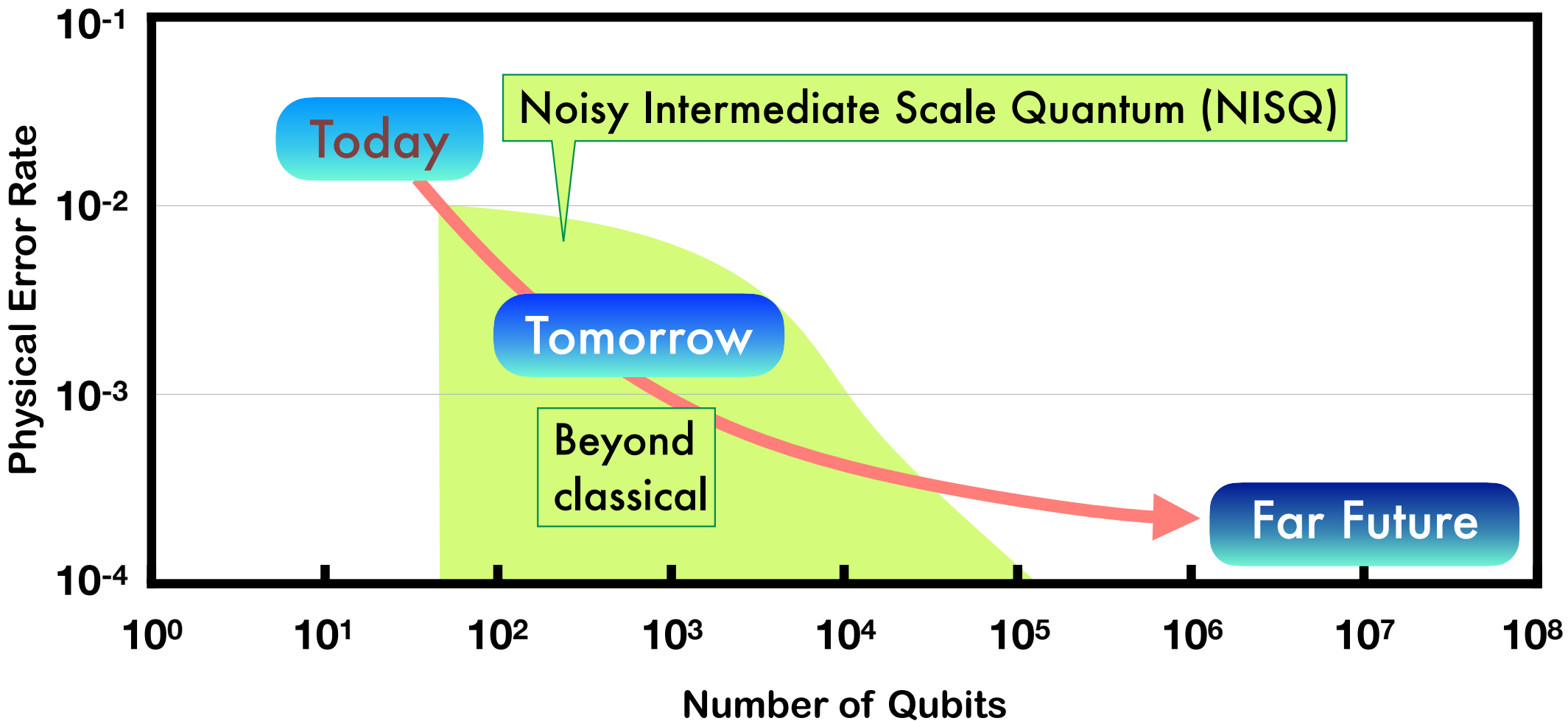


Quantum computers are hard to build

- Qubits, unlike classical bits, need to interact strongly among themselves to form entangled states, which in turn form the basis for computation in quantum computers. But to achieve this experimentally is incredibly hard.



Quantum Hardware Roadmap



A quantum computing partnership with the University of Chicago and the University of Tokyo

100,000-qubit system by 2033

The commitment, to be signed May 21 on the sidelines of the G7 and U.S.-Japan Leaders' Meeting, aims to advance development of a fault-tolerant quantum computer by supporting research, entrepreneurship and workforce training.

May 17, 2023 · 2 min read

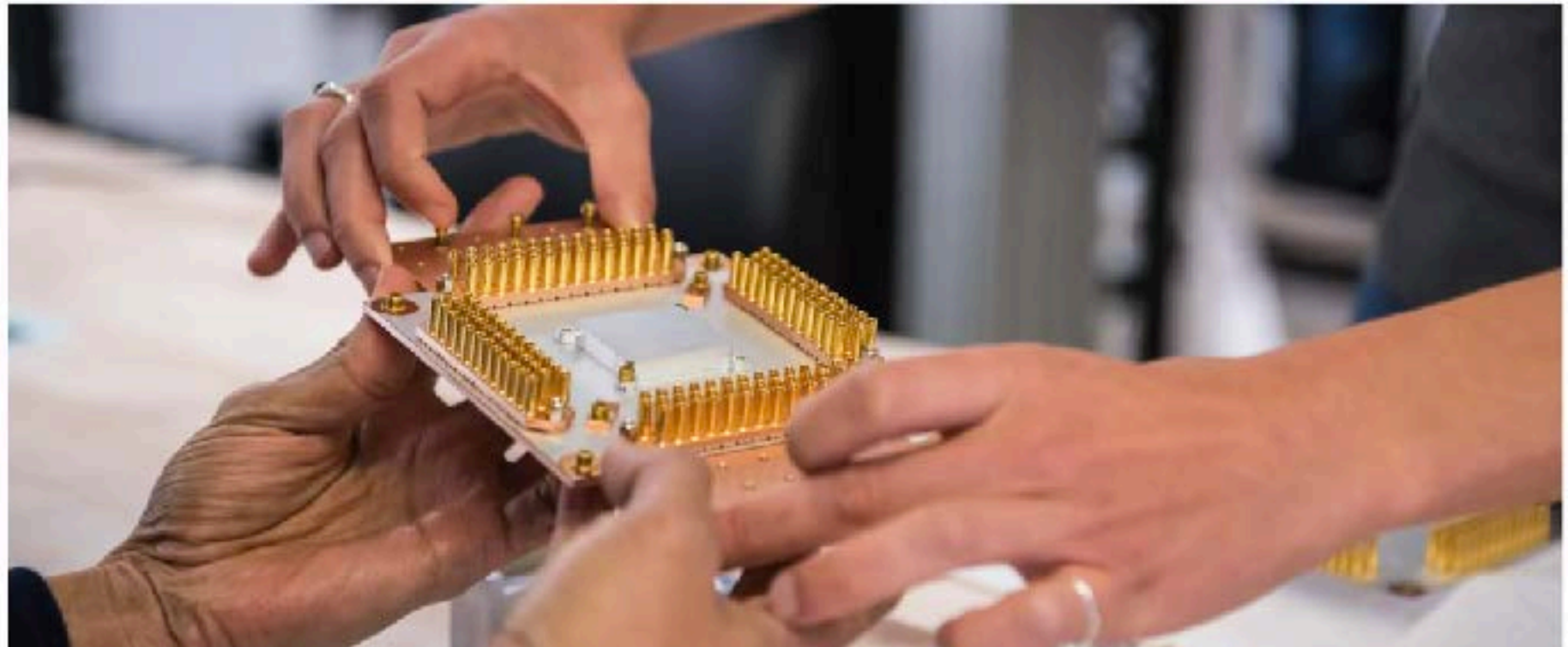


Charina Chou
Director and COO,
Google Quantum AI



Hartmut Neven
VP, Google Quantum AI

Share



Is there a “Moore's law” for quantum computing?

- <https://arxiv.org/pdf/2303.15547>

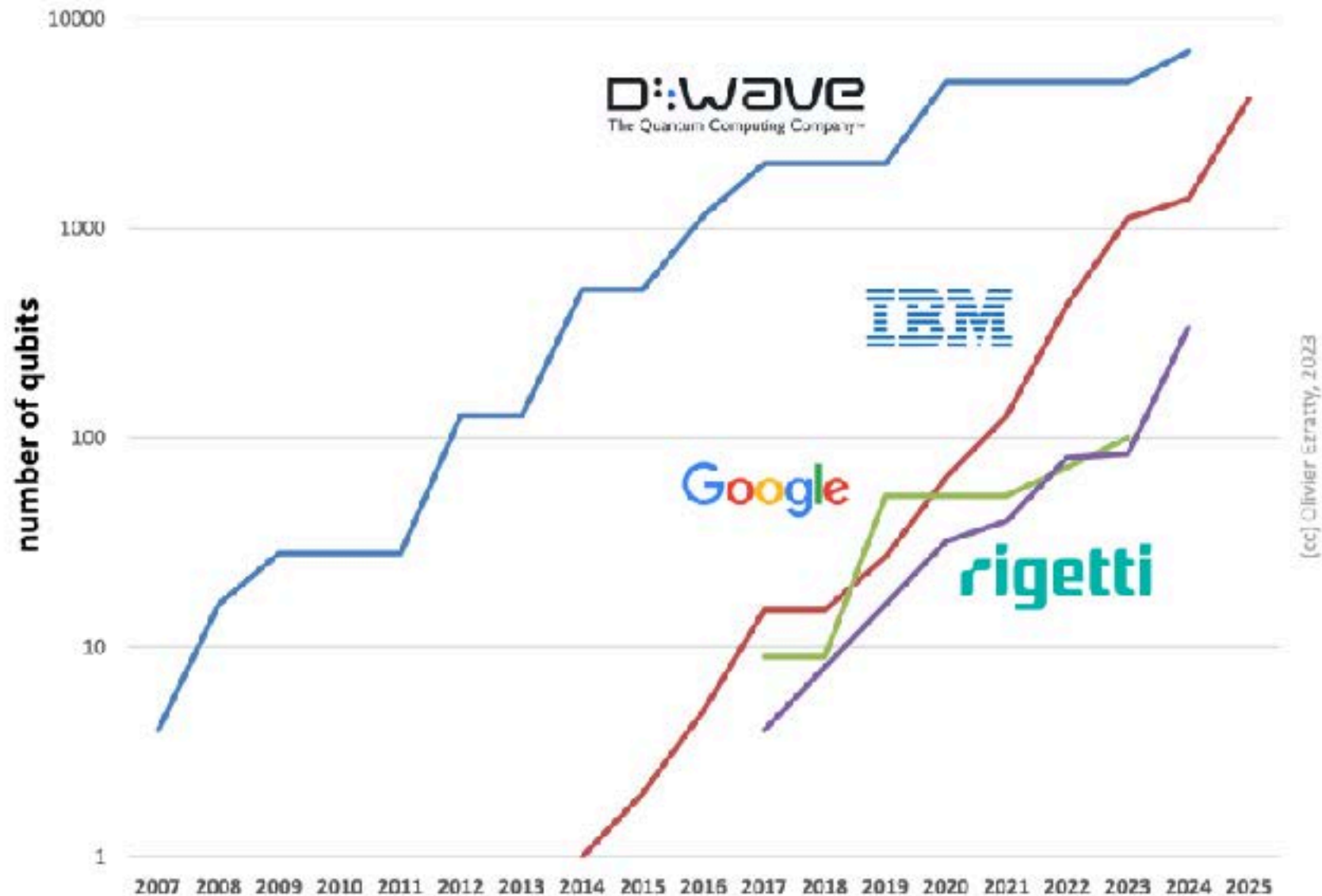


Figure 6: evolution of the number of physical qubits with D-Wave, IBM, Google and Rigetti. Compilation: Olivier Ezratty.

Is there a “Moore's law” for quantum computing?

- <https://arxiv.org/pdf/2303.15547>

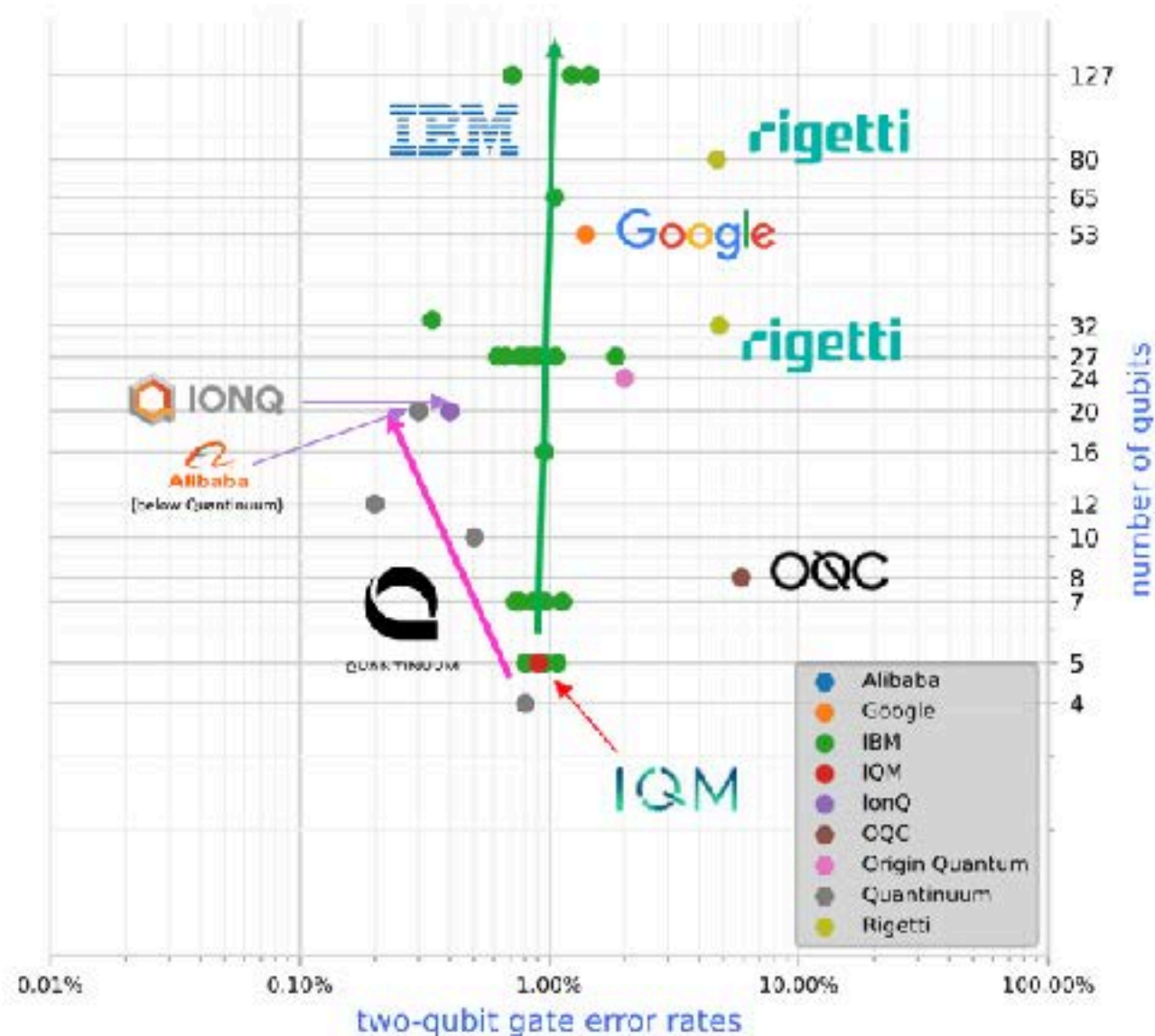


Figure 9: scatter plot of qubit numbers and two-qubit gate error rates for commercial vendors. Source: vendor fidelities numbers compiled by Olivier Ezratty as of March 2023.

Willow System Metrics

Number of qubits

105

Average connectivity

3.47
(4-way typical)

Quantum Error Correction (Chip 1)

Single-qubit gate error¹
(mean, simultaneous)

$0.035\% \pm 0.029\%$

Two-qubit gate error¹
(mean, simultaneous)

$0.33\% \pm 0.18\%$
(CZ)

Measurement error
(mean, simultaneous)

$0.77\% \pm 0.21\%$
(repetitive, measure qubits)

Reset options

Multi-level reset ($|1\rangle$ state and above)
Leakage removal ($|2\rangle$ state only)

T_1 time
(mean)

$68 \mu\text{s} \pm 13 \mu\text{s}^2$

Error correction cycles
per second

909,000
(surface code cycle = 1.1 μs)

Application performance

$\Lambda_{3,5,7} = 2.14 \pm 0.02$

Random Circuit Sampling (Chip 2)

Single-qubit gate error¹
(mean, simultaneous)

$0.036\% \pm 0.013\%$

Two-qubit gate error¹
(mean, simultaneous)

$0.14\% \pm 0.052\%$
(iswap-like)

Measurement error
(mean, simultaneous)

$0.67\% \pm 0.51\%$
(terminal, all qubits)

Reset options

Multi-level reset ($|1\rangle$ state and above)
Leakage removal ($|2\rangle$ state only)

T_1 time
(mean)

$98 \mu\text{s} \pm 32 \mu\text{s}^2$

Circuit repetitions per second

63,000

Application performance

XEB fidelity depth 40 = 0.1%

Estimated time on Willow
vs classical supercomputer

5 minutes vs. 10^{25} years

- Involvement with National labs and government agencies
 - NASA, DoD, DOE, NSF, . . .
 - Many private foundations: Templeton, Keck . . .



NASA Quantum Artificial Intelligence Laboratory (QuAIL)

QuAIL is the space agency's hub for assessing and advancing the potential of quantum computers to impact computational challenges faced by the agency in the decades to come.

Overview

Quantum computing is an emerging computational paradigm with the potential to revolutionize diverse areas of future-generation computational systems. QuAIL's mandate is to determine and advance the potential for quantum computation to enable more ambitious, efficient, and safer NASA missions in the future. The team focuses on hard computational problems whose solution would impact NASA's aeronautics, Earth and space sciences, and space exploration missions.

The QuAIL team tracks and contributes to advances in algorithms, both near- and longer-term, for optimization, machine learning, simulation of condensed-matter, high-energy physics, chemistry and materials, and differential equations and computational fluid dynamics. While the group is a theory and numerical team, a key component of this work is close collaboration with quantum hardware groups. Hardware-algorithm co-design is central to the team's work, with algorithmic needs feeding into the hardware design and prioritization of hardware capabilities, hardware limitations feeding into algorithm design, and hardware noise characterization feeding into the design of protocols for error mitigation and correction. Key to this research has been advancing tools, including classical circuit simulation and open quantum system modeling, leading to an improved understanding of co-design principles across multiple architectures, including gate-model, adiabatic, and measurement-based architectures.



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[Home](#) : [Our Work](#) : [Areas of Research](#) : [Quantum Research](#)

Quantum Research

Email: nrlpao@nrl.navy.mil

History

The beginning of research into Quantum Information Science (QIS) can be traced to theoretical physicist, Richard Feynman, Ph.D. In 1982 he proposed that computers that take advantage of quantum mechanical principles may have certain advantages over classical computers. Since then, researchers have proposed or demonstrated various methods of using quantum information to achieve results that cannot be obtained with classical physics. One of the most striking proposals is the quantum computer itself.

Army designates Quantum Information Science Research Center

By U.S. Army DEVCOM Army Research Laboratory Office of Strategic Communications | July 5, 2023



TOP STORIES

JANUARY 20, 2025

National Guard Bureau leaders meet with Guardsmen supporting 60th Presidential Inauguration

JANUARY 17, 2025

42nd Infantry Division heads to Warfighter exercise

JANUARY 16, 2025

Bradley improvement tested at US Army Yuma Proving Ground

JANUARY 1, 2025

Army provides Soldiers, Families variety

AFRL – Quantum Information Science & Technology



AFRL/RITQ - Quantum Networking and Computing

- Integrated Photonics and Photon Qubits
- Quantum Algorithms
- Superconducting and Hybrid Quantum Systems
- Trapped Ions

Contact Information:

Email: ri.quantum.gov@afrl.af.mil

- Active collaboration between industry, quantum experts and quantum computing companies.

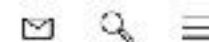


Q+Crypto™ Cryptocurrency in the age of quantum computers
OCT 19-21 | NEW YORK CITY

REGISTER NOW

Germany's Deutsche Bahn Netz AG (DB) partnering with Cambridge Quantum to explore how quantum computers can improve rail traffic





Bosch Quantum Sensing

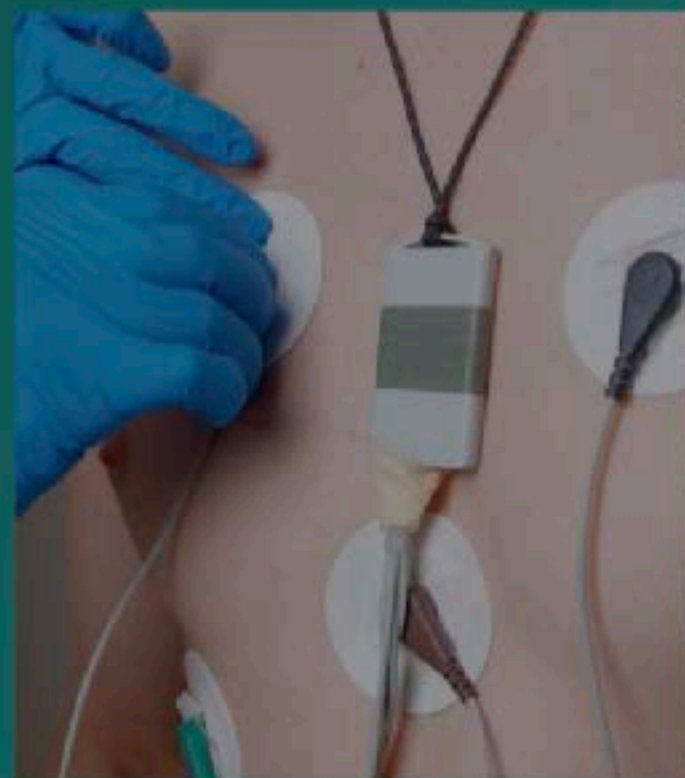
Revolutionary technology

Bosch Quantum Sensing

#CreateBeyondPossible

Our application fields >

Our technology >



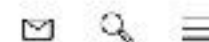


JPMorganChase

Overview | Publications

Quantum Computing	+
Networking and Security	+
Immersive Technology and Spatial Computing	+
Cloud Computing and Networking	+
Internet of Things and Ubiquitous Computing	+
Blockchain and Privacy	+





JPMorgan Chase

Samsung
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Press Resources



Overview | P

The First Step to a Quantum-Safe Future With Samsung Knox

on February 4, 2025



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양자기술연구소



연구활동

물리측정본부



화학소재측정본부



바이오의약품측정본부

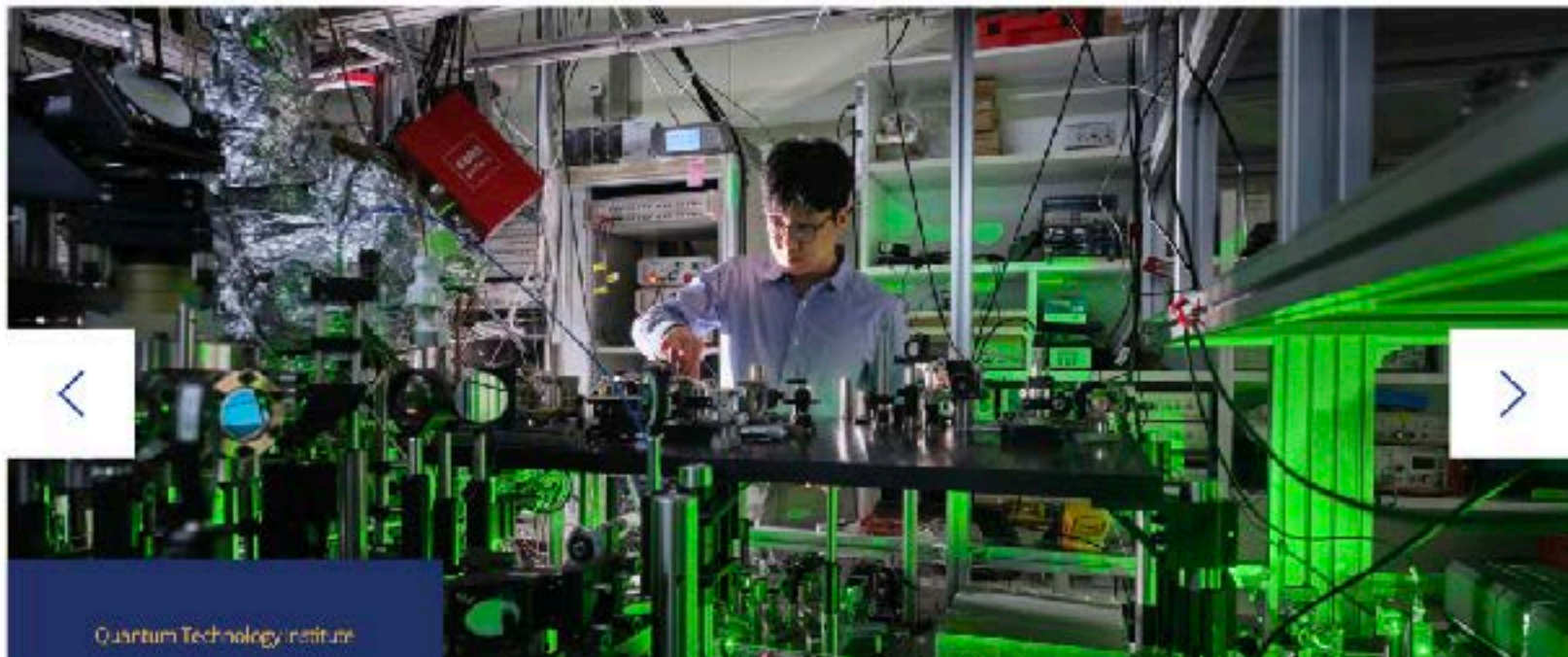


양자기술연구소



- 양자저기센싱그룹
- 양자광학그룹
- 양자소자그룹
- 양자전기자기측정그룹
- 양자질량속정그룹
- 원자양자센서그룹

↑

10³

Quantum Technology Institute

Our ap

Our tec



- Many schools and internship opportunities for graduate, undergraduate and high school students!
 - REU program
 - Workshops and conferences



Quantum, Computing, Mathematics, & Physics Summer Camp

WHAT IS QCaMP?

- Two summer camps introducing high school teachers and students to quantum concepts and careers

QCaMP GOALS AND GUIDING PRINCIPLES

Goal: Reduce barriers. Stipends for all. No prerequisites! Hands-on puzzles/experiments throughout.

Goal: Give teachers tools to introduce quantum topics to their students, allowing us to **reach more students**.

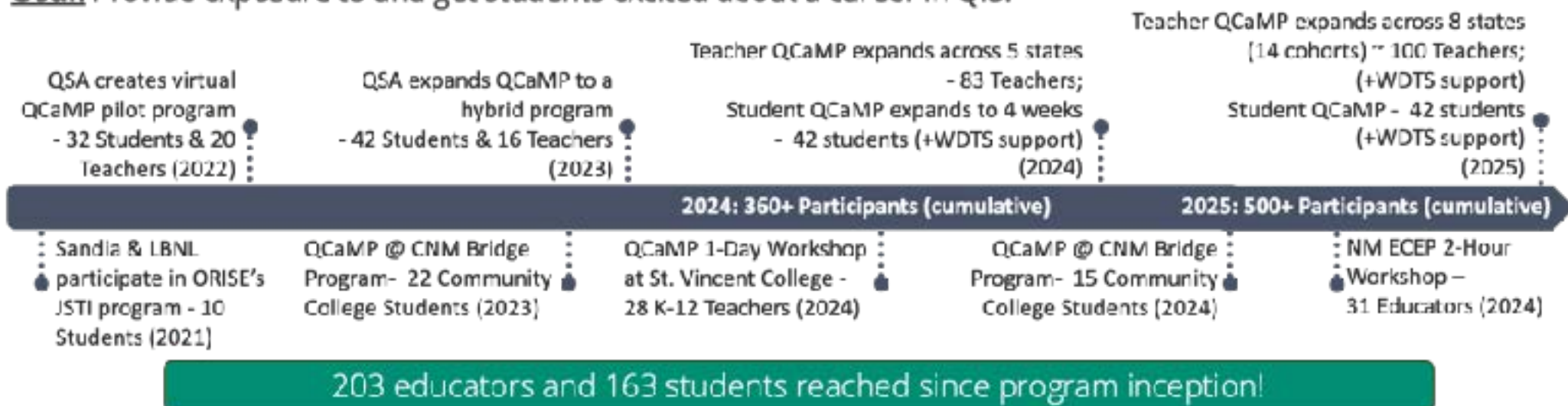
Goal: Provide exposure to and get students excited about a career in QIS.

2025 Camps

Teachers: June 16-July 18, 2025

Students: June 30-July 25, 2025

sandia.gov/quantum/qcamp/



Taken from Megan Ivory's presentation at QIST workshop 2025

Quantum Ideas Summer School

The Quantum Ideas Summer School is open to Undergraduates, Graduate Students, and Industry participants. Support for Undergraduate and Graduate students is funded by the National Science Foundation (PHY-2325080).

[2025 Summer School policy guidelines](#)

2025 Summer School

Application are now closed for 2025 Summer School June 9-13 at Duke University.

Application notifications will be sent no later than Friday 11 April.

Confirmed Instructors:

[Kenneth Brown \(Duke\)](#)

[Emily Edwards \(Duke\)](#)

[Natalie Kleo \(Duke\)](#)

[Huangfan Loh \(Duke\)](#)

[Akimasa Miyake \(UNM\)](#)

[Crystal Noel \(Duke\)](#)

[Gokul Ravi \(U Michigan\)](#)

more instructors to be announced

Duke, since 2019

For advanced
undergraduate and
graduate students

Undergraduate and Graduate student support funded by the National Science Foundation award PHY-2325080 and outlined in [Participant Travel Guidelines](#). Summer STAQ 2019-2024 was funded under National Science Foundation award PHY-1818914.

For more details, contact the [STAQ Administrator](#).

2023 Session

Lecture Notes and Session Recordings

[Day 1 | Quantum Information Basics](#)

[Day 2 | Ions and QECC](#)

[Day 3 - Offsite programming. No lectures](#)

[Day 4 | Superconductors and Algorithms](#)

[Day 5 | Quantum Architecture and Optimization](#)

2022 Session

Lecture Notes

[DAY 1 | Basics with Aram Harrow](#)

[DAY 2 | Basics with Akimasa Miyake](#)

[DAY 3 | Algorithms with Peter Love](#)

[DAY 4 | Grover's Algorithm and Quantum Hype with Shelby Kimmel](#)

[DAY 5 | Quantum Error Correction with Ken Brown](#)

2024 Session

Undergraduate Quantum Summer School

Aug 14 and 15, 2024 Northwestern University



<https://forms.gle/BaNkooRL3ozeQPit8>

What to expect:

The IQ-PARC project, funded by the Defense Department's National Defense Education Program, is extending an invitation to underrepresented students from Illinois and nationwide to participate in a workshop focused on Quantum technologies, to be held at Northwestern University. Recognizing the importance of diversity in the advancement of these critical technological fields, the project aims to foster an inclusive environment that encourages participation from all corners of the academic community. Successful applicants will receive a stipend of \$1,000, in reimbursement, to cover travel and lodging expenses in Evanston, IL, ensuring that financial barriers do not hinder the opportunity to engage in this transformative educational experience. There will be presentations by faculty and industry partners introducing quantum technologies, cutting-edge research areas, and related job opportunities particularly in the areas of national security.

Registration:

Application: Space is limited, up to 1000 USD will be available to selected participants. Please use the QR code, to apply by May 15, 2024. Decisions will be sent by June 15, 2024.



ORNL

The 2025 **U.S. Quantum Information Science Summer School (USQIS)** will be hosted by the Quantum Science Accelerator at Lawrence Berkeley National Laboratory. Dates and application information TBA.



FNAL

For undergraduate and graduate students

Quantum Computing Summer School Fellowship



Developing new leaders in the theory, application, and programming of quantum computers

For undergraduate and
graduate students

Applications are open until January 19th 2025

The Quantum Computing Summer School is an immersive 10-week curriculum that includes tutorials from world-leading experts in quantum computation as well as one-on-one mentoring from LANL staff scientists who are conducting cutting-edge quantum computing research. Summer school fellowship recipients will be exposed to the theoretical foundations of quantum computation and will become skilled at programming commercial quantum computers, such as those developed by D-Wave Systems, Quera, Quantinuum, and IBM. Roughly twenty students (with the precise number determined based on the applicant pool) will be awarded a fellowship for the summer school. The fellowship amount ranges from \$10,000 to \$20,000, based on academic rank. All students (undergraduate and graduate) are encouraged to apply.

The school will be held from June 2nd to August 8th 2025.

C²QA QIS 102: Applied Quantum Computing Summer School

June 10 – 28, 2024

*“There is a distinct need to teach students core scientific computing,
and then stitch quantum computing concepts to enhance intuition and skills.”*

David Biersach
Senior Technology Architect
Brookhaven National Laboratory

Brookhaven National Laboratory offers a three-week virtual workshop to introduce rising college juniors, seniors, and recent college graduates to the exciting world of quantum information science (QIS). Led by **Dr. David Biersach**, through a series of demonstrations and hands-on programming labs, students will learn how quantum algorithms, when applied to specific problem domains, can outperform classical computers. Students will learn **IBM Qiskit**, a world-class software package for working with quantum computers at the level of pulses, circuits, and application modules. Students will download and install the 100% open-source courseware and development tools on their personal computers, which they will use during the program. Accepted students will be provided with a weekly stipend of \$500 and those who successfully complete the workshop will be presented with a certificate from Brookhaven National Laboratory attesting to their skills in **Applied Quantum Computing**. The workshop will run each day, Monday – Friday, for three weeks from June 10 – 28, 2024 from 10:00 AM – 6:00 PM ET (two 1-hour breaks) and be conducted entirely via Zoom web conferencing.

STIPEND: \$500 per week

ELIGIBILITY: Minimum 18 years of age. Must be a U.S. citizen or Legal Permanent Resident. Must successfully complete the Brookhaven National Laboratory Guest Registration process.

PREREQUISITES: Students must have completed all (6) courses listed below prior to program commencement on Jun 10, 2024 with a grade B (3.0 GPA) or better.

- **Computer Programming:** A course covering an “Introduction to Programming” and a course covering “Data Structure & Algorithms”.
- **Math:** A course covering “Linear Algebra with Vectors & Matrices” and a course covering “Differential Equations”.
- **Physics:** A course covering “Physics Using Calculus including Classical Mechanics & Electromagnetism” and a course covering either “Thermodynamics” or “Mathematical Methods in Physics”.

In addition to the above, only applicants who are rising college juniors or seniors (or recent college graduates) and who will have completed the spring 2024 semester by June 10, 2024 will be considered.

QIS 100: Applied Quantum Computing Virtual Program, June 2025

Application Period Opens February 2025

Virginia Tech & C2QA QIS & Engineering High School Level Virtual Program, August 2025

Application Period Opens March 2025



Google Summer of Code 2025

Introduction

In 2025 ML4SCI plans to participate in the program as a GSoC umbrella organization. The ML4SCI organization plans to partner with the [Google Summer of Code](#) in 2025 to broaden student participation in machine learning projects over a wide variety of scientific fields. ML4SCI participants will be mentored by scientists at top research universities and laboratories on research projects at the cutting edge of science. Projects span a wide range of scientific domains, including physics, astronomy, planetary science, quantum information science and others.

For Students

In 2025 GSoC students work with their mentors for 175 hrs to produce open-source codes that apply machine learning solutions to solve science problems. Projects span three evaluation periods that allow for students and mentors to collaborate on their project and evaluate student progress. Detailed rules for the GSOC program can be found [here](#). Interested students should look at the ideas page and contact the mentors. Candidates will be asked to complete an evaluation test for each project they apply to demonstrate the skills needed for the respective projects. In the next step, students will produce a proposal which will be evaluated for final student selection.

Please see the [official GSoC Timeline](#)

Projects in 2025



[CMS](#) is a high-energy physics experiment at the [Large Hadron Collider](#) (LHC) at [CERN](#). It is a general-purpose detector that is designed to observe any new physics phenomena that the LHC might reveal. CMS acts as a giant, high-speed camera, taking 3D "photographs" of particle collisions from all directions up to 40 million times each second. The CMS collects few tens of Peta-Bytes of data each year and processes them through Worldwide LHC Computing Grid infrastructure around the globe.

[List of proposals](#)



[DeepLense](#) is a deep learning pipeline for particle dark matter searches with strong gravitational lensing.

[List of proposals](#)



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Please see the [official GSoC Timeline](#)

Projects in 2025



Google Summer of Code program is intended for advanced students who are already familiar with AI or quantum computing.

[CMS](#) is a high-energy physics experiment at the [Large Hadron Collider \(LHC\)](#) at [CERN](#). It is a general-purpose detector that is designed to observe and study new physics phenomena. CMS acts as a giant, high-speed camera, taking 3D "photographs" of particle collisions from all directions up to 40 million times each second. The CMS collects few tens of Peta-Bytes of data each year and processes them through Worldwide LHC Computing Grid infrastructure around the globe.

[List of proposals](#)



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[List of proposals](#)

- QCenter SKKU
- KIAS

2025 퀀텀챌린지

2025 Quantum Challenge

2025.10.21.(화) ~ 12.2(화)

이온트랩 기반 양자컴퓨터 클라우드 기업 IonQ의 전문가가
직접 출제한 3개의 양자컴퓨팅 문제!
출제자의 특별 강연과 함께하는 이번 챌린지에 지금 바로 도전하세요!

참가 대상

국내 소재 대학원 대학원생(원생 포함)

주요 일정

(등록 신청) 2025.10.15.(수) ~ 11.13.(목)

(문제 공개) 2025.10.21.(화) 14:00

(양자정보연구지원센터 홈페이지 IonQ 클라우드)

(문제 풀이 마감) ~ 2025.11.14.(금) 11:00까지

(신청서 2025.11.17.(월) ~ 11.21.(금)

(결과 발표 및 수료) 2025.12.2.(화)

특전

전문가 해설 강연, IonQ 양자클라우드 제공(IonQ 1GB,
참가인원수에 따라)

참가대상	참가비
Master Level	20만원 상당 장학금
Challenger Level	20만원 상당 장학금
Rookie Level	10만원 상당 장학금

참가 안내



* 참가 신청서 제출 후 QR코드: <https://qcenter.kr> (참가비 확인)

참가 신청



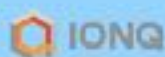
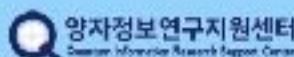
문의처

양자정보연구지원센터 후원 장학금 담당

• qcloud@kku.edu
• 031-299-6438



※ 상기 일정은 변동될 수 있으며, 자세한 내용은 홈페이지(<http://qcenter.kr>) 확인



QCenter & IonQ 2025 IonQ 여름 리서치 멘토링 프로그램

2025.8.12.(화) ~ 2025.10.1.(수)

모집 대상



양자정보에 관심 있는
대학(원)생 또는 일반인
(3~5인 팀 구성)

신청 방법

프로젝트 주제 선택 후 신청서 접수
* 주제 선정 시 양자정보연구지원센터 유튜브에
업로드 된 멘토 제안 프로젝트 참고

활동 내용

- 주 1회 실시간 온라인 멘토링 참석(영어 진행)
- 6주간 주간 보고서 및 멘토링 결과를 제출
- 멘토와의 정기적인 소통 및 피드백

활동 혜택



멘토링 기간 동안
IonQ 양자컴퓨터 클라우드 제공
(\$1,000)



리서치 멘토링
참가인원서 지급

신청 바로가기

QR코드 스캔

참가 신청서 바로가기

<https://forms.gle/qf8pAQWKSJCP16>



IonQ 멘토



Denny Dahl



Oliver Knitter



Panagiotis
Barkoutsos

모집 절차

01 7/23 ~ 8/4 참가 접수	>	02 8/5 ~ 8/7 신청 절차	>	03 8/8 신청서 발표 및 안표
04 8/12 ~ 8/22 리포트 미팅 이전 1회실 진행 예정	>	05 8/18 ~ 9/28 2주 멘토링 진행	>	06 9/30 ~ 10/1 프로그램 결과 공개 발표

* 멘토링 기간 내 모든 프로그램 필수 참석 필수

* 해당 프로그램은 영어 / 실시간 온라인으로 진행 됩니다.

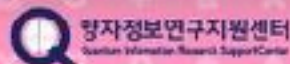
* 신청서 발표 및 안표 발표 이후 모든 인원은 디스코드를 통해 진행됩니다.

* 해당신청은 상황에 따라 변동될 수 있습니다.

문의처

전화번호
이해 일

031-299-6438
qcloud@kku.edu



QCenter x Pasqal

양자컴퓨팅 교육 및 실습 프로그램

2025 11.10.(월) / 11.12.(수) / 11.14.(금) 14:00 - 15:30 KST
(총 3회 진행)

성균관대학교 양자정보연구지원센터(QCenter)와 Pasqal이 함께하는
3일 과정의 '온라인 양자컴퓨팅 교육 및 실습 프로그램'입니다.
양자정보기술에 관심있는 분들의 많은 참여 바랍니다.

모집 기간

2025년 11월 5일(수)까지
참가신청서(구글폼) 작성

* 모집 기간 내 참가 신청을 하지 못한 경우,
교육 기간 중에도 참가 신청서 제출 후 교육 참가 가능

교육 방식

zoom을 통한 온라인 공개 강의

(참가신청서 내상 이메일로 세부 일정 안내)

모집 대상

양자정보기술관련 분야의 연구자, 개발자,
기술 엔지니어, 관리자 및 정책 수립 관계자, 학생 등
(전공 무관)

신청 방법

1. 하단에 있는 QR을 통해 참가신청서 작성(구글폼)
2. 양자정보연구지원센터 홈페이지(www.qcenter.kr) 접속
→ 커뮤니티 → 행사정보

교육 일정

회차	일차	시간	주제
1회차	11/10(월)	14:00 - 15:30	Introduction and Pre-program access
2회차	11/12(수)	14:00 - 15:30	MS
3회차	11/14(금)	14:00 - 15:30	QEL

▼ 참가신청



문의 | 031-299-6437 / q.edu@skku.edu

The Adjoint Is All You Need: Characterizing Barren Plateaus in Quantum Ansätze

Enrico Fontana,^{1,2} Dylan Herman,^{1,*} Shouvanik Chakrabarti,¹ Niraj Kumar,¹
Romina Yalovetzky,¹ Jamie Heredge,^{1,3} Shree Hari Sureshbabu,¹ and Marco Pistoia¹

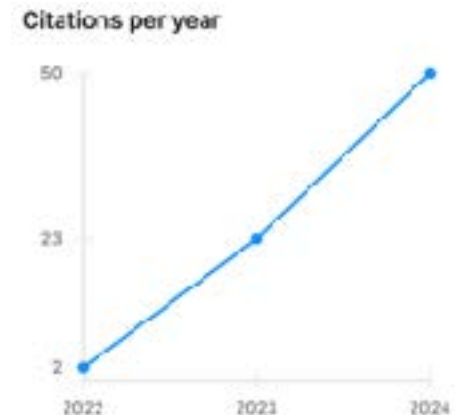
¹*Global Technology Applied Research, JPMorgan Chase*

²*Computer and Information Sciences, University of Strathclyde*

³*School of Physics, The University of Melbourne*

Using tools from the representation theory of compact Lie groups, we formulate a theory of Barren Plateaus (BPs) for parameterized quantum circuits whose observables lie in their dynamical Lie algebra (DLA), a setting that we term Lie algebra Supported Ansatz (LASA). A large variety of commonly used ansätze such as the Hamiltonian Variational Ansatz, Quantum Alternating Operator Ansatz, and many equivariant quantum neural networks are LASAs. In particular, our theory provides, for the first time, the ability to compute the variance of the gradient of the cost function of the quantum compound ansatz. We rigorously prove that, for LASA, the variance of the gradient of the cost function, for a 2-design of the dynamical Lie group, scales inversely with the dimension of the DLA, which agrees with existing numerical observations. In addition, to motivate the applicability of our results for 2-designs to practical settings, we show that rapid mixing occurs for LASAs with polynomial DLA. Lastly, we include potential extensions for handling cases when the observable lies outside of the DLA and the implications of our results.

Nature Commun. 15 (2024) 1, 7171
2309.07902



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What You'll Do

Who We're Looking For

What We Offer

Valued qualities

We are seeking colleagues with excellent analytical, quantitative and problem solving skills and demonstrated research ability. We value strong communication skills and the ability to present findings to a non-technical audience.

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 - Implementing modern encryptions algorithms
 - *Delivering key management systems, key distribution protocols, or working with hardware security modules*
 - *Identifying and mitigating real-world security vulnerabilities and attack vectors*
- You've managed or technically led a team of developers and owned application development from planning to delivery.
- You bring hands-on engineering experience and have tackled complex, ambiguous problems through code.
- You use your technical background to coach developers and elevate team performance

Bonus points for:

- Experience with post-quantum cryptography (e.g., lattice-based algorithms) in theory or practice
- Familiarity with evolving cryptographic standards and bodies (e.g., NIST), and how they shape implementation decisions

Summary

- Quantum Information Science (QIS) and Quantum Computing (QC) open new opportunities by connecting diverse fields: physics, mathematics, computer science, and engineering.
- Although it may take time to mature, QIS/QC will eventually become an integral part of our daily lives, much like AI/ML have become today.
- Challenges remain: scalability, noise mitigation, algorithmic development, and workforce training
 - But progress is accelerating rapidly.
- Quantum computing today is where classical computing was in the 1950s: slow, expensive, and limited, yet full of promise to redefine science and technology.
- It is crucial to explore where quantum computers can make a real impact from fundamental science to applications in our society.
- We need young, creative, and imaginative minds to shape this emerging quantum future.

Quantum Frontiers in Particle Physics: From Sensing Rare Decays to Solving Collider Challenges



공경철 / K.C. Kong
University of Kansas



Pusan National University
October 27, 2025



부산대학교
PUSAN NATIONAL UNIVERSITY

Quantum Computing for High-Energy Physics

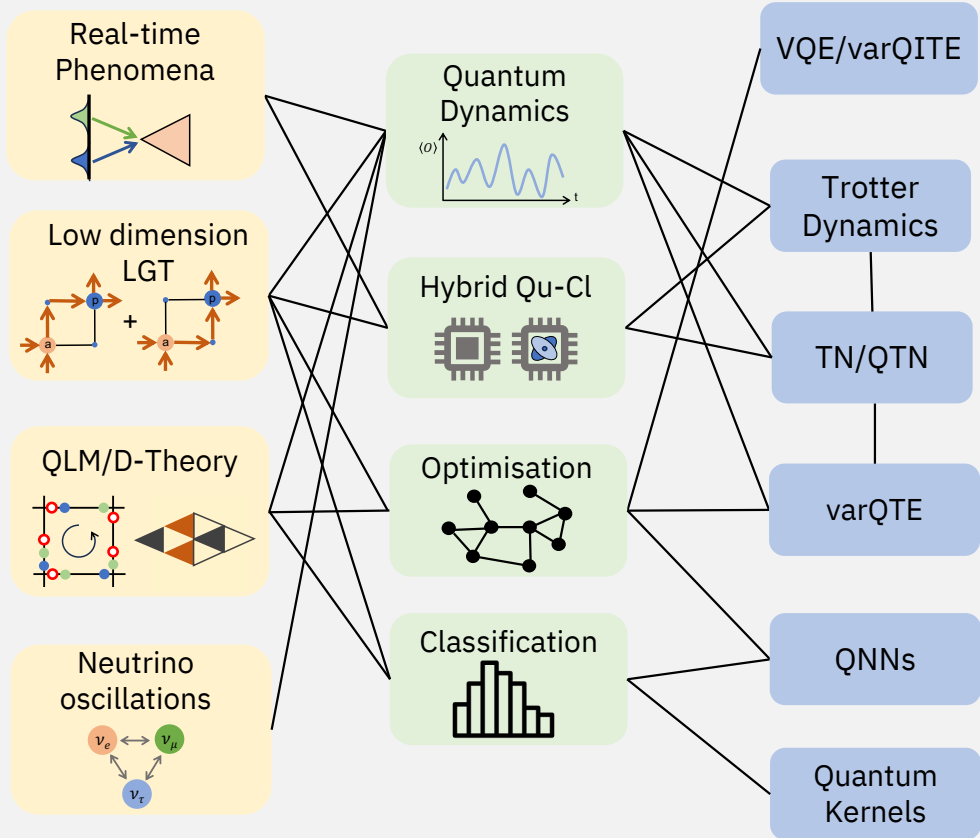
State of the Art and Challenges

Summary of the QC4HEP Working Group

2307.03236

Theoretical
physical
model
systems

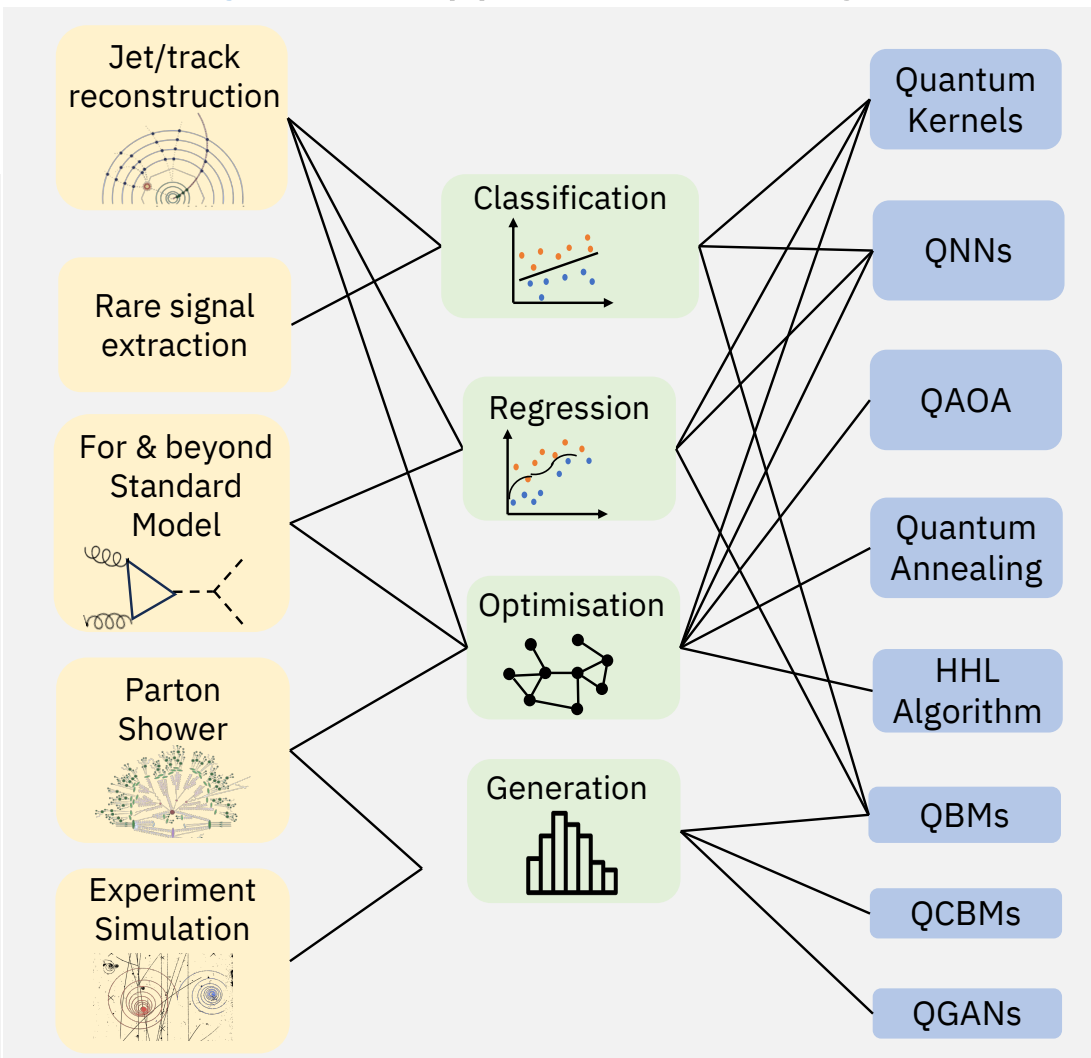
corresponding
approaches



Experimental
challenges

corresponding
approaches

quantum
algorithms

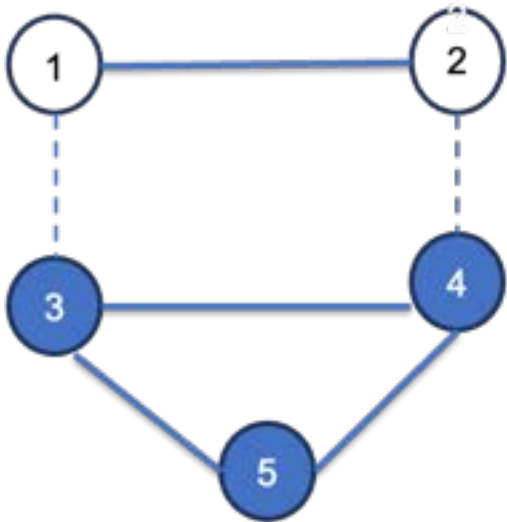


Where quantum computers are useful

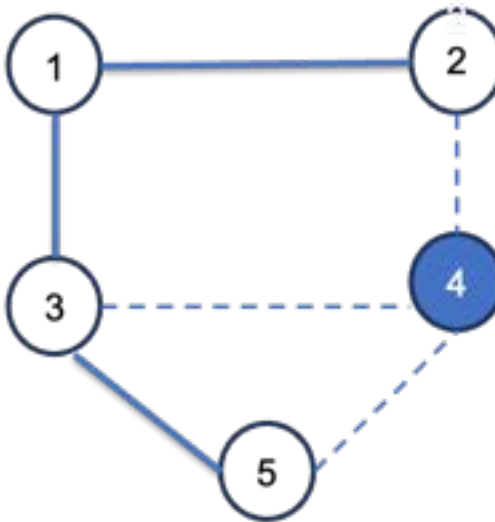
- Quantum optimization: combinatorial problems at the LHC [2410.22417](#)
 - Finance, physics, materials discovery, science [2508.09139](#)
- Quantum devices (qubits) as sensors: detection of dark matter and neutrino decays
 - Dark matter detection, quantum magnetometry + electrometry, atomic clocks
- Quantum simulation/Hamiltonian simulation
 - Physics, chemistry
- Quantum machine learning [2510.10501](#), [2411.15315](#), [2412.21082](#), [2411.13520](#)
 - Classification, generative models, anomaly detection
- Quantum cryptography and security
 - Quantum key distribution, post-quantum cryptography
- Quantum linear algebra and numerical methods
 - HHL algorithms, eigenvalue problems
- Quantum communication and networks
 - Entanglement and violation of Bell's inequality [2407.01663](#), [2407.07147](#), [2305.07075](#)
 - Teleportation, quantum internet
- Quantum sampling
 - Supremacy tasks, random circuits

MaxCut problem

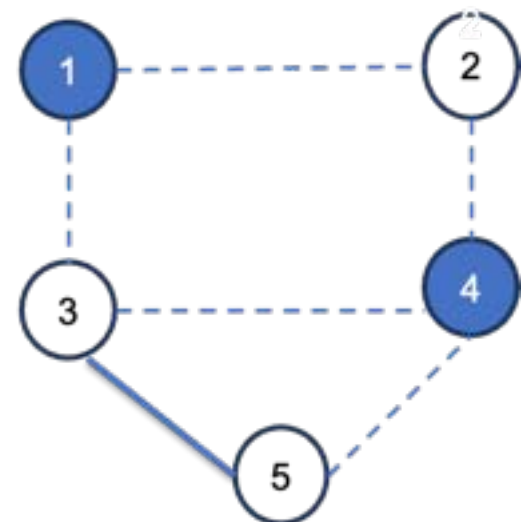
- Objective:
 - Maximize the number of cut edges in a graph when split into 2 parts
 - Divides the set of nodes in the graph into two subsets so that we have as many edges as possible that go between the two sets



Cut size = 2



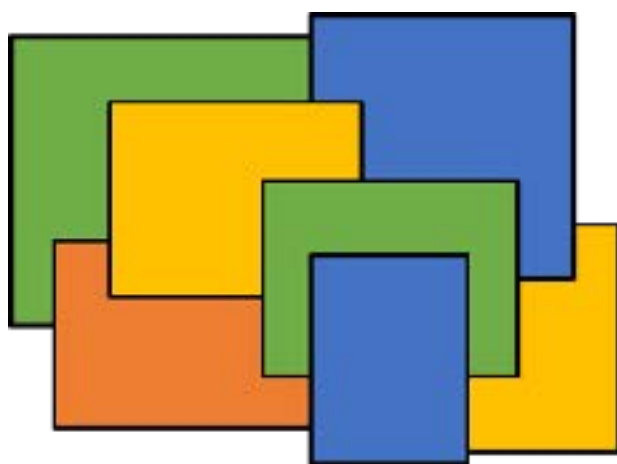
Cut size = 3



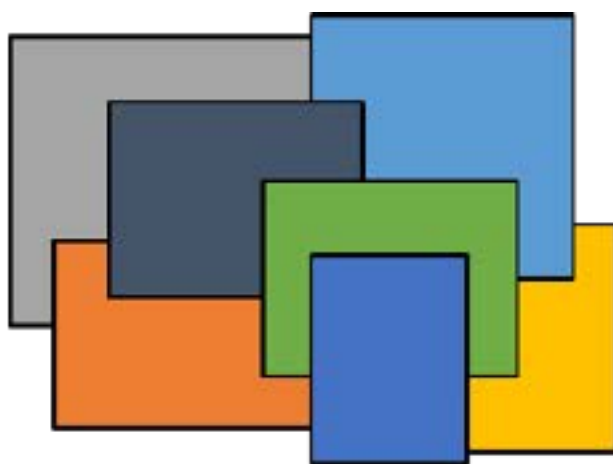
Cut size = 5

Map Coloring

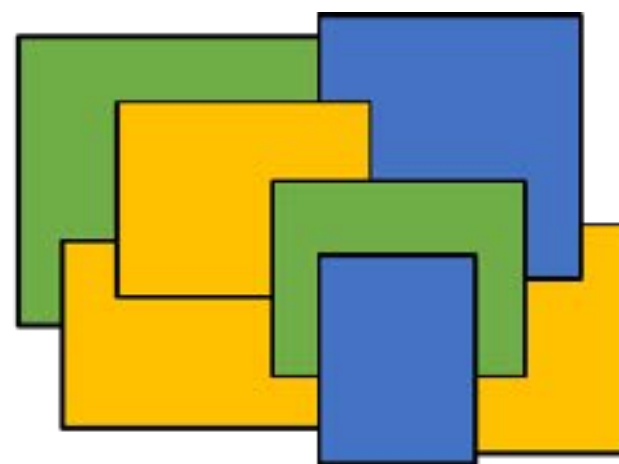
- Problem: Color the regions of a given map such that
 - at most four colors are used
 - no two adjacent* regions have the same color



Valid four-coloring



Invalid; more than 4 colors

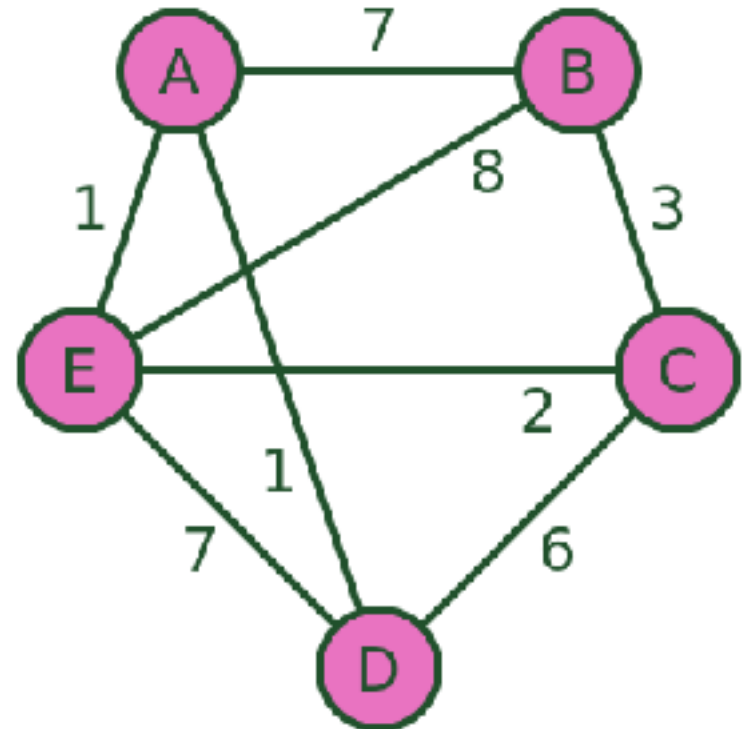


Invalid; yellows touch

* "adjacent" means "shares an edge"; sharing a corner is OK

Traveling salesman problem (TSP)

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?
- It is an NP-hard problem in combinatorial optimization, important in theoretical computer science and operations research.



Traveling salesman problem (TSP)

- Label vertices $1, \dots, N$ and the edge set (uv) to be directed. (For undirected, add (uv) and (vu)).
- Introduce N^2 bits $x_{v,i}$, where v represents the vertex and i represents its order to the edge set.
- Every vertex can only appear once in a cycle, and that there must be a j -th node in the cycle for each j .
- For the nodes in our prospective ordering, if $x_{u,j}$ and $x_{u,j+1}$ are both 1, then there should be an energy penalty if $(uv) \notin E$.

$$H_A = A \sum_{v=1}^n \left(1 - \sum_{j=1}^N x_{v,j} \right)^2 + A \sum_{j=1}^n \left(1 - \sum_{v=1}^N x_{v,j} \right)^2 + A \sum_{(uv) \notin E} \sum_{j=1}^N x_{u,j} x_{v,j+1}$$

- $A > 0$ is a constant. The ground state of this system has $H = 0$, only if we have an ordering of vertices where each vertex is only included once, and adjacent vertices in the cycle have edges on the graph – i.e., we have a **Hamiltonian cycle**.

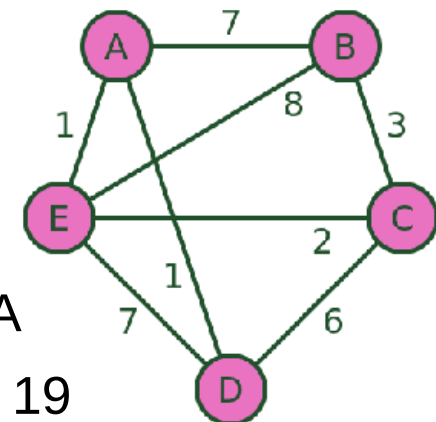
- The traveling salesman also needs to minimize the total traveling distance (weight)

$$H_B = B \sum_{(uv) \in E} W_{uv} \sum_{j=1}^N x_{u,j} x_{v,j+1}$$

- B should be small enough that it is never favorable to violate the constraints of H_A . $0 < B \max(W_{uv}) < A$

$$H_{\text{full}} = H_P = H_A + H_B$$

	1	2	3	4	5
A	1				
B			1		
C				1	
D					
E		1			1



AEBCDA

$$1+8+3+6+1 = 19$$

Breaking limitation of quantum annealer in solving optimization problems under constraints

Masayuki Ohzeki^{1,2,3*}

2002.05298

[*Nature, Scientific Reports* volume 10, Article number: 3126 \(2020\)](#)

Quantum annealing is a generic solver for optimization problems that uses fictitious quantum fluctuation. The most ground-breaking progress in the research field of quantum annealing is its hardware implementation, i.e., the so-called quantum annealer, using artificial spins. However, the connectivity between the artificial spins is sparse and limited on a special network known as the chimera graph. Several embedding techniques have been proposed, but the number of logical spins, which represents the optimization problems to be solved, is drastically reduced. In particular, an optimization problem including fully or even partly connected spins suffers from low embeddable size on the chimera graph. In the present study, we propose an alternative approach to solve a large-scale optimization problem on the chimera graph via a well-known method in statistical mechanics called the Hubbard-Stratonovich transformation or its variants. The proposed method can be used to deal with a fully connected Ising model without embedding on the chimera graph and leads to nontrivial results of the optimization problem. We tested the proposed method with a number of partition problems involving solving linear equations and the traffic flow optimization problem in Sendai and Kyoto cities in Japan.

Supply chain logistics

Route Optimization

Portfolio management

Nurse scheduling problem

Image recognition

Remote sensing imagery

Classifier optimization

Drug discovery

Material science

Seismic inversion

Quantum simulation

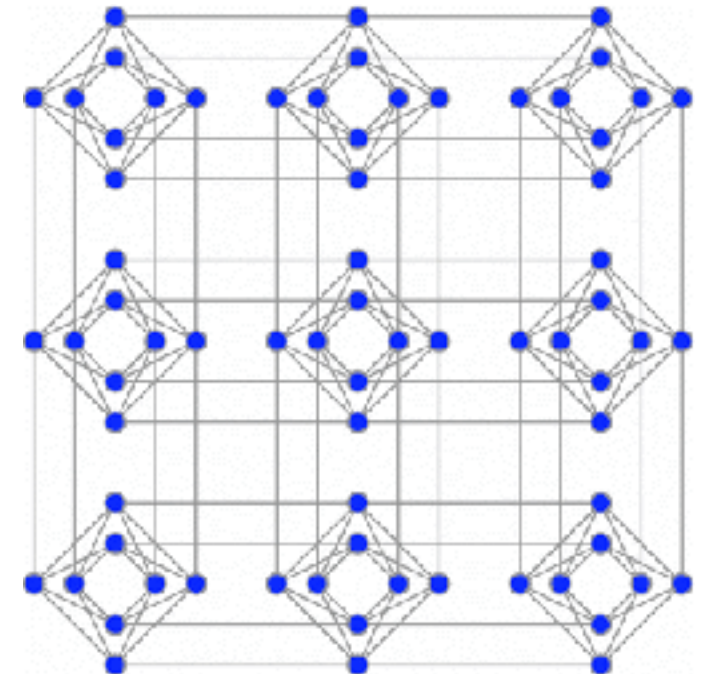
Financial modeling

Cryptography

Better batteries

Cleaner fertilization

Electronic material discovery



Quadratic Unconstrained Binary Optimization (QUBO)

- QUBO: combinatorial optimization problem with a wide range of applications from finance to ML (partitioning, graph coloring, task allocation, max-sat, max-cut etc)

$$f: \mathbb{Z}_2^n \longrightarrow \mathbb{R}$$

Quadratic polynomial over binary variable

$$f(x) = \sum_{i=1}^n \sum_{j=1}^i q_{ij} x_i x_j + \sum_{i=1}^n h_i x_i$$

$$x_i \in \mathbb{Z}_2 = \{0,1\}, \quad h_i, q_{ij} \in \mathbb{R}$$

$$x = (x_n x_{n-1} \cdots x_2 x_1)$$

- Find a binary vector x^* which minimizes f (binary strings of n-bits)

$$x^* = \underset{x \in \mathbb{Z}_2^n}{\operatorname{argmin}} f(x)$$

- In matrix notation, $f(x) = x^T Q x$, where $Q \in \mathbb{R}^{n \times n}$
- NP-hard problem: no polynomial-time algorithms are known.

Ising formulations of many NP problems

1 Introduction

- 1.1 Quantum Adiabatic Optimization
- 1.2 Ising Spin Glasses
- 1.3 The Goal of This Paper
- 1.4 What Problems Are Easy (to Embed) on Experimental AQO Devices?

Andrew Lucas

1302.5843

2 Partitioning Problems

- 2.1 Number Partitioning
- 2.2 Graph Partitioning
- 2.3 Cliques
- 2.4 Reducing N to $\log N$ Spins in Some Constraints

$$H = - \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z$$

Fully connected (mean field) Ising model

3 Binary Integer Linear Programming

4 Covering and Packing Problems

- 4.1 Exact Cover
- 4.2 Set Packing
- 4.3 Vertex Cover
- 4.4 Satisfiability
- 4.5 Minimal Maximal Matching

5 Problems with Inequalities

- 5.1 Set Cover
- 5.2 Knapsack with Integer Weights

6 Coloring Problems

- 6.1 Graph Coloring
- 6.2 Clique Cover
- 6.3 Job Sequencing with Integer Lengths

7 Hamiltonian Cycles

- 7.1 Hamiltonian Cycles and Paths
- 7.2 Traveling Salesman

8 Tree Problems

- 8.1 Minimal Spanning Tree with a Maximal Degree Constraint
- 8.2 Steiner Trees
- 8.3 Directed Feedback Vertex Set
- 8.4 Undirected Feedback Vertex Set
- 8.5 Feedback Edge Set

9 Graph Isomorphisms

10 Conclusions

1. Quantum annealers

2. Universal Quantum Computers

Portfolio optimization

Markowitz model

$$F = \sum_{ij} C_{ij} n_i n_j - \zeta \sum_i R_i n_i$$

Diagram illustrating the Markowitz model equation for portfolio optimization:

- F : risk minus returns
- C_{ij} : correlations between assets
- $n_i n_j$: no. of assets of type i
- ζ : risk tolerance
- R_i : return on i 'th asset

Spin glasses

Sherrington-Kirkpatrick model

$$H = \sum_{ij} J_{ij} s_i s_j + \sum_i h_i s_i$$

Diagram illustrating the Sherrington-Kirkpatrick model equation for spin glasses:

- H : Hamiltonian
- J_{ij} : couplings between sites
- $s_i s_j$: spin at site i
- h_i : external magnetic field at i 'th site

Neural networks

Hopfield model

$$E = \sum_{ij} w_{ij} v_i v_j + \sum_i \theta_i v_i$$

Diagram illustrating the Hopfield model equation for neural networks:

- E : energy function
- w_{ij} : weights of the learning rule
- $v_i v_j$: state of neuron i
- θ_i : activation threshold for i 'th neuron

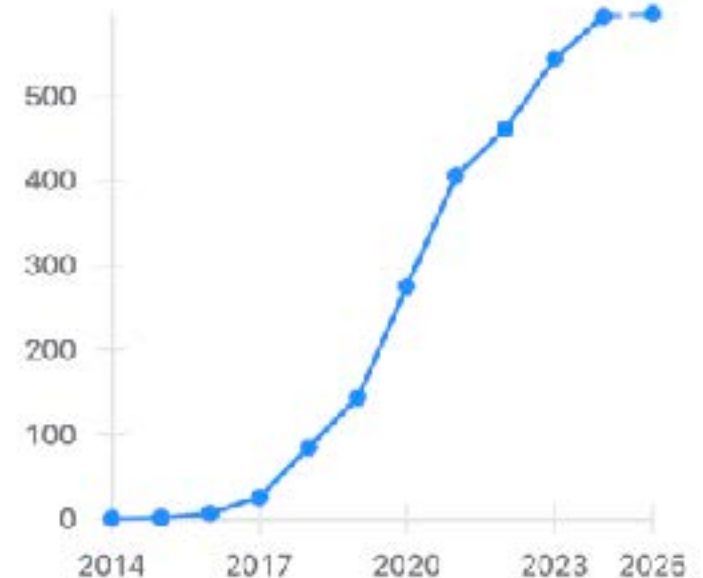
Quantum Approximate Optimization Algorithm (QAOA)

- Abstract:** We introduce a quantum algorithm that produces approximate solutions for combinatorial optimization problems. The algorithm depends on a positive integer p and the quality of the approximation improves as p is increased. The quantum circuit that implements the algorithm consists of unitary gates whose locality is at most the locality of the objective function whose optimum is sought. The depth of the circuit grows linearly with p times (at worst) the number of constraints. If p is fixed, that is, independent of the input size, the algorithm makes use of efficient classical preprocessing. If p grows with the input size a different strategy is proposed. We study the algorithm as applied to MaxCut on regular graphs and analyze its performance on 2-regular and 3-regular graphs for fixed p . For $p = 1$, on 3-regular graphs the quantum algorithm always finds a cut that is at least 0.6924 times the size of the optimal cut.

1411.4028

E. Farhi,
J. Goldstone,
S. Gutmann

Citations per year



Quantum Approximate Optimization Algorithm (QAOA)

Farhi et al 2014

$$H_P = C(s) = H_C = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right) : \text{Problem Hamiltonian}$$

(i, j) : the edge index

i : vertex index

$$H_M = B = H_B = \sum_j \sigma_j^X : \text{Mixer Hamiltonian (Initial Hamiltonian)}$$

Full Hamiltonian: $H(t) = \left(1 - \frac{t}{T} \right) H_M + \frac{t}{T} H_P$

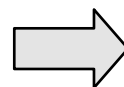
$$|\psi\rangle = \exp \left[-i \int_0^t H(t') dt' \right] |\psi_0\rangle = \exp \left[-i \sum_{j=1}^p H(j\Delta t) \Delta t \right] |\psi_0\rangle$$

$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left[\left(1 - \frac{j\Delta t}{T} \right) H_M + \frac{j\Delta t}{T} H_P \right] \right] |\psi_0\rangle$$

$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left(1 - \frac{j\Delta t}{T} \right) H_M \right] \exp \left[-i\Delta t \frac{j\Delta t}{T} H_P \right] |\psi_0\rangle$$

Works in the adiabatic limit
or $p \rightarrow \infty$

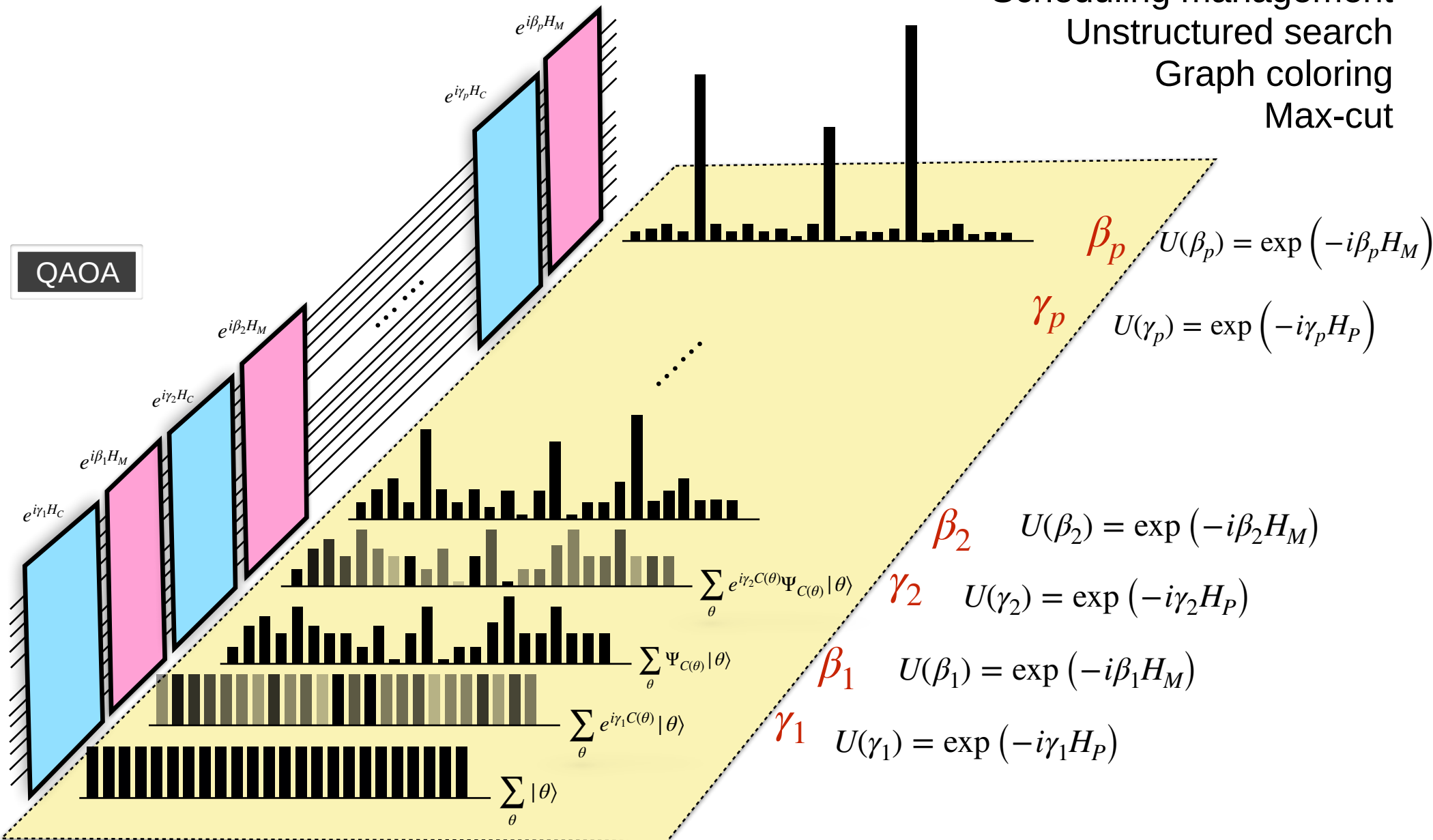
$$= \prod_{j=1}^p \underbrace{\exp \left[-i\beta_j H_M \right]}_{U(H_M, \beta_j)} \underbrace{\exp \left[-i\gamma_j H_P \right]}_{U(H_P, \gamma_j)} |\psi_0\rangle$$



$$|\gamma, \beta\rangle = \prod_{j=1}^p U(H_M, \beta_j) U(H_P, \gamma_j) |+\rangle^{\otimes n}$$

QAOA

Maximum Likelihood detection
 Traveling salesman problem
 Scheduling management
 Unstructured search
 Graph coloring
 Max-cut



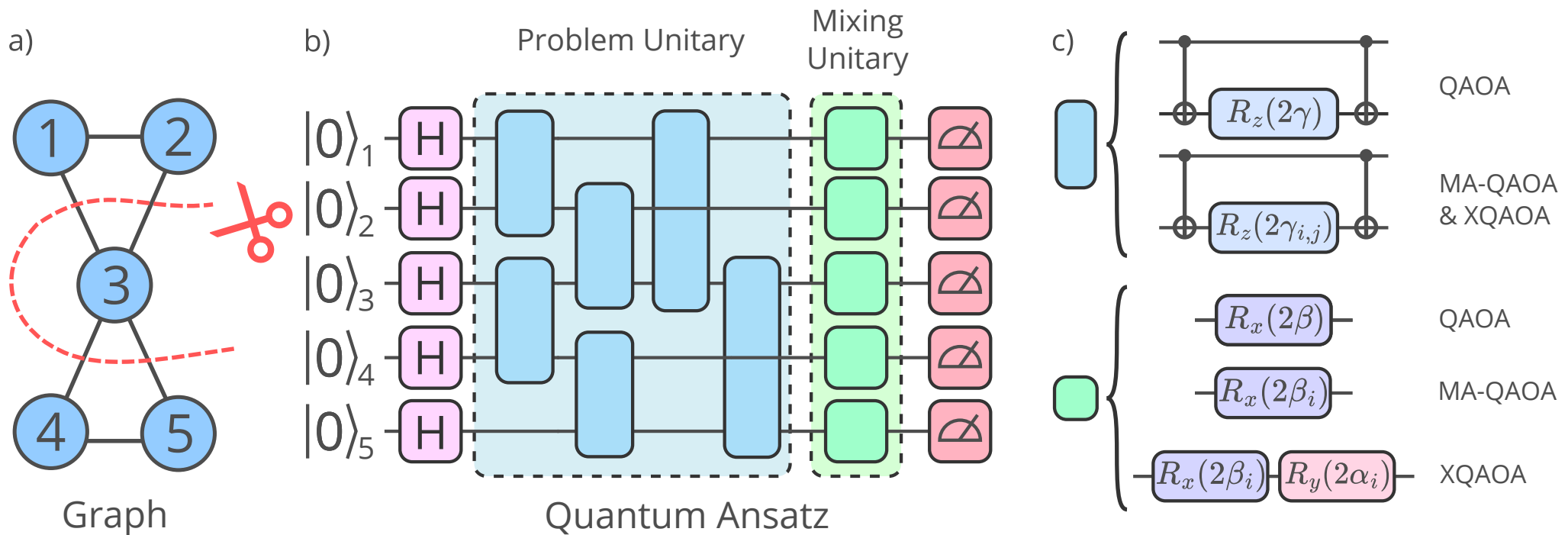


FIG. 1: a) A specific instance of a graph for which we want to identify a set of vertices that maximises the number of edges that are cut. b) A quantum circuit with a single iteration of a quantum ansatz applied to it. The quantum ansatz consists of a unitary operation specific to the problem being solved and a problem-independent mixing unitary. c) Decomposing the problem and mixing unitaries for QAOA, MA-QAOA, and XQAOA into CNOT and single-qubit rotation gates.

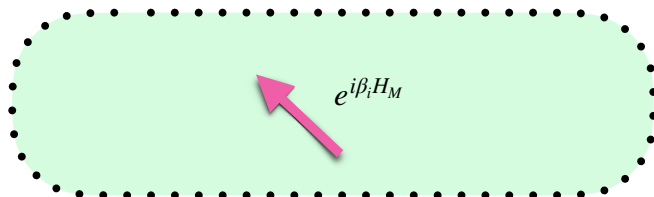
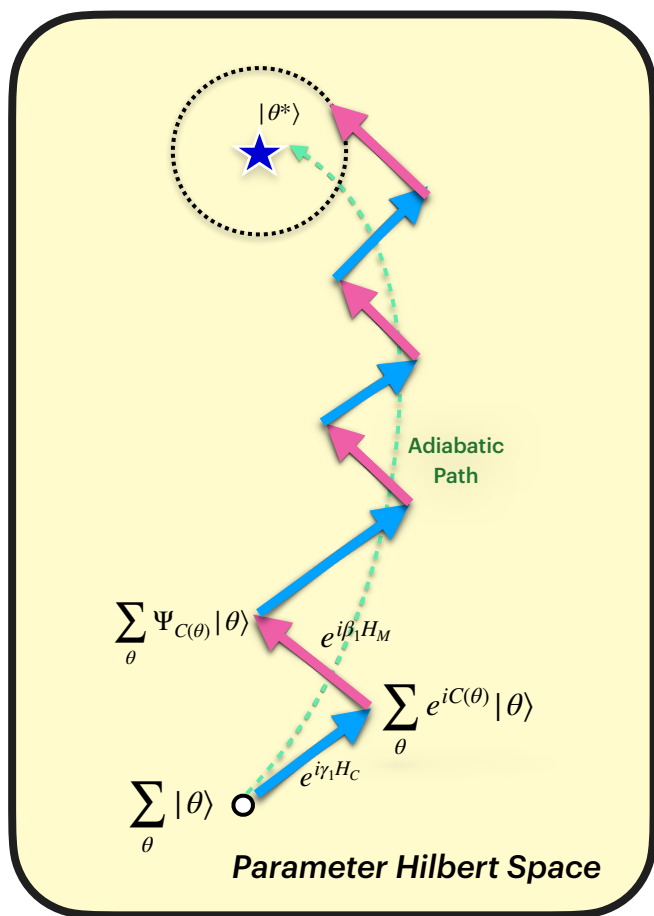
- QAOA: one parameter for each mixer layer
- ma-QAOA: n_q parameters for each mixer layer
- XQAOA: $2n_q$ parameters for each mixer layer

Multi-angle Quantum Approximate Optimization Algorithm: 2109.11455

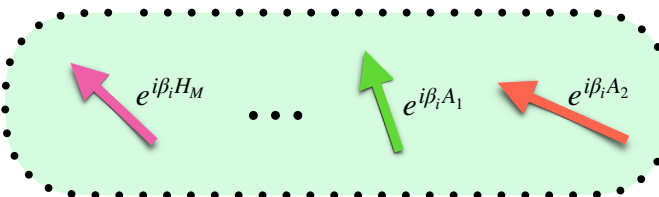
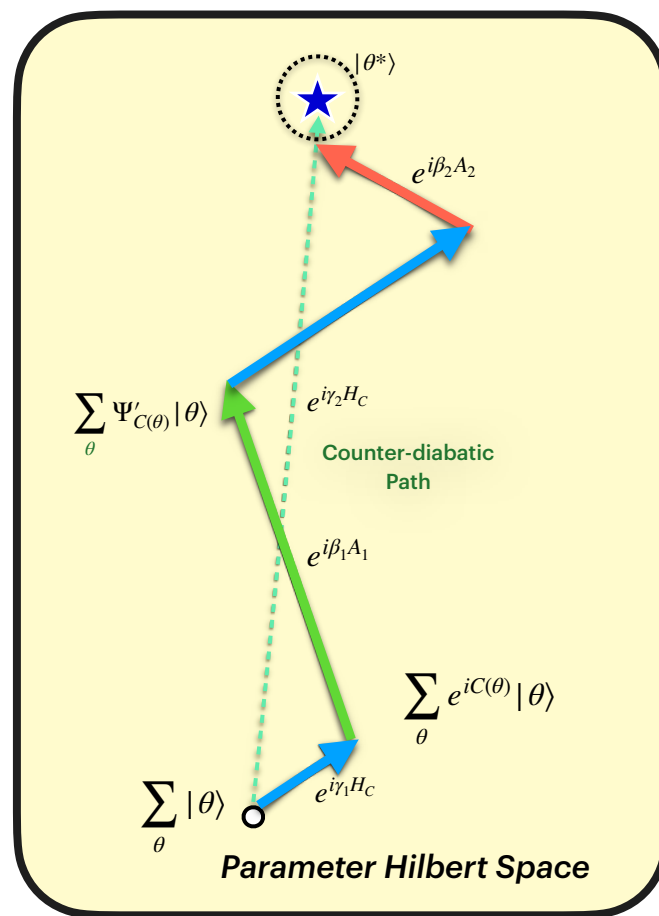
An Expressive Ansatz for Low-Depth Quantum Approximate Optimisation: 2302.04479

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)

QAOA



Adaptive QAOA



Mixers Pool

Feedback-based ALgorithm Quantum Optimization (FALQON)

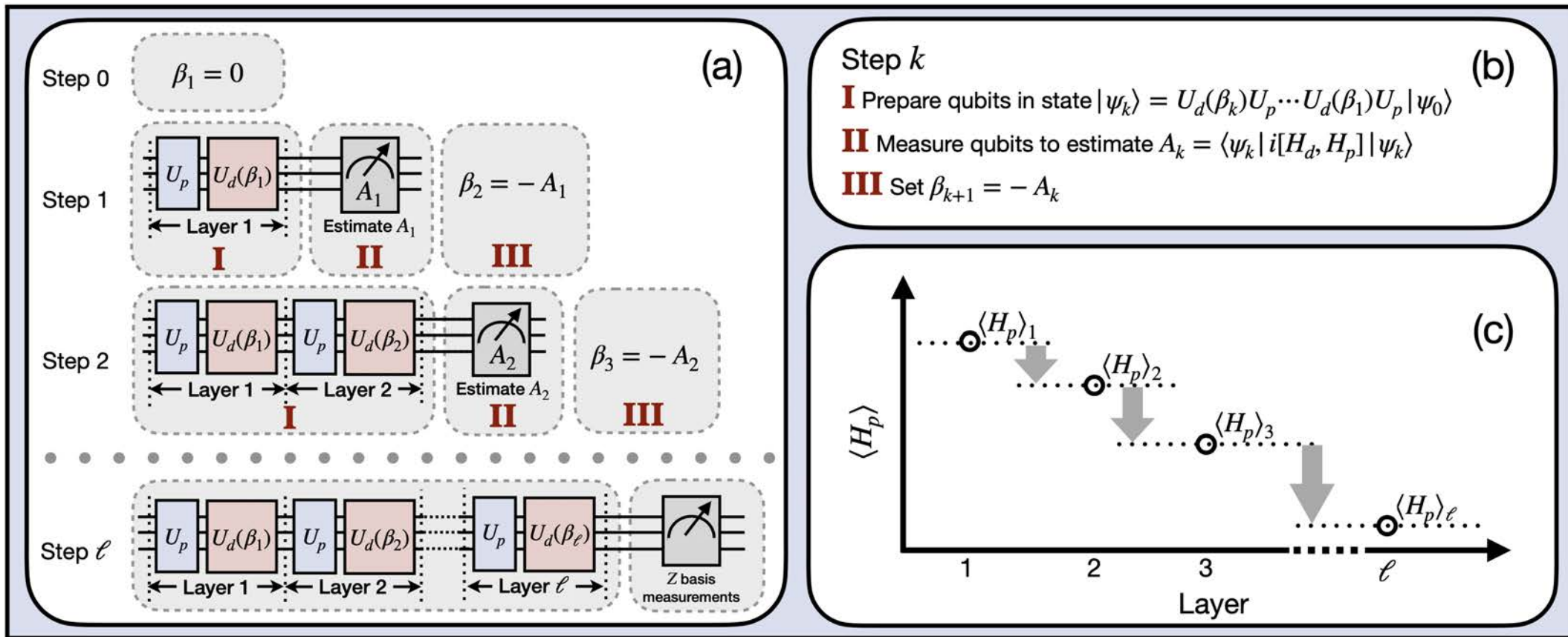


Figure 1. (a) The procedure for implementing FALQON. The initial step is to seed the procedure by setting $\beta_1 = 0$. The qubits are then initialized in the state $|\psi_0\rangle$, and a single FALQON layer is implemented to prepare $|\psi_1\rangle = U_d(\beta_1)U_p|\psi_0\rangle$. The qubits are then measured to estimate A_1 , whose result is fed back to set $\beta_2 = -A_1$, up to sampling error. For subsequent steps $k = 2, \dots, \ell$, the same procedure is repeated, as shown in (b): the qubits are initialized as $|\psi_0\rangle$, after which k layers are applied to obtain $|\psi_k\rangle = U_d(\beta_k)U_p \cdots U_d(\beta_1)U_p|\psi_0\rangle$, and then the qubits are measured to estimate A_k , and the result is fed back to set the value of β_{k+1} . This procedure causes $\langle H_p \rangle$ to decrease layer-by-layer as per $\langle \psi_1 | H_p | \psi_1 \rangle \geq \langle \psi_2 | H_p | \psi_2 \rangle \geq \dots \geq \langle \psi_\ell | H_p | \psi_\ell \rangle$, as shown in (c), such that the quality of the solution to the combinatorial optimization problem monotonically improves with circuit depth. The protocol can be terminated when the value of $\langle H_p \rangle$ converges or a threshold number of layers ℓ is reached. Then, after the final step, Z basis measurements on $|\psi_\ell\rangle$ can be used to determine a best candidate solution to the combinatorial optimization problem of interest, by repeatedly sampling from the probability distribution over bit strings induced by $|\psi_\ell\rangle$ and selecting the outcome associated with the best solution.

Variational Quantum Imaginary Time Evolution (VarQITE)

- State time evolution under a Hamiltonian: $|\psi(t)\rangle = \sum_m c_m \exp\left(-iE_m \frac{t}{\hbar}\right) |m\rangle$
- Imaginary time, $t \longrightarrow -i\beta$

$$|\psi(\beta)\rangle = \sum_m c_m \exp\left(-E_m \frac{\beta}{\hbar}\right) |m\rangle \underset{\beta \rightarrow \infty}{\sim} c_0 \exp\left(-E_0 \frac{\beta}{\hbar}\right) |0\rangle$$

All coefficients decay 

$$|0\rangle = \lim_{\beta \rightarrow \infty} \frac{|\psi(\beta)\rangle}{\| |\psi(\beta)\rangle \|}$$

$$|\psi_{n+1}\rangle = \frac{e^{-\Delta\tau H \hbar} |\psi_n\rangle}{\| e^{-\Delta\tau H \hbar} |\psi_n\rangle \|}$$

- In VarQITE, $|\psi(\beta)\rangle$ is replaced with $|\psi(\theta(\beta))\rangle$ with variational parameters θ . Using McLachlan variational principle, the algorithm update variational parameters θ by minimizing

$$\left\| \left(\frac{\partial}{\partial \beta} + H - E_\beta \right) |\psi(\theta(\beta))\rangle \right\|$$

2404.16135, Performant near-term quantum combinatorial optimization

1901.07653, Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution

1812.08767, Theory of variational quantum simulation

Combinatorial problems at the LHC

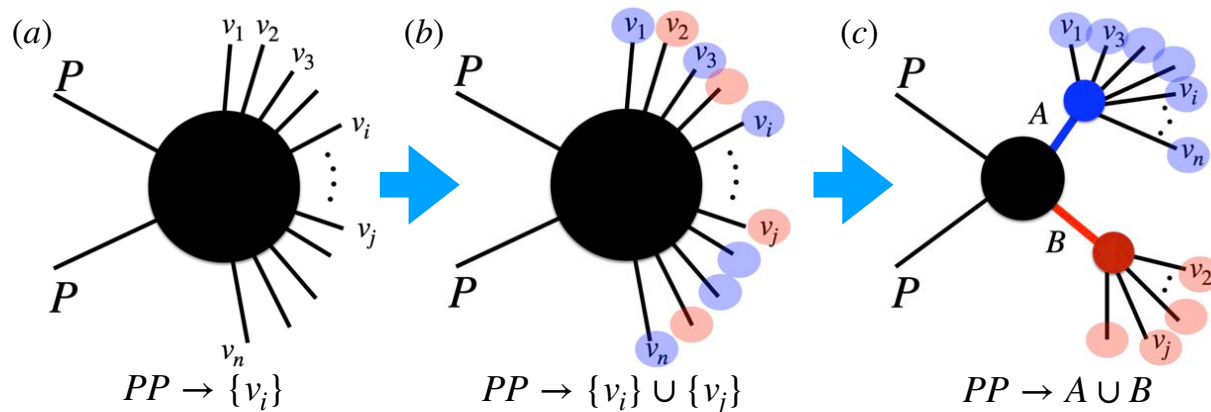


FIG. 1. (a) n -observed particles (b) Dividing n particles into two groups for $2 \rightarrow 2$ process (c) Identified event-topology with A and B .

- Assuming $2 \rightarrow 2$ production with subsequent decays, identification of an event-topology becomes a binary classification, with 2^{n-1} possibilities.
- Combinatorial problem: What would be an efficient way of assigning all observed particles in two decay chains?**

p_i is the momentum of constituent of A if $x_i = 1$

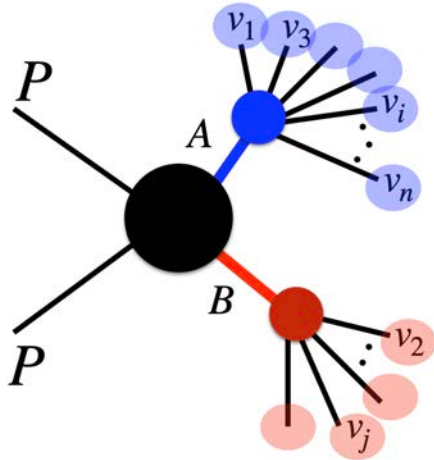
$$P_1 = \sum_i p_i x_i$$

p_i is the momentum of constituent of B if $x_i = 0$

$$P_2 = \sum_i p_i (1 - x_i)$$

Minimize the mass difference: $H = (P_1^2 - P_2^2)^2$ for all possible combinations of x_i

Combinatorial problems in the top quark production



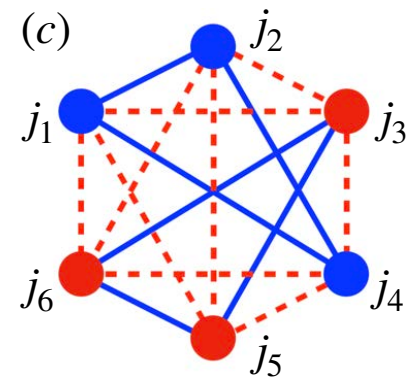
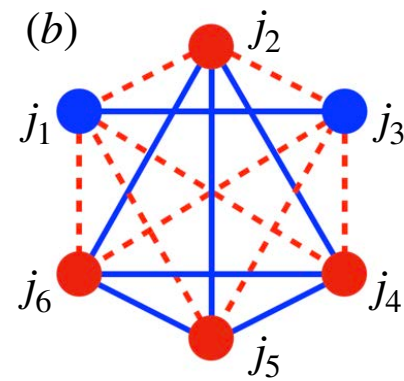
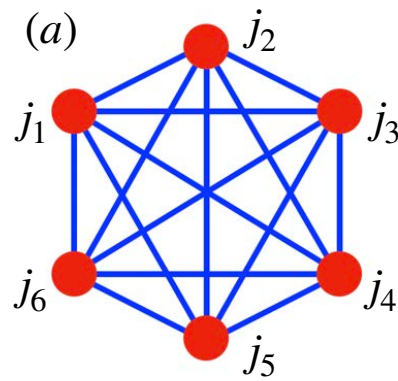
$$PP \rightarrow A \cup B$$

$$H_0 = (P_1^2 - P_2^2)^2$$

$$P_1 = \sum_{i=1}^n p_i x_i,$$

$$P_2 = \sum_{i=1}^n p_i (1 - x_i),$$

$$H_P = H_0 + \lambda H_1$$



$$H_0 = \left(\sum_{ij} P_{ij} x_i x_j - \sum_{ij} P_{ij} (1 - x_i)(1 - x_j) \right)^2$$

$$= \left(\frac{1}{4} \sum_{ij} P_{ij} [(1 + s_i)(1 + s_j) - (1 - s_i)(1 - s_j)] \right)^2$$

$$= \left(\sum_{ij} P_{ij} s_i \right)^2 = \sum_{ij} J_{ij} s_i s_j,$$

$$H_1 = (P_1^2 + P_2^2)$$

$$= \frac{1}{4} \sum_{ij} P_{ij} [(1 + s_i)(1 + s_j) + (1 - s_i)(1 - s_j)]$$

$$\rightarrow \frac{1}{2} \sum_{ij} P_{ij} s_i s_j,$$

$$x_i = \frac{1 + s_i}{2}$$

$$J_{ij} = \sum_{k\ell} P_{ik} P_{j\ell},$$

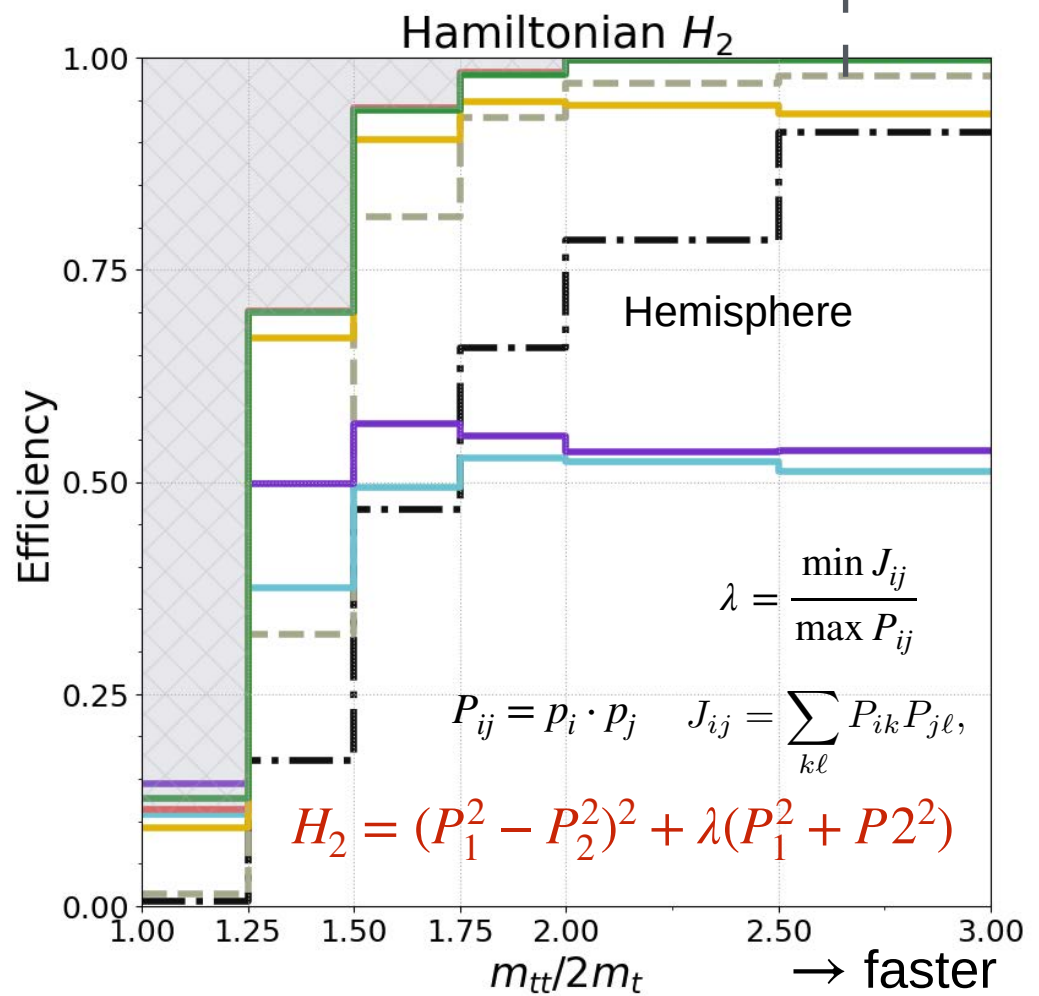
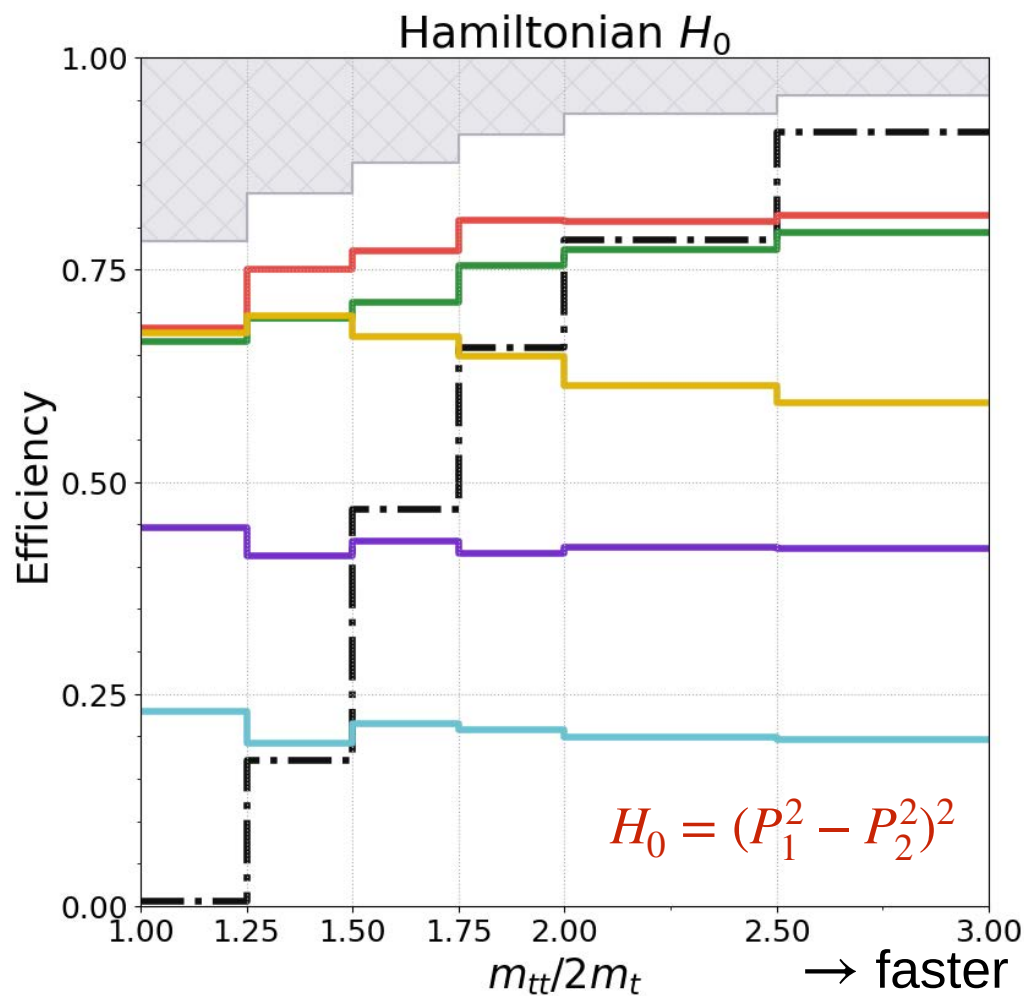
$$P_{ij} = p_i \cdot p_j$$

$$\lambda = \frac{\min J_{ij}}{\max P_{ij}}$$

- Simulated various quantum algorithms using PennyLane to resolve combinatorial ambiguity in the top quark production (6 jets)
 - QAOA, ma-QAOA: Adam optimizer used
 - VarQITE, FALQON: no classical optimization needed
- Compared performance against hemisphere method and classical ML (using SPANet)
- Event generation with MG5 with proper off-shell effects (t and W)
 - Parton-level events and smeared events (for detector effects)

$$\frac{v}{c} = \sqrt{1 - \frac{4m_t^2}{m_{t\bar{t}}}}$$

Comparison



QAOA ($p=8$) $\varepsilon = 42.36\%$	XQAOA ($p=8$) $\varepsilon = 47.28\%$	ma-QAOA ($p=8$) $\varepsilon = 78.91\%$	FALQON ($p=250$) $\varepsilon = 74.83\%$	VarQITE $\varepsilon = 78.98\%$	Hemisphere Method $\varepsilon = 49.98\%$	Quantum Annealer $\varepsilon = 67.10\%$
---	--	--	---	------------------------------------	--	---

$$\frac{m_{t\bar{t}}}{2m_t} = 1$$

$$v = 0$$

$$\frac{m_{t\bar{t}}}{2m_t} = 1.5$$

$$v = 0.75c$$

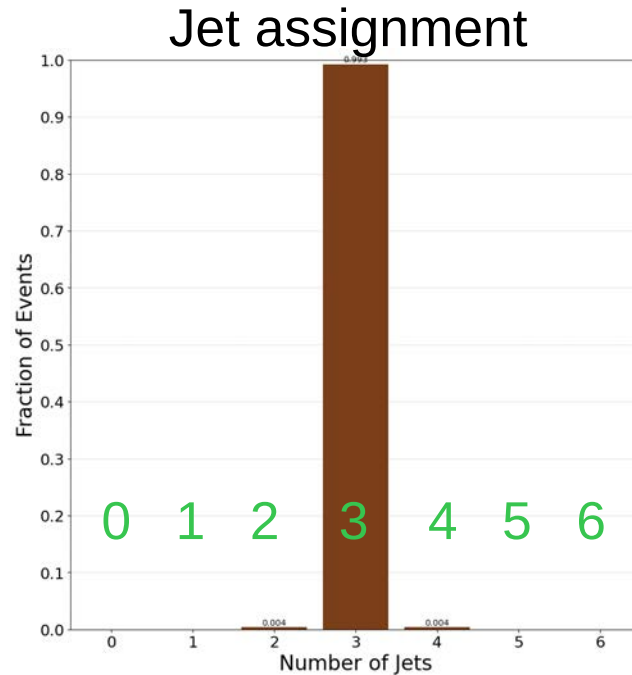
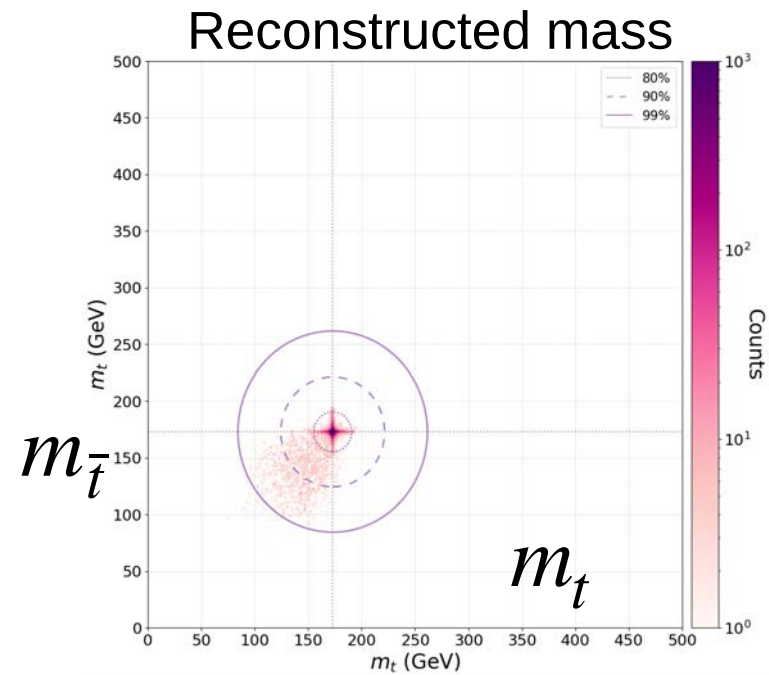
$$\frac{m_{t\bar{t}}}{2m_t} = 2$$

$$v = 0.87c$$

$$\frac{m_{t\bar{t}}}{2m_t} = 3$$

$$v = 0.94c$$

Parton-level truth and hemisphere method

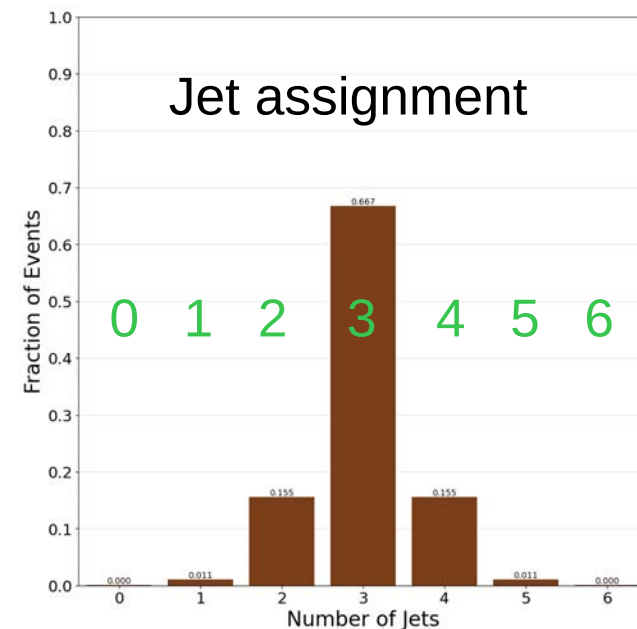
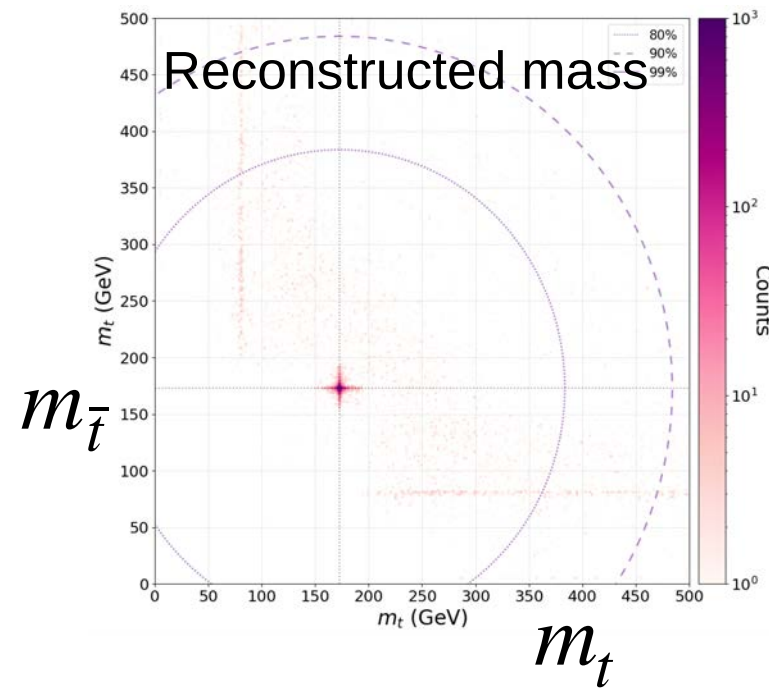


parton-level truth

79% overall efficiency

No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417



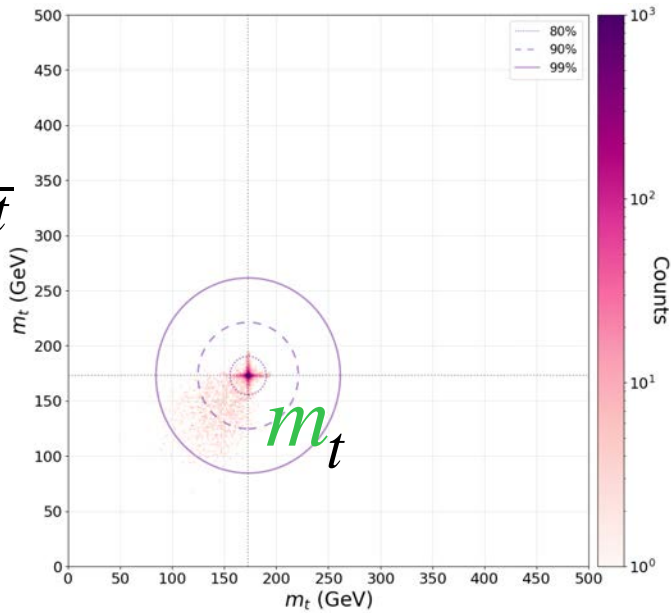
Hemisphere method

50% overall efficiency

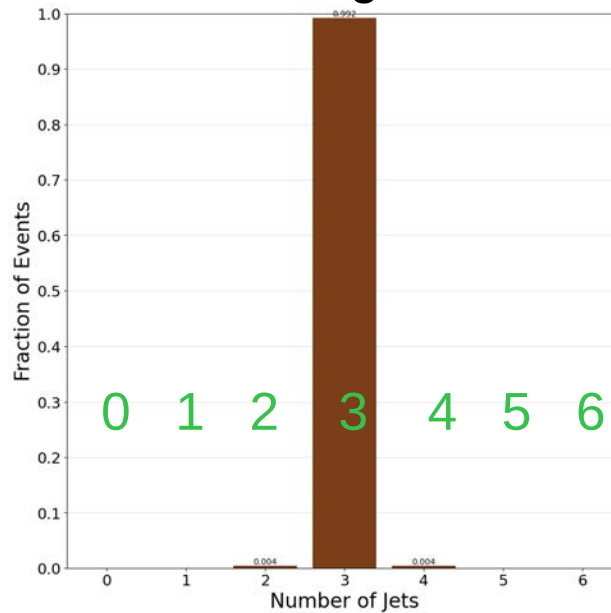
No mass information used.

ma-QAOA and FALQON

Reconstructed mass



Jet assignment



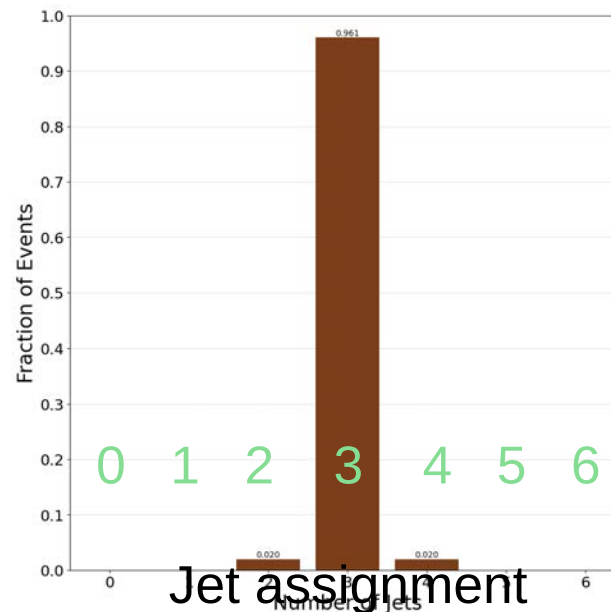
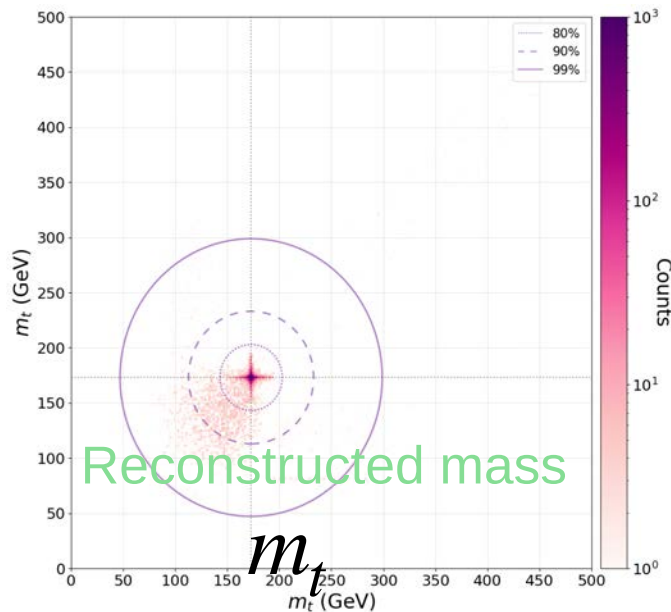
ma-QAOA / VarQITE

79% overall efficiency

No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

$m_{\bar{t}}$



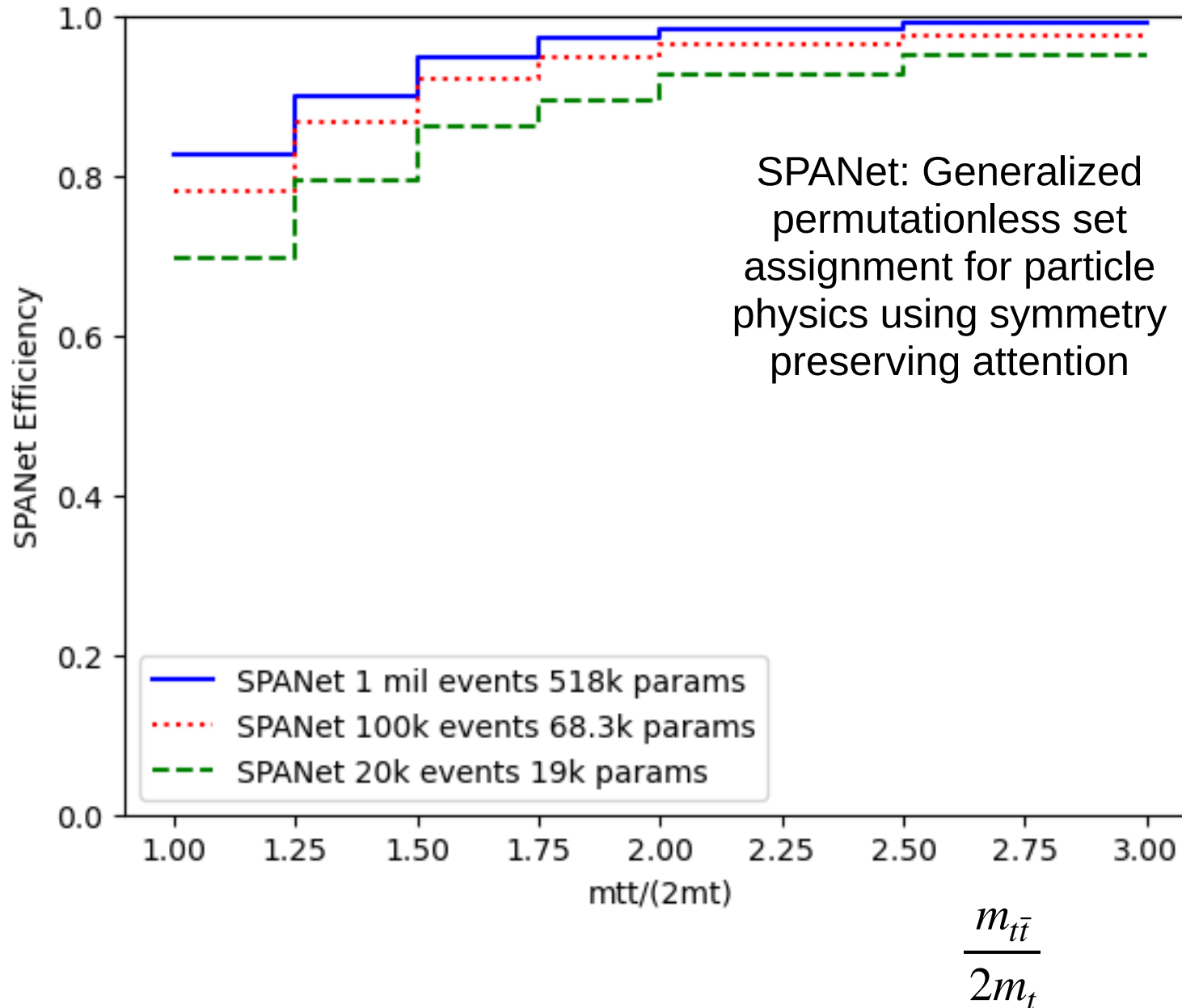
FALQON

78% overall efficiency

No mass information used.

Jet assignment

Combinatorial problems in the top quark production



Combinatorial problems in the top quark production

Methods	matching accuracy (efficiency)		n_{train}	$n_{\text{parameters}}$	depth (p)	n_{CNOT}	n_{R_Z}	n_{R_X}	n_{R_Y}
	parton-level	smeared events							
Hemisphere	50%	48%	N/A						
QAOA	43%	41%	N/A	16	8	240	120	48	0
ma-QAOA	79%	77%		168					
XQAOA	47%	46%		216					48
FALQON	78%	76%		2	250	7,500	3,750	1,500	0
VarQITE	79%	77%		16	1	30	15	30	0
SPANet	91%	70%	5×10^5	10^6	N/A				
	81	62%	2×10^4	1.9×10^3					

TABLE I. Summary of the performance of various methods and the corresponding parameters.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

Currently running these algorithms on IonQ

Discussion

- Real hardware run: IonQ with $n=6$
- Scalability: $pp \rightarrow AB \rightarrow j_1 \cdots j_n, n > 6$
- Can we do this with hadrons ($n \sim O(100)$)?

High-energy reaction

aa

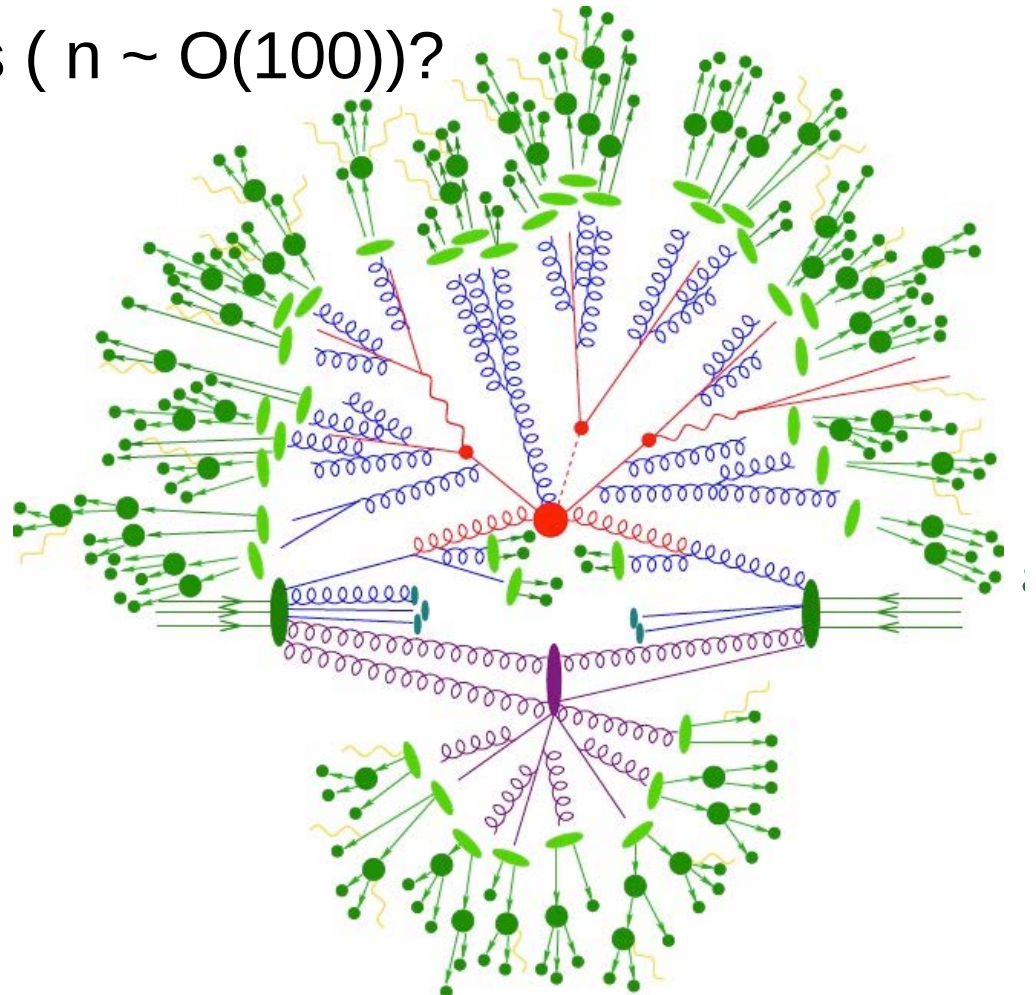
Radiation cascade

aa

Secondary scatterings

aa

Hadronization

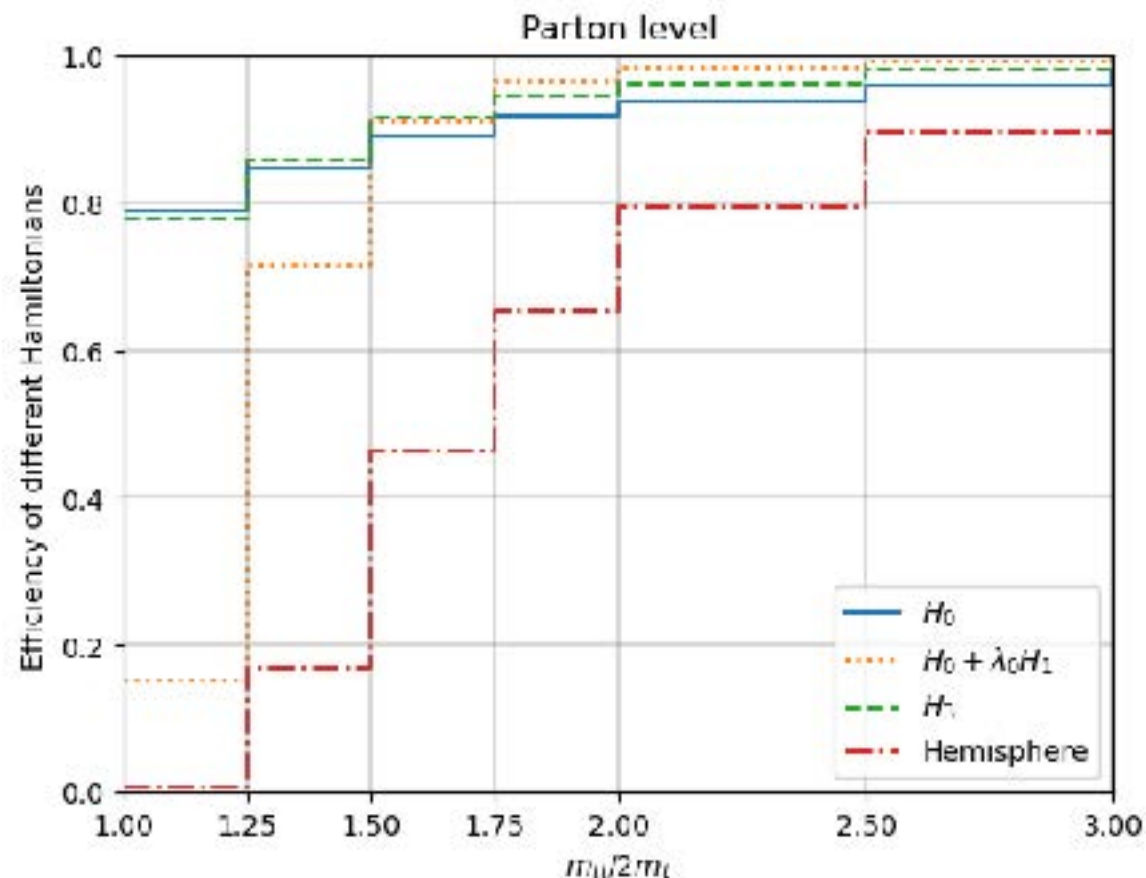


Discussion

- Real hardware run: IonQ with $n=6$
- Scalability: $pp \rightarrow AB \rightarrow j_1 \cdots j_n, n > 6$
- Can we do this with hadrons ($n \sim O(100)$)?
- Higher order terms

$$\begin{aligned}
 H_3 &= H_0 H_1 \\
 &= (P_1^2 - P_2^2)^2 \cdot (P_1^2 + P_2^2) \\
 &= \frac{1}{2} \left[\left(\sum_{k,\ell} P_{k\ell} \right) \sum_{i,j} J_{ij} s_i s_j + \sum_{i,jk\ell} J_{ij} P_{k\ell} s_i s_j s_k s_\ell \right]
 \end{aligned}$$

- QUBO is extended to a problem with higher order (quartic) terms.
- 85% of the time, the ground state of H_3 resolves the combinatorial problem.
- What quantum algorithm can simulate this Hamiltonian?



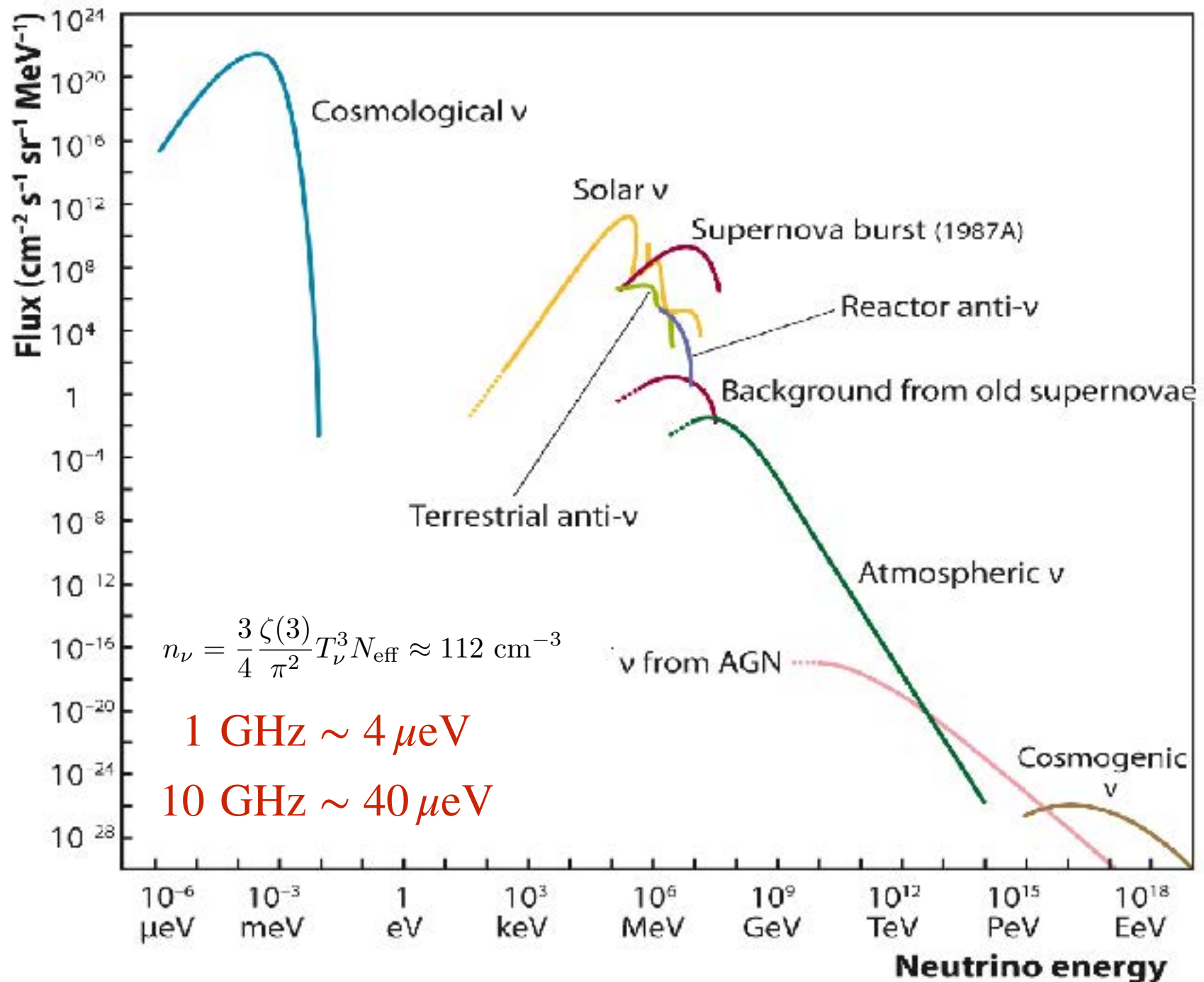
Discussion

- Real hardware run: IonQ with $n=6$
- Scalability: $pp \rightarrow AB \rightarrow j_1 \cdots j_n, n > 6$
- Can we do this with hadrons ($n \sim O(100)$)?
- Higher order terms
- Full error correction
- Combinatorial problems with ISR
 - Naive extension requires twice more qubits for n particles (two binary variables, x_i and y_i for $i = 1, \dots, n$)
 - Qubits with efficient embedding: $2 \otimes 2 = 3 \oplus 1$
 - Qutrits
- Combinatorial problems with missing transverse momentum
 - Multi-jets + MET

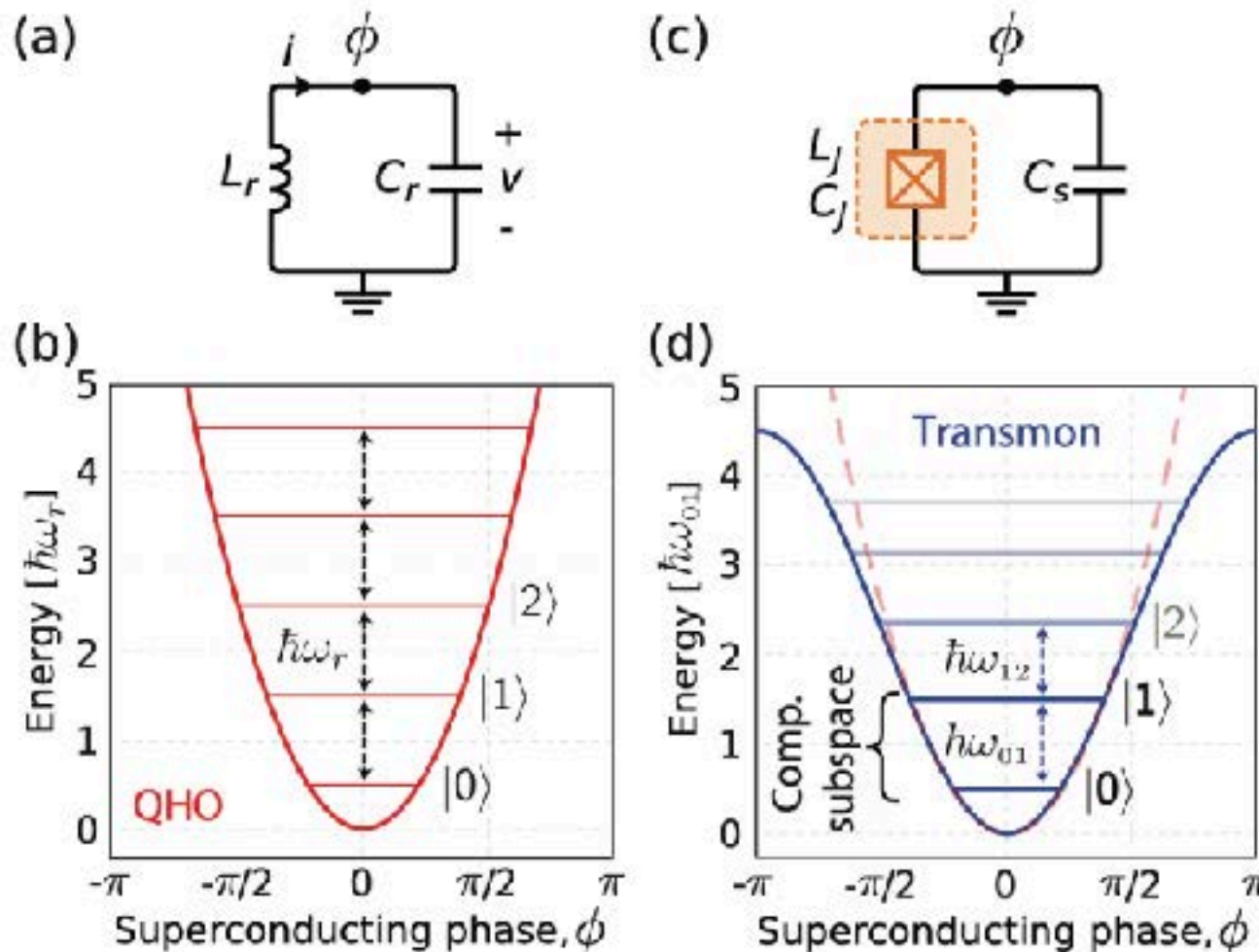
Where quantum computers are useful

- Quantum optimization: combinatorial problems at the LHC
 - Finance, physics, materials discovery, science
- Quantum devices as sensors: detection of dark matter and neutrino decays
 - Dark matter detection, quantum magnetometry + electrometry, atomic clocks
- Quantum simulation/Hamiltonian simulation
 - Physics, chemistry
- Quantum machine learning
 - Classification, generative models, anomaly detection
- Quantum cryptography and security
 - Quantum key distribution, post-quantum cryptography
- Quantum linear algebra and numerical methods
 - HHL algorithms, eigenvalue problems
- Quantum communication and networks
 - Entanglement, teleportation, quantum internet
- Quantum sampling
 - Supremacy tasks, random circuits

Neutrino fluxes from different sources as a function of the neutrino energy



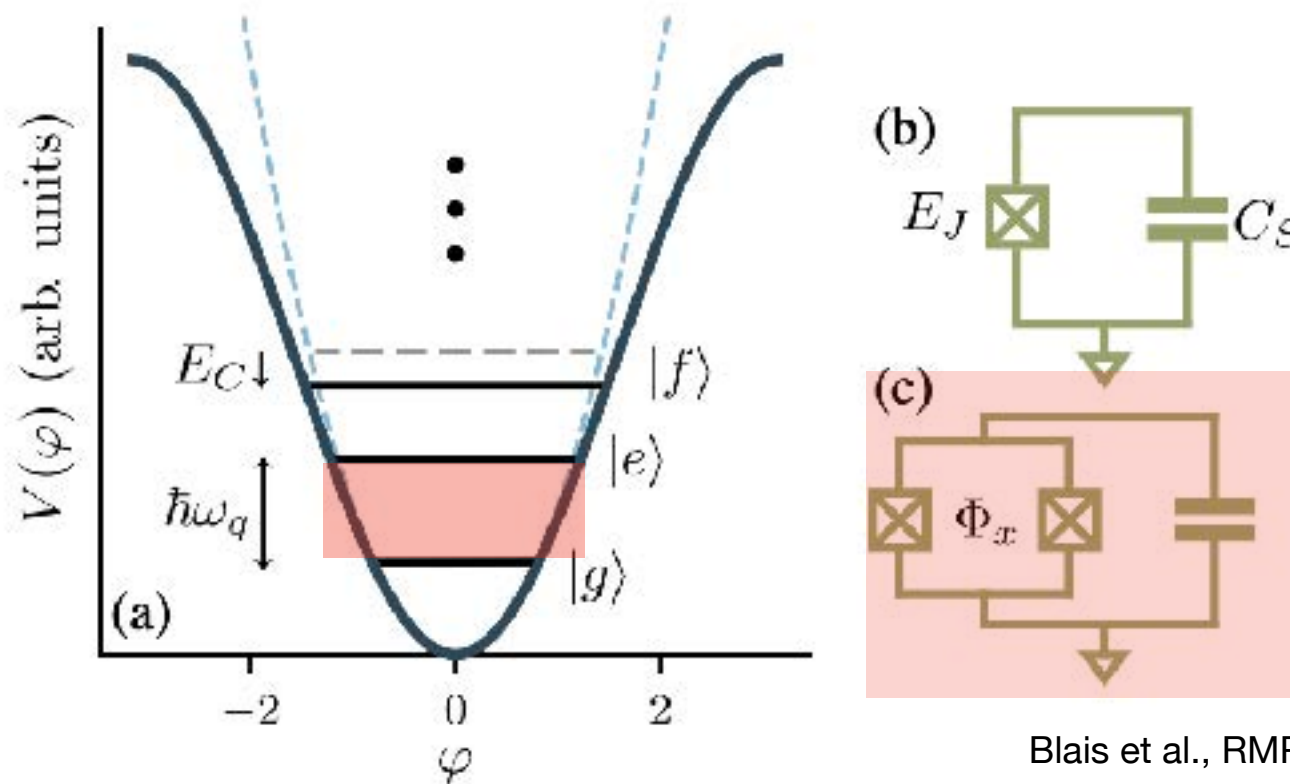
Transmon qubit = Capacitor + Josephson junction



P. Krantz et.al Applied Physics Reviews 6, 021318 (2019)

- Josephson nonlinearity breaks the harmonic spectrum, leading to discrete, unequal energy levels.
- A system of two lowest states can be considered as qubit with ω_{01} .

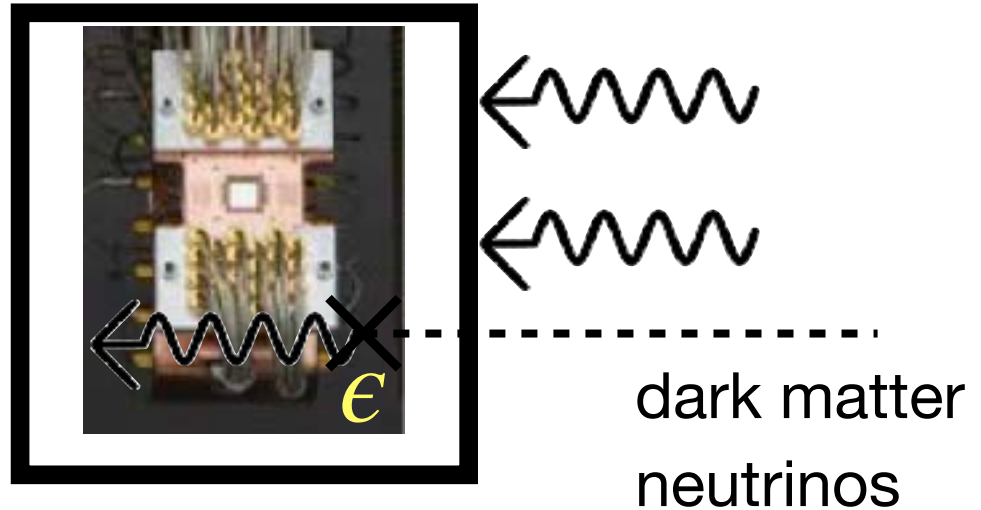
Transmon qubit = Capacitor + Josephson junction



Blais et al., RMP 93 025005 (2021)

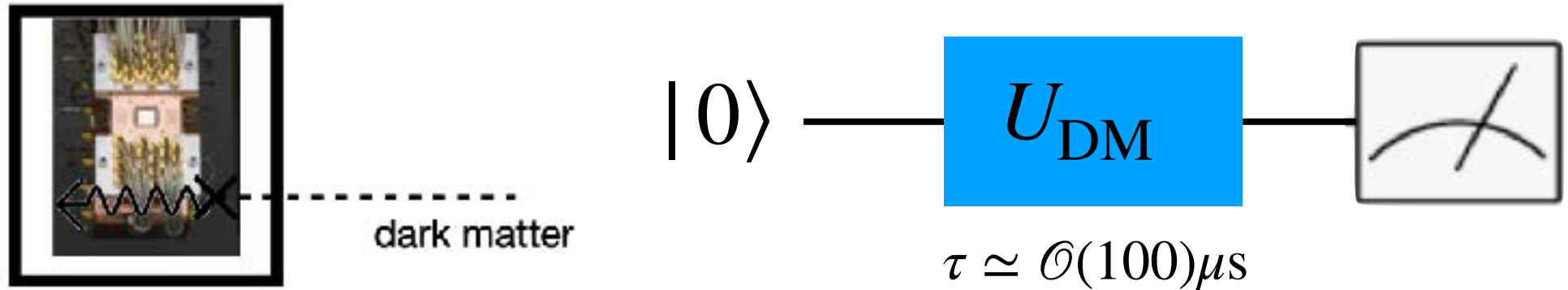
- "By using a superconducting quantum interference device (**SQUID**) rather than a single junction, the **frequency of the transmon qubit becomes flux tunable**."
- Transmon qubits have discrete energy levels that can be controlled.
- One can scan the frequency ω_q in the (c) SQUID-based system.

$$\omega \simeq \mathcal{O}(1 - 10) \text{ GHz} \quad \lambda \simeq \mathcal{O}(3 - 30) \text{ cm}$$



- For typical device parameters, $\omega \simeq \mathcal{O}(1 - 10)\text{GHz}$ with $\lambda \simeq \mathcal{O}(3 - 30)\text{cm}$
- At refrigerator temperatures $\mathcal{O}(1 - 10)\text{mK}$, a few-GHz transition satisfies $k_B T \ll \hbar\omega$.
 - Thus thermal excitation is strongly suppressed, and the qubit stays in the ground state.
- “Dark Matter” could penetrate the shield, and induce a weak electric field that couples to qubit, $E^{(\text{ext})} = \epsilon\sqrt{2\rho_{\text{DM}}}\sin m_X t$, here ϵ is a kinetic-mixing parameter, ρ_{DM} is the local DM density.
- **If the DM-induced photons fall within this frequency band, they can resonantly excite the qubit - effectively turning dark-matter interactions into observable qubit errors.** For a de Broglie wavelength in this range, $m_{\text{DM}} \sim \mathcal{O}(4 - 40)\mu\text{eV}$

Dark Matter as an operator U_{DM}



- Signal: a weak classical field: wave-like DM, high-frequency gradational waves
- The interaction between DM and qubits, $H_{\text{DM}} = \epsilon(\cos \alpha \sigma_X + \sin \alpha \sigma_Y)$ will drive transitions from $|0\rangle$ to $|1\rangle$.
- The Goal to estimate the signal strength ϵ .

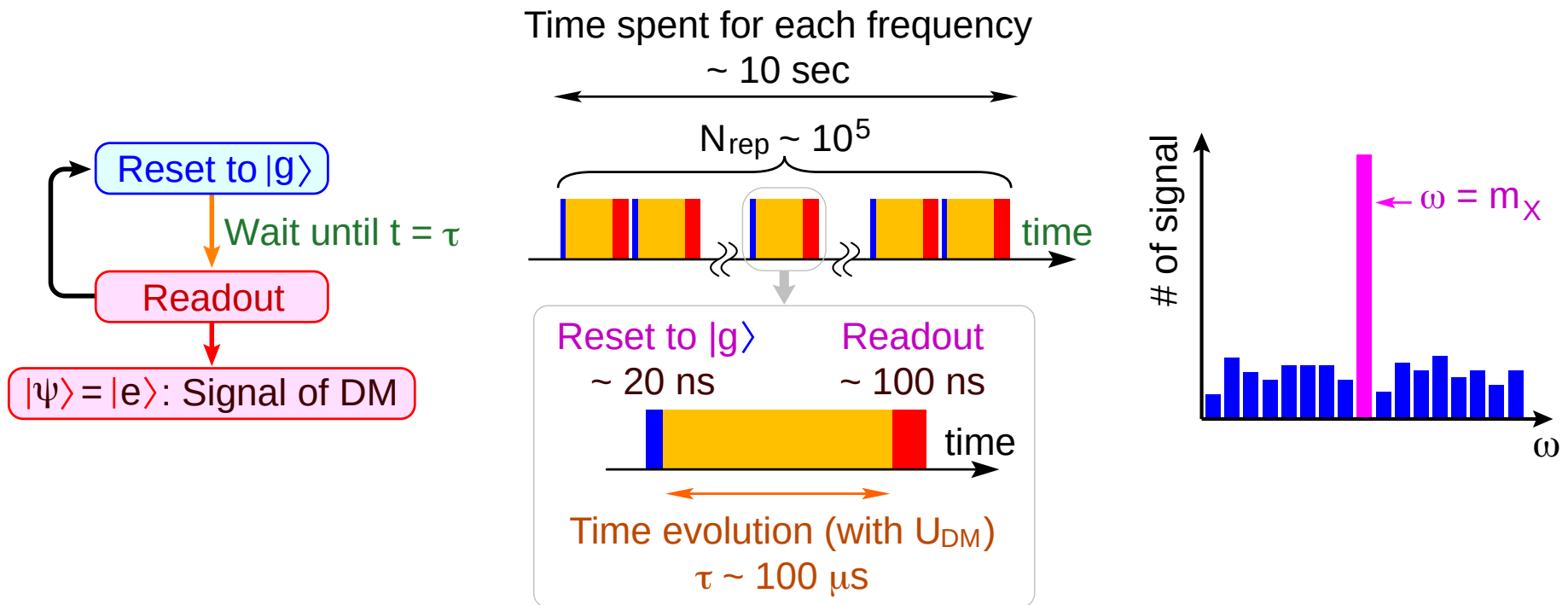
1. Evolve: System evolves from $|0\rangle$ (ground state of free Hamiltonian) under H_{DM} . For a weak signal $\epsilon t \ll 1$,

$$|\psi(0)\rangle = |0\rangle \longrightarrow |\psi(t)\rangle = U(t)|\psi(0)\rangle \approx |0\rangle - ie^{i\alpha} \epsilon t |1\rangle$$

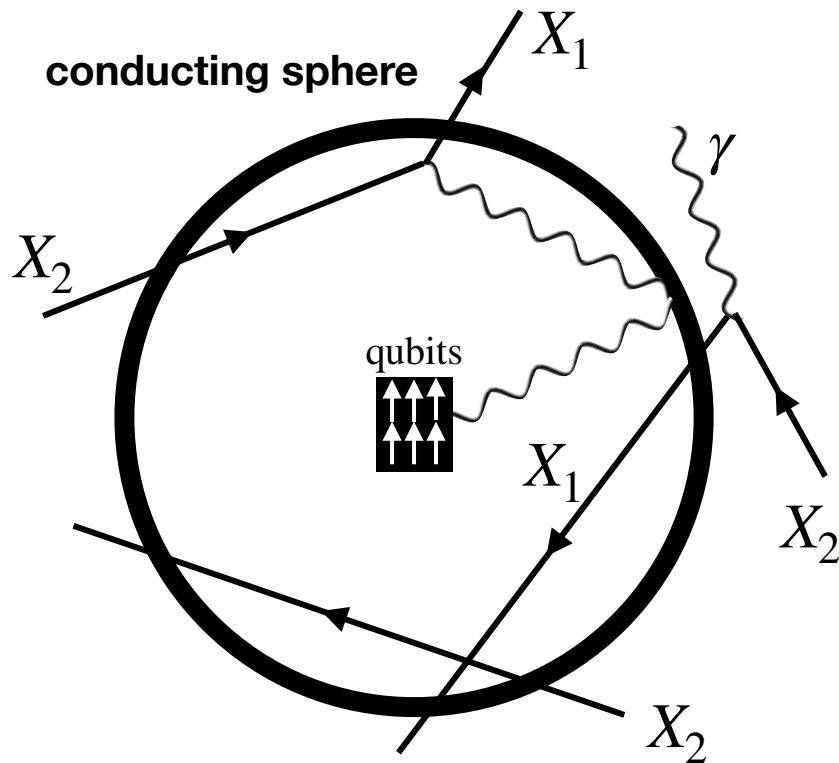
2. Measure: Project $|\psi(t)\rangle$ onto $|0\rangle$ and $|1\rangle$. Probability of finding $|1\rangle$:

$$p_1 = \left| \langle 1 | \psi(t) \rangle \right|^2 = \epsilon^2 t^2$$

Dark Matter as an operator U_{DM}



- Qubits are exposed to microwaves from dark matters over the coherence time $\mathcal{O}(100)\mu\text{s}$
- One can repeat a cycle of (Reset $\rightarrow U_{\text{DM}} \rightarrow$ Measurement) as many as $\mathcal{O}(10^5)$ during 10 seconds.
 - Time for reset qubits $\sim \mathcal{O}(10)\mu\text{s}$
 - Time for measurement $\simeq \mathcal{O}(100)\text{ns}$



- Photon from the transition process of a heavier state into a lighter state will couple to qubits.
- A conducting cavity with reflective inner walls confines photons.

- Two scenarios :

1) Heavier "dark" particle X_2 decays into lighter particle X_1 and a photon γ , with an interaction Lagrangian $\mu X_1^\mu X_2^\nu \tilde{F}^{\mu\nu}$

2) Cosmic neutrino background $C\nu B$, with transition via neutrino magnetic moment interaction, $L \ni (\mu_\nu^{ij}/2) \bar{\nu}_i \sigma^{\mu\nu} \nu_j F_{\mu\nu}$

2508.09139

(Many thanks to Prof. Kyu Jung Bae and Prof. Takeo Moroi for useful discussion.)

Radiative Decays of Neutrinos and Dark Matter Particles

- Photon from a decay process is described by a plane-wave

$$\vec{E}_{\text{eff}} = \bar{E}_{\text{eff}} \vec{n}_E \sin(E_\gamma t) \text{ with a photon energy } E_\gamma = \frac{m_{X_2}^2 - m_{X_1}^2}{2m_{X_2}}.$$

- Energy density of photons $\rho_\gamma = E_\gamma n_\gamma = \epsilon_0 \bar{E}_{\text{eff}}^2$ with $n_\gamma \propto \Gamma_{X_2} n_{X_2}$
- Interaction between a photon from decay process and qubit,

$$H_{\text{int}} = CV d\bar{E}_{(\text{eff})} \cos \Theta \sin(E_\gamma t) \equiv 2\eta \sin(E_\gamma t)(\hat{a} + \hat{a}^\dagger),$$

so that the total Hamiltonian

$$H = \omega |e\rangle\langle e| + 2\eta \sin(E_\gamma t) (|e\rangle\langle g| + |g\rangle\langle e|)$$

- If one tunes $\omega \simeq E_\gamma$, then with rotating-wave approximation which neglects the rapid oscillation with averaging into 0,

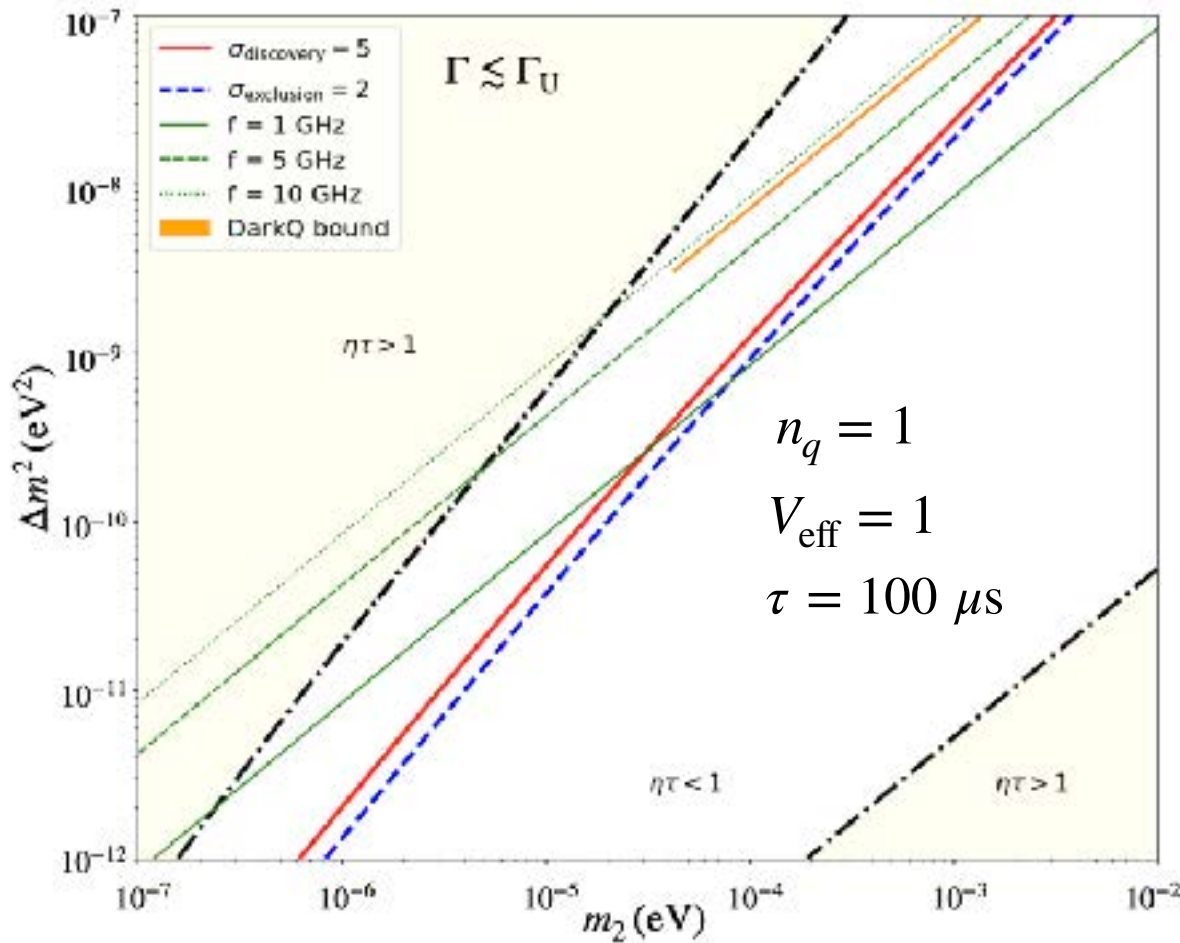
$$i \frac{d}{dt} \begin{pmatrix} \psi_g(t) \\ \psi_e(t) \end{pmatrix} \simeq \begin{pmatrix} 0 & -i\eta \\ -i\eta & 0 \end{pmatrix} \begin{pmatrix} \psi_g(t) \\ \psi_e(t) \end{pmatrix} \Rightarrow \begin{aligned} \psi_g(t) &\simeq \cos \eta t \\ \psi_e(t) &\simeq \sin \eta t \end{aligned}$$

with initial condition of $\psi_g(0) = 1$ and $\psi_e(0) = 0$

- Thus, a transition probability $p_{g \rightarrow e}(t)$ from $|g\rangle \rightarrow |e\rangle$

$$p_* \equiv p_{g \rightarrow e}(\tau) = |\psi_e(\tau)|^2 \simeq \sin^2(\eta\tau) \simeq (\eta\tau)^2 \text{ within a coherent time}$$

- Signals: $N_{\text{sig}} = p_* \times N_{\text{try}}$
- Backgrounds
 - Uniform 0.1% readout error: $10^{-3} \times N_{\text{try}}$
 - Thermal noise: $e^{-\omega/T} \times N_{\text{try}}$, where $T \sim 30$ mK



- Scanning the frequency range from **1 - 10 GHz** with a quality factor $Q = 10^6$, which sets the frequency resolution to $\delta f = f/Q$.
- This corresponds to about 2.3×10^6 resolvable frequency bins across the **1-10GHz** range
- One year continuous scan (3.15×10^7 s) yields 14s of integration time per frequency bin, corresponding to $N_{\text{try}} = \mathcal{O}(10^5)$ / bin

$$p_* = 0.069$$

$$p_* \simeq p_*^0 \times \cos^2 \Theta \left(\frac{V_{\text{eff}} \rho_{\text{DM}}}{0.45 \text{ GeV cm}^{-3}} \right) \left(\frac{\Delta m^2}{m_2^2} \right) \left(\frac{d}{100 \text{ } \mu\text{m}} \right)^2 \left(\frac{C}{0.1 \text{ pF}} \right) \left(\frac{f}{1 \text{ GHz}} \right) \left(\frac{\tau}{100 \text{ } \mu\text{s}} \right)^3$$

- Assuming a local dark matter density $\rho_{\text{DM}} = 0.45 \text{ GeV/cm}^3$ and decay rate $\Gamma \lesssim \Gamma_{\text{U}} = 2.299 \times 10^{-18} \text{ s}^{-1}$ ensuring that X_2 remains stable over the age of the universe.

Quantum Enhancement of the Signal Rate

- Suppose we have N identical sensors. What is the maximum sensitivity?

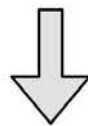
Without entanglement

$$|0\rangle \xrightarrow{\text{DM}} |0\rangle - i\epsilon\tau |1\rangle \Rightarrow p = \epsilon^2\tau^2$$

$$|0\rangle \xrightarrow{\text{DM}} |0\rangle - i\epsilon\tau |1\rangle \Rightarrow p = \epsilon^2\tau^2$$

$\vdots$

$$|0\rangle \xrightarrow{\text{DM}} |0\rangle - i\epsilon\tau |1\rangle \Rightarrow p = \epsilon^2\tau^2$$



$$p_{\text{total}} = 1 - (1 - p)^N \approx Np = N\epsilon^2\tau^2$$

Classical

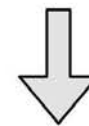
- N independent system are prepared and separately detected

With entanglement

$|\psi\rangle \equiv$ some entangled state

$$|\psi\rangle \xrightarrow{\text{DM}} |\psi\rangle + O(N\epsilon\tau) |\psi_{\perp}\rangle$$

An example of such a state is the GHZ state.



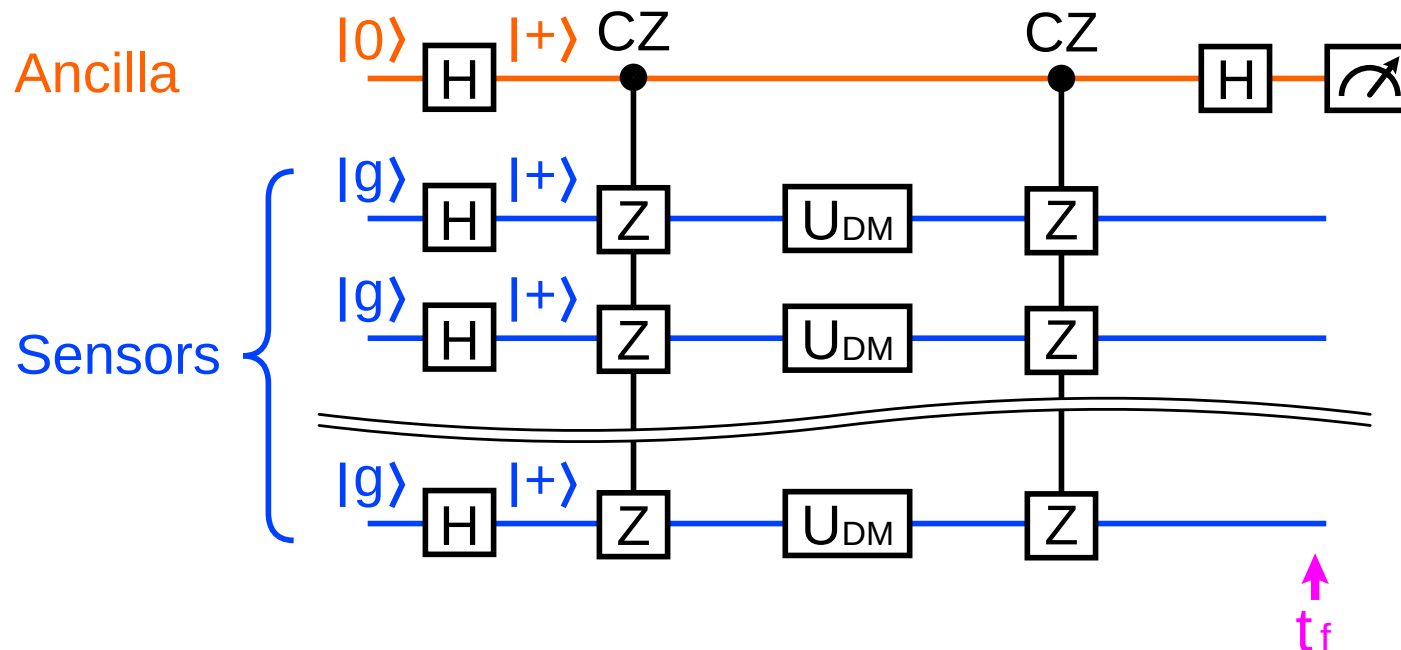
$$p \sim N^2\epsilon^2\tau^2 \quad ?$$

Quantum

- Highly correlated configurations are measured collectively with a single measurement that encompasses all the systems.

Quantum Enhancement of the Signal Rate

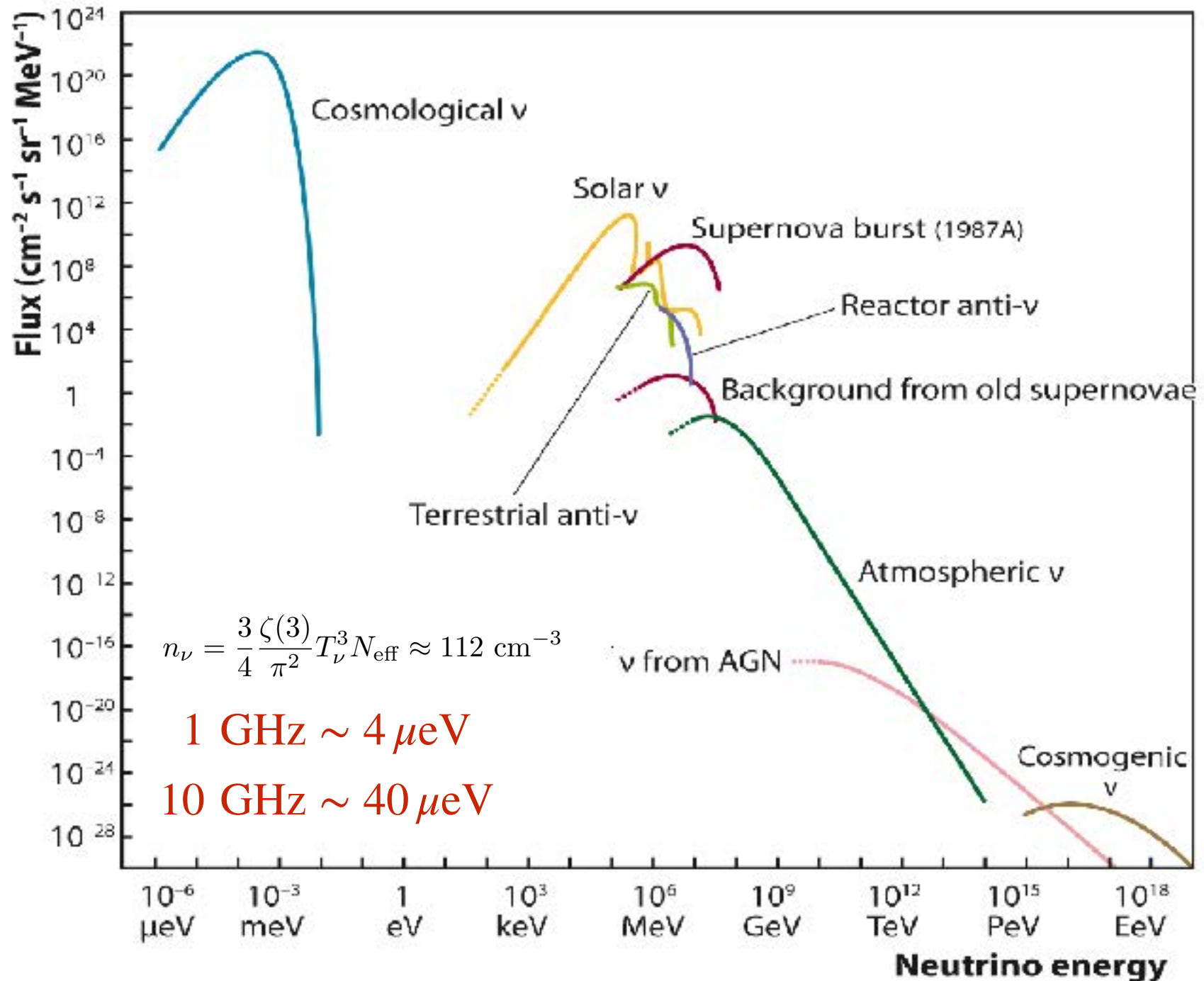
One measurement cycle for the signal enhancement



$$|\Psi(t_f)\rangle = (\cos N_q \delta |0\rangle + i \sin N_q \delta |1\rangle) \otimes |+\rangle^{\otimes N_q}$$

$\Rightarrow$ Ancilla qubit can be excited: $P_{0 \rightarrow 1} \simeq \sin^2 N_q \delta \simeq N_q^2 \delta^2$

Neutrino fluxes from different sources as a function of the neutrino energy



Neutrino transition magnetic moment

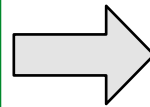
$$E_{C\nu B} \sim 10^{-6} - 10^{-4} \text{ eV}$$

$$n_\nu = \frac{3}{4} \frac{\zeta(3)}{\pi^2} T_\nu^3 N_{\text{eff}} \approx 112 \text{ cm}^{-3}$$

$$\frac{\mu_\nu^{ij}}{2} \bar{\nu}_i \sigma^{\mu\nu} \nu_j F_{\mu\nu} + h.c.$$

$$\mu_\nu = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} \approx 3 \times 10^{-19} \mu_B \left(\frac{m_\nu}{1 \text{ eV}} \right)$$

$$\Gamma_{ij} = \Gamma(\nu_i \rightarrow \nu_j + \gamma) = \frac{\mu_{ij}}{8\pi m_i} \frac{\Delta m_{ij}^2}{2m_i}$$



$$\Delta m_{21}^2 = 7.59 \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{32}^2 = 2.32 \times 10^{-3} \text{ eV}^2$$

$$m_2 \simeq \sqrt{\Delta m_{21}^2} \sim 8.6 \times 10^{-3} \text{ eV}$$

$$m_3 \simeq \sqrt{\Delta m_{32}^2 + \Delta m_{21}^2} \sim 0.05 \text{ eV}$$

NH

$$m_1 \simeq \sqrt{|\Delta m_{32}^2 + \Delta m_{21}^2|} \sim 0.0492 \text{ eV}$$

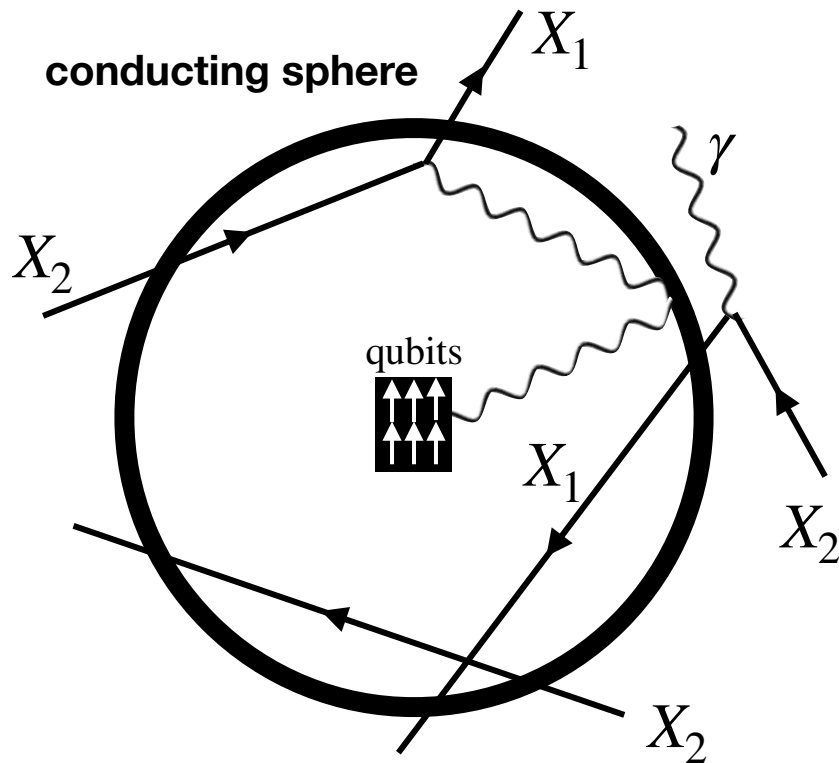
$$m_2 \simeq \sqrt{|\Delta m_{32}^2|} \sim 0.05 \text{ eV},$$

IH

$$\bar{E}_{(\text{eff})} = \sqrt{\frac{\Gamma_{ij} \mathcal{T} V_{\text{eff}} n_\nu \Delta m_{ij}^2}{2m_i \epsilon_0}}$$

$$p_* \simeq p_*^0 \times \left(\frac{d}{100 \text{ } \mu\text{m}} \right)^2 \left(\frac{C}{0.1 \text{ pF}} \right) \left(\frac{f}{1 \text{ GHz}} \right) \left(\frac{\tau}{100 \text{ } \mu\text{s}} \right)^3 \left(\frac{\mu}{10^{-11} \mu_B} \right)^2 \left(\frac{\Delta m_{ij}^2}{10^{-5} \text{ eV}^2} \right)^4 \left(\frac{V_{\text{eff}} n_\nu}{112 \text{ cm}^{-3}} \right) \left(\frac{0.05 \text{ eV}}{m_i} \right)^4$$

$$p_*^0 = 2.1 \times 10^{-27} \quad f \simeq 24 \text{ GHz} \times \left(\frac{\Delta m_{ij}^2}{10^{-5} \text{ eV}^2} \right) \left(\frac{0.05 \text{ eV}}{m_i} \right) \quad \mu_B = \frac{e\hbar}{2m_e}$$



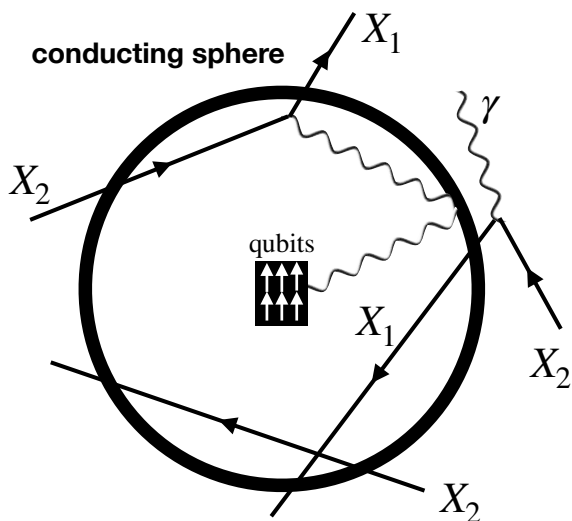
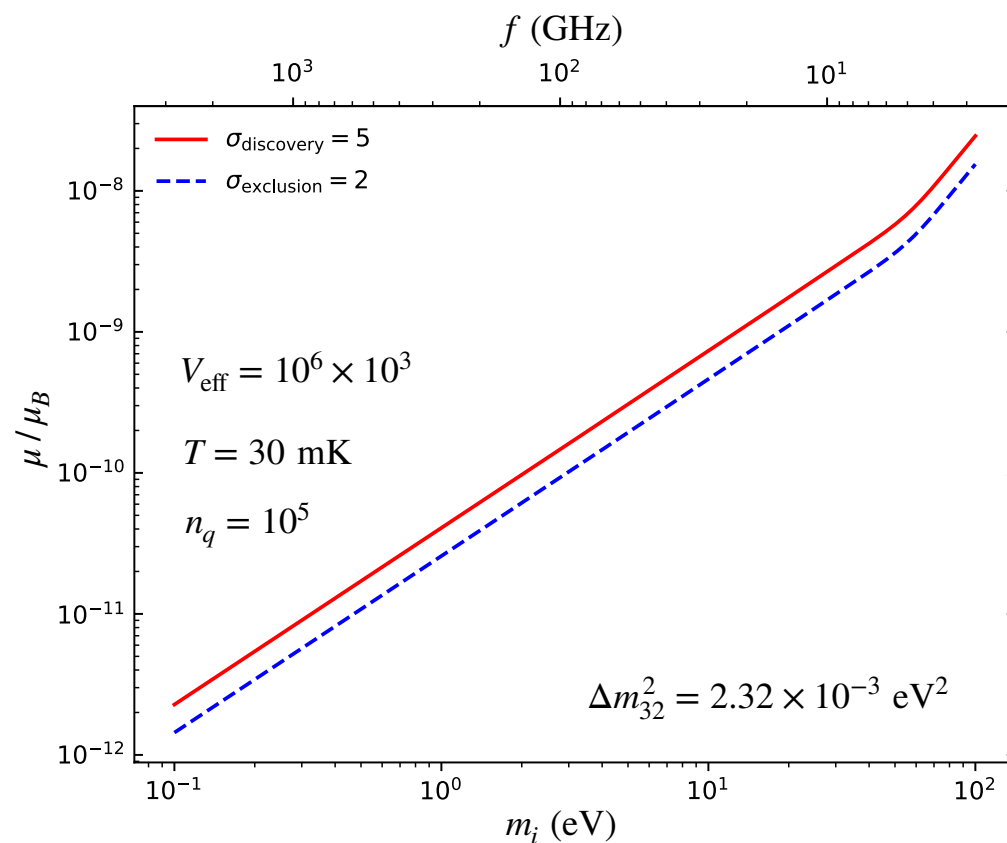
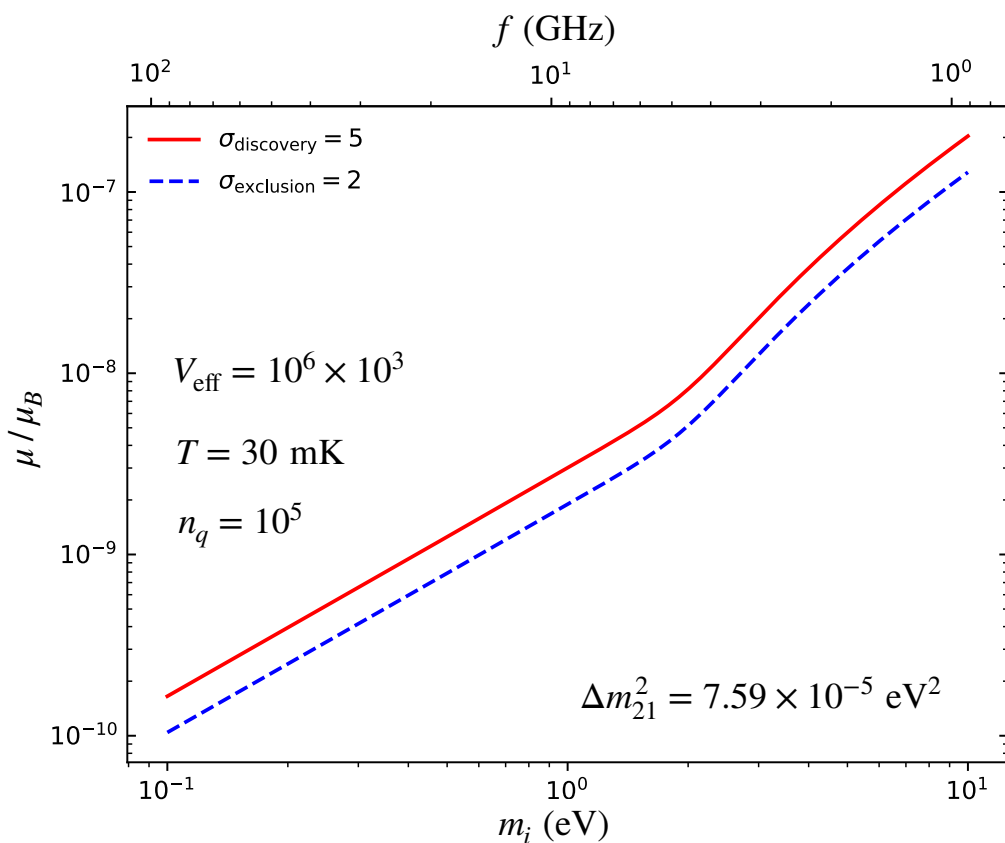
Polished aluminum and copper typically reflect 98–99% of incident radiation

In this configuration, DM or neutrinos can freely enter the cavity, but once they decay into photons, the photons are confined. Eventually, these trapped photons will interact with the quantum device.

- This setup significantly enhances the chance of detecting a signal by effectively increasing the detection volume and thereby the expected as

$$n_t \rightarrow n_e \left(\frac{V_{\text{eff}}}{\text{cm}^3} \right) \rightarrow n_2 \times 10^6 \text{ for } 1 \text{ m}^3 \text{ cavity.}$$

Bounds on neutrino transition magnetic moment



Decay volume

$$n_2 \rightarrow n_2 \left(\frac{V_{\text{eff}}}{\text{cm}^3} \right)$$

$$V_{\text{eff}} = 1 \text{ m}^3 = 10^6 \text{ cm}^3$$

Entanglement

$$P_* \simeq n_q^2 (\eta \tau)^2$$

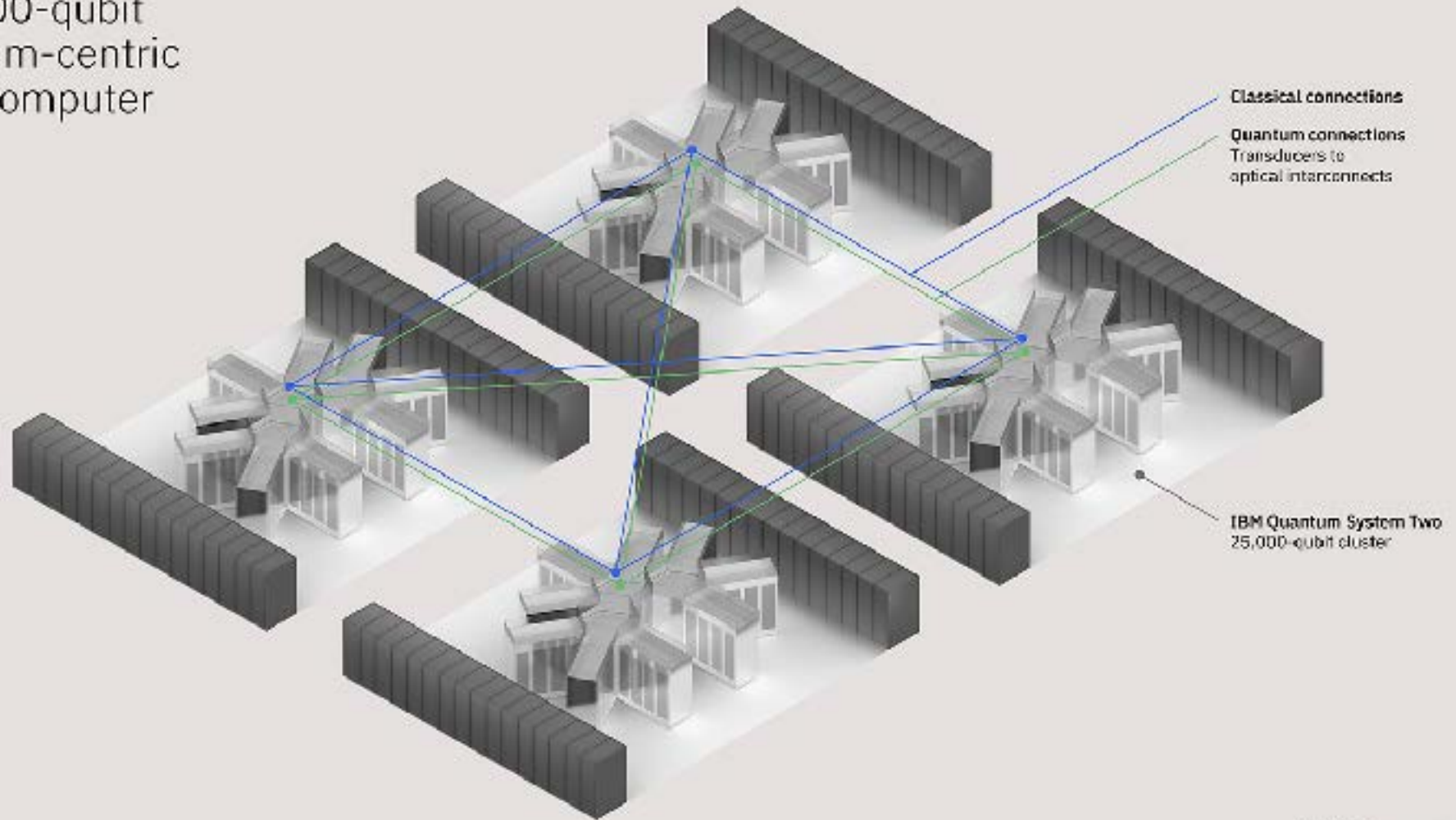
$$n_q = 10^5$$

Current reactor bounds are $\mu < 10^{-11} \mu_B$ 2508.09139

Charting the course to 100,000 qubits

100,000-qubit
quantum-centric
supercomputer

—
2033

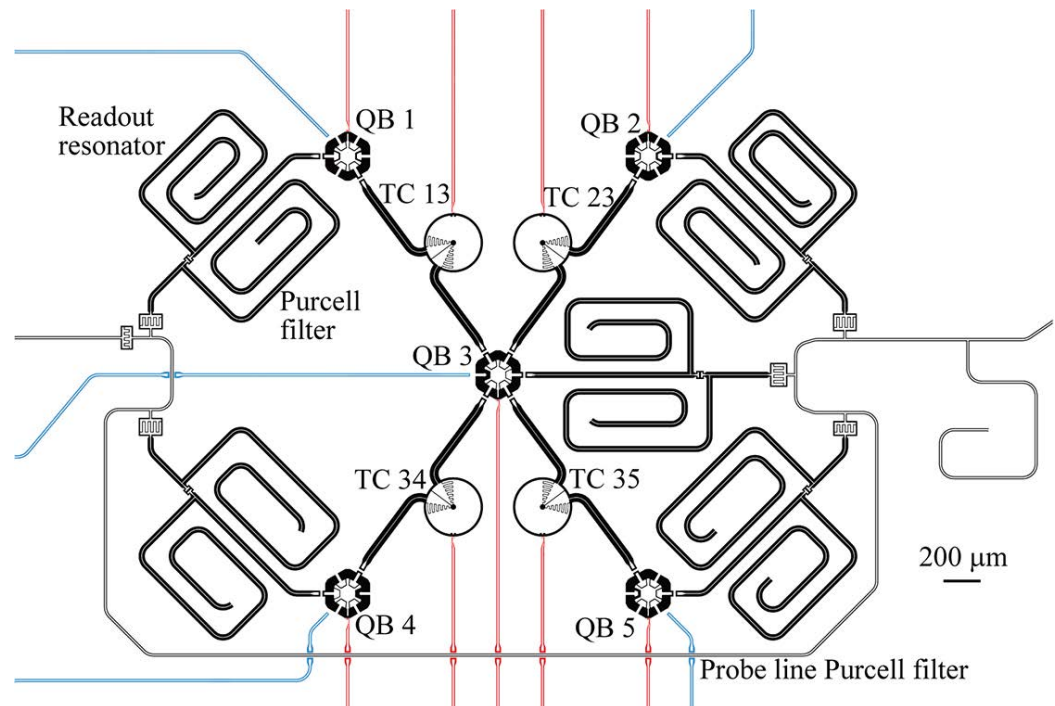


IBM Quantum

- All other quantum computing companies (D-wave, IonQ, Google etc) are also targeting Large-Scale, Fault-Tolerant Quantum Computers for next 10-20 years.
- The future may arrive sooner than we expect.

Discussion

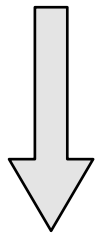
- High frequency gravitational wave detection
- What about other quantum devices
 - Transmon, trapped ion, Rydberg, what else?
 - Ge-based spin qubits
- Quantum algorithms that could enhance beyond n_q^2
- IQM Spark 5 qubits



Hamiltonian Simulation: Gross-Neveu

$$\mathcal{L} = \sum_{b=0}^{N-1} i\bar{\psi}_b \not{\partial} \psi_b - m \sum_{b=0}^{N-1} \bar{\psi}_b \psi_b + \frac{g^2}{2} \left(\sum_{b=0}^{N-1} \bar{\psi}_b \psi_b \right)^2$$

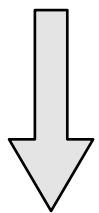
A toy model for strong force (quantum chromodynamics, QCD) with N fermions on (1+1) dimension, 1974



Kogut-Susskind staggered fermion formalism

$$\gamma^0 = Z, \gamma^1 = iY, \gamma^5 = \gamma^0 \gamma^1 = X \quad \text{in (1+1) dim}$$

$$H = \sum_{j=0}^{L-2} \sum_{b=0}^{N-1} \frac{i}{2a} (-\xi_{j,b}^\dagger \xi_{j+1,b} + \xi_{j+1,b}^\dagger \xi_{j,b}) + \sum_{j=0}^{L-1} \sum_{b=0}^{N-1} (-1)^j m n_{j,b} - \frac{g^2}{2a} \sum_{j=0}^{L-1} \sum_{b,c=0}^{N-1} n_{j,b} n_{j,c}$$



$$\xi_{j,b} \mapsto \frac{1}{2} Z_{0,b} \otimes Z_{1,b} \otimes \dots \otimes Z_{j-1,b} \otimes (X_{j,b} + iY_{j,b}) \quad \text{Jordan-Wigner transformation}$$

$$\xi_{j,b}^\dagger \mapsto \frac{1}{2} Z_{0,b} \otimes Z_{1,b} \otimes \dots \otimes Z_{j-1,b} \otimes (X_{j,b} - iY_{j,b})$$

$$H_{\text{qubit}} = \sum_{j=0}^{L-2} \sum_{b=0}^{N-1} \frac{1}{4a} (X_{j,b} Y_{j+1,b} - Y_{j,b} X_{j+1,b}) + \frac{m}{2} \sum_{j=0}^{L-1} \sum_{b=0}^{N-1} (-1)^j (I - Z)_{j,b} - \frac{g^2}{8a} \sum_{j=0}^{L-1} \sum_{b,c=0}^{N-1} (I - Z)_{j,b} (I - Z)_{j,c}$$

$$|\psi(t)\rangle = e^{-iHt/\hbar} |\psi_0\rangle$$

$$O_{\text{density}}(t) = \langle \psi(t) | \bar{\psi}(x_1) \psi(x_1) \bar{\psi}(x_2) \psi(x_2) | \psi(t) \rangle$$

$$O_{\text{string}}(t) = \langle \psi(t) | \bar{\psi}(x_1) \psi(x_2) | \psi(t) \rangle$$

$$O_{\text{chiral}}(t) = \langle \psi(t) | \bar{\psi} \psi | \psi(t) \rangle_{\text{avg}}$$

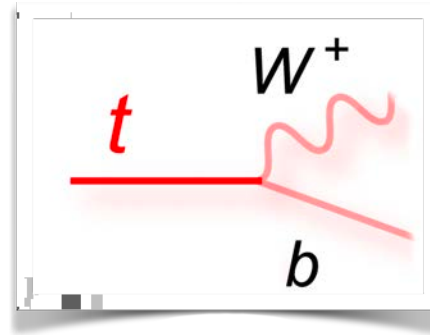
Top quark is special

- Decays before it hadronizes or its spin flips

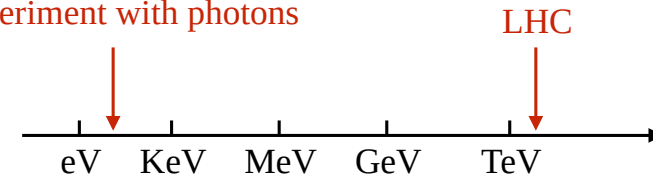
$$\tau_{top} \approx 5 \times 10^{-25} s$$

$$\tau_{had} \approx 2 \times 10^{-24} s \sim \frac{1}{\Lambda_{QCD}}$$

$$\tau_{flip} \approx 10^{-21} s \sim \frac{m_t}{\Lambda_{QCD}^2}$$



Typical entanglement experiment with photons

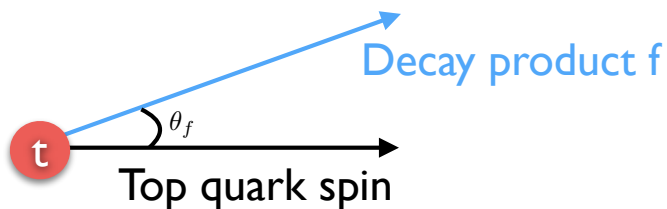


- Bottom quark is several orders of magnitude behind
- Top polarization directly observable via angular distributions of its decay products

$$\frac{1}{\Gamma_f} \frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2} (1 + \omega_f \cos\theta_f)$$

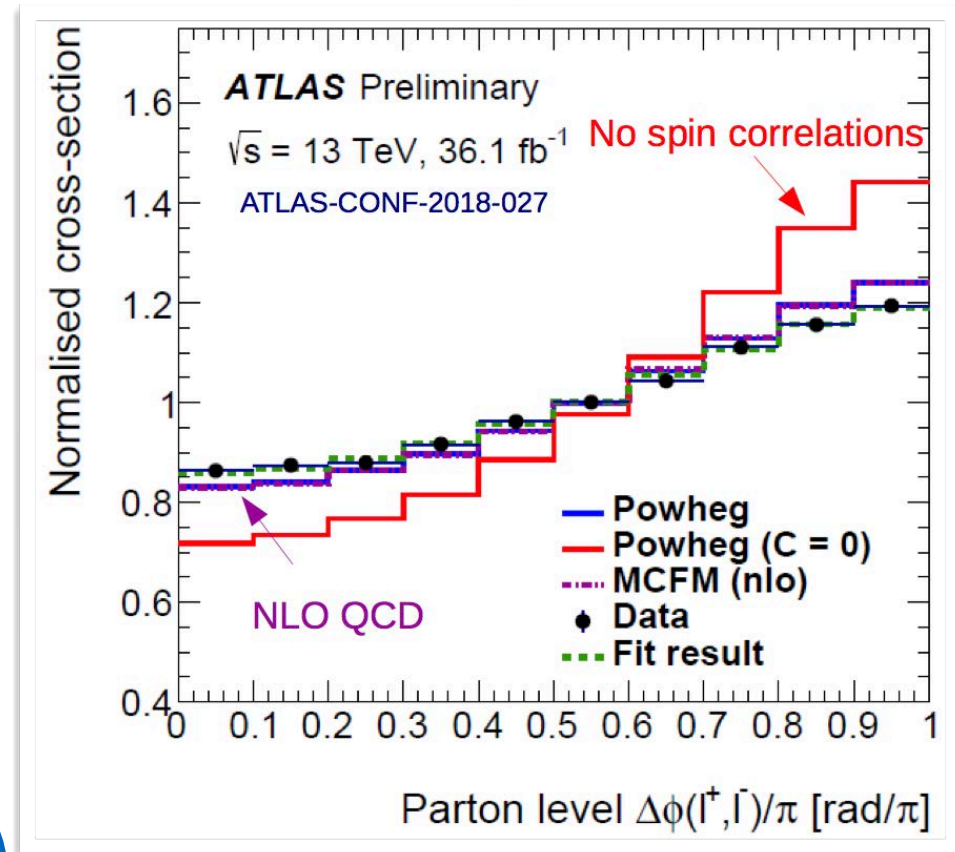
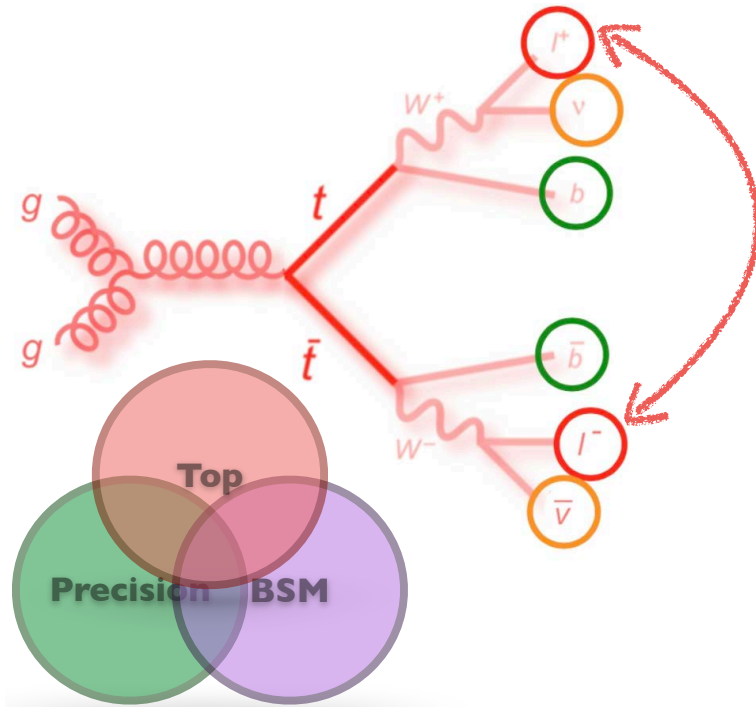
	$l^+, \bar{d}$	b	$\bar{\nu}, u$
ω_f	1	-0.4	-0.3

Spin analyzing power: maximum for charged leptons and down-type quarks

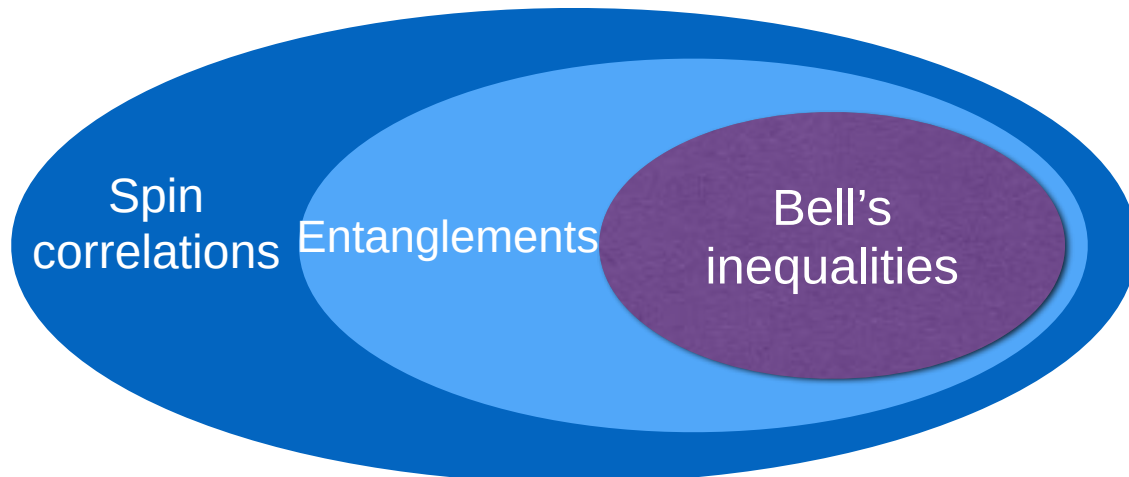


Probing entanglement and violation of Bell's inequality

- We are probing the spin correlations of top and anti-top since the Tevatron era and continue to study them at the LHC in different forms: relevant precision observable



Parke, Mahlon '95



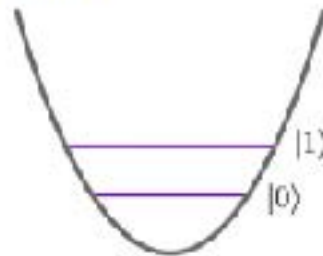
Summary

- Quantum Information Science (QIS) and Quantum Computing (QC) open new opportunities by connecting diverse fields: physics, mathematics, computer science, and engineering.
- Although it may take time to mature, QIS/QC will eventually become an integral part of our daily lives, much like AI/ML have become today.
- Challenges remain: scalability, noise mitigation, algorithmic development, and workforce training
 - But progress is accelerating rapidly.
- Quantum computing today is where classical computing was in the 1950s: slow, expensive, and limited, yet full of promise to redefine science and technology.
- It is crucial to explore where quantum computers can make a real impact from fundamental science to applications in our society.
- We need young, creative, and imaginative minds to shape this emerging quantum future.

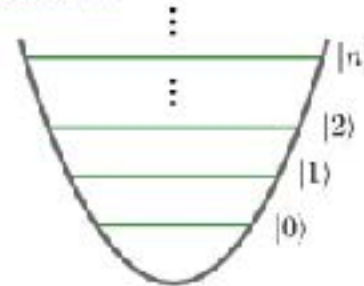
Extra slides

(a)

Qubit



Qumode



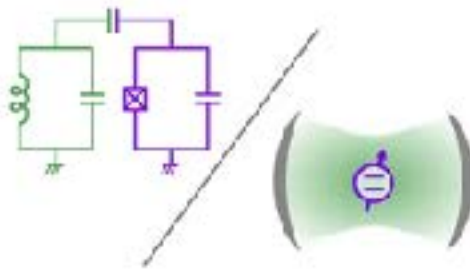
Different quantum systems
have different gates.
(ex) conditional displacement

$$U_{CD}(\alpha) = e^{\alpha \sigma_z (a^\dagger - a)}$$

(b)

Experimental platforms with qumodes

Circuit/Cavity QED



Both circuit and cavity QED platforms involve an electromagnetic mode of a circuit/cavity interacting with a natural or "artificial" atom, such as a transmon or quantum dot.

Qumodes: electromagnetic circuit/cavity modes

Qubits: transmon, fluxonium, etc. (circuit QED)
quantum dot, color center, cold atoms, etc. (cavity QED)

Trapped Ion



Trapped ion architectures consist of qubits encoded in the electronic states of individual ions that couple through the collective motional modes of the ion chain/lattice.

Qumodes: collective vibrational modes of the ions

Qubits: (a pair of) electronic levels of an ion

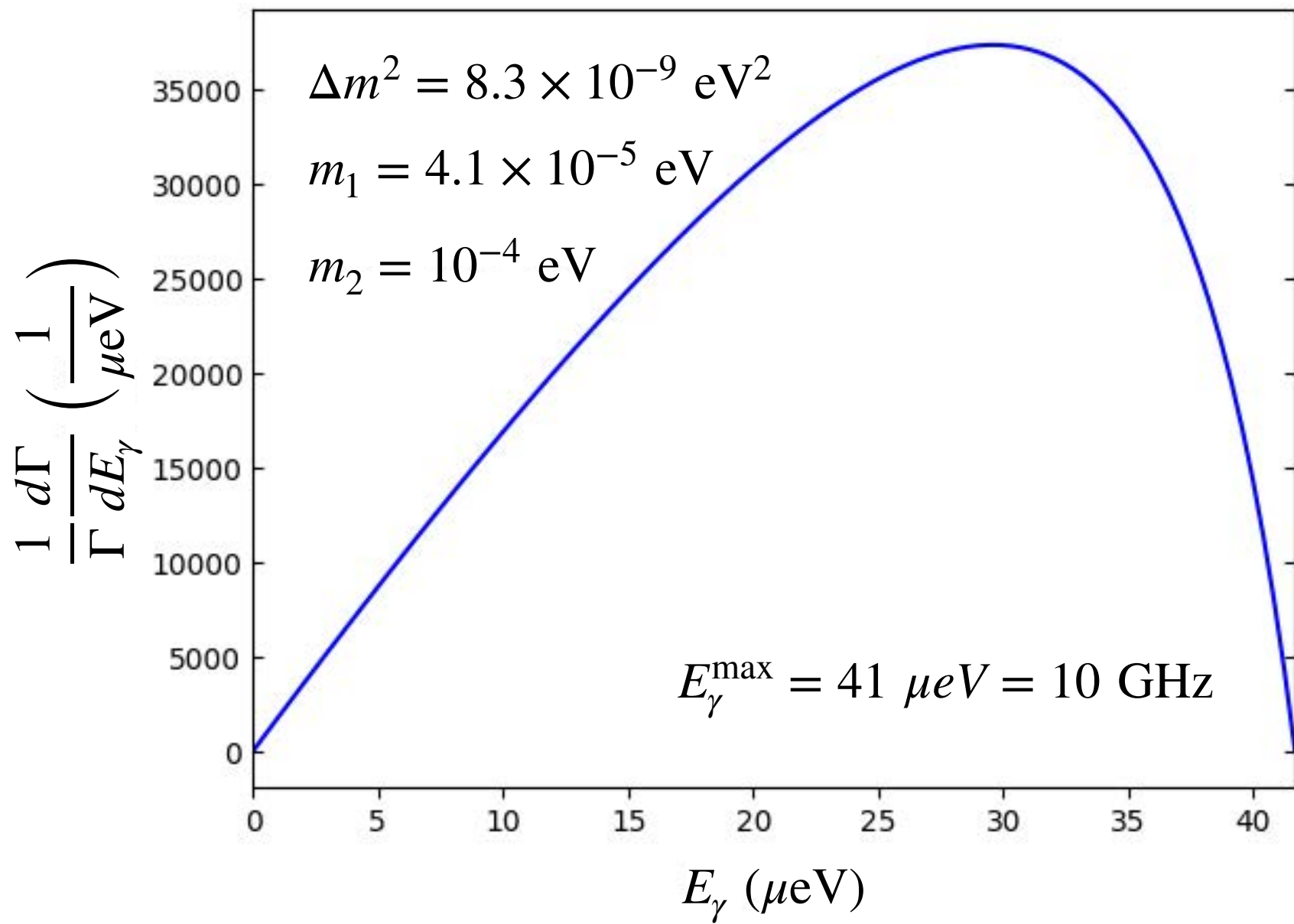
Photonic

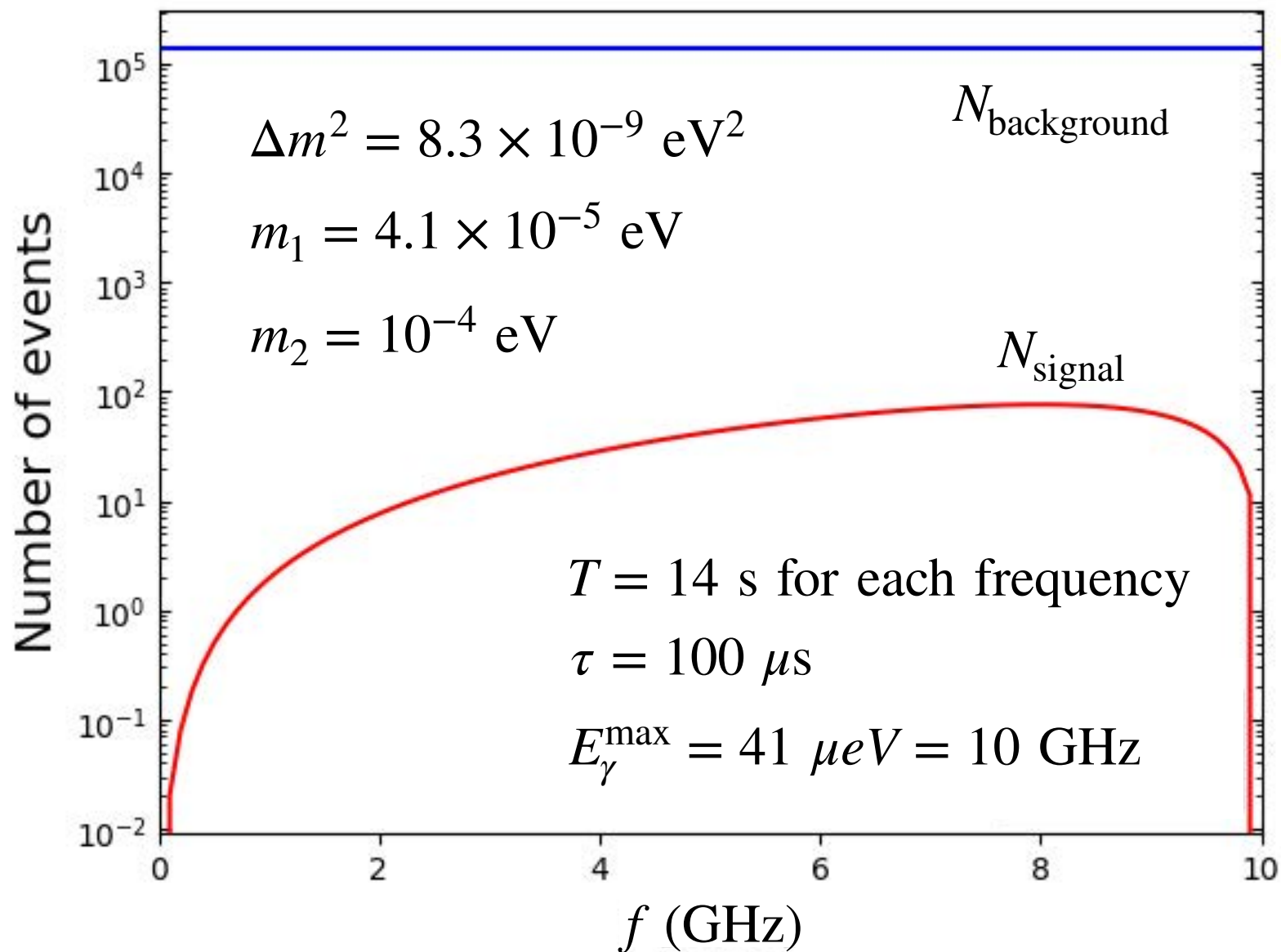


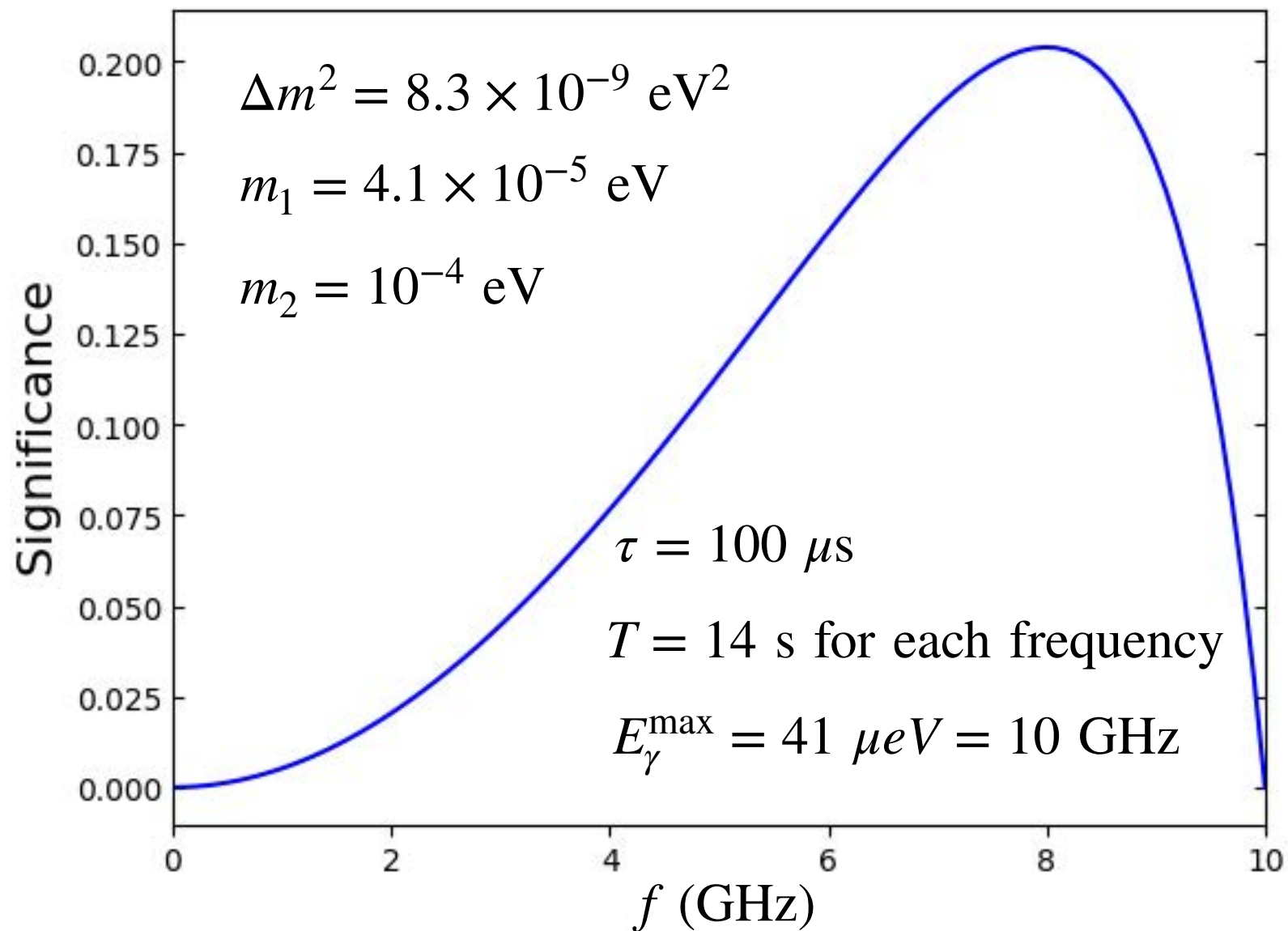
Photonic quantum processors encode information using a network of guided optical modes, which can be leveraged either as a qumode or used to encode a qubit. Entangling gates are achieved through measurement.

Qumodes: photonic modes

Qubits: photon-encoded qubit (polarization, dual-rail, etc.)







P vs NP

- In Theoretical Computer Science, the two most basic classes of problems are P and NP.
- **P** includes all problems that **can be solved “efficiently”**.
 - For example: add two numbers. The formal definition of "efficiently" is in **time that's polynomial in the input's size**.
- **NP** (nondeterministic polynomial (time)) includes all problems that given a solution, one can efficiently verify that the solution is correct.
 - An example is the following problem: given a bunch of numbers, can they be split into two groups such that the sum of one group is the same as the other. Clearly, if one is given a solution (two groups of numbers), it's simple to verify that the sum is the same. (This is a partitioning problem).
- **What's unknown is whether problems such as the one above have an efficient algorithm for finding the solution.** This is the (in)famous (unsolved) **P = NP problem**, and the common conjecture is that no such algorithm exists.
- Now, **NP hard problems** are such problems that were shown that if they can be efficiently solved (which, as mentioned, is believed to not be the case), then each and every problem in NP (each and every problem whose results can be efficiently verified) can be efficiently solved. **In other words, if you're up to showing that P=NP, you might want to take a stand at any of those NP-hard problems since they are "equivalent" in some way to all others.**

Quantum Annealing

(Gradient-Free quantum optimization)

- H_p is the problem Hamiltonian whose ground state encodes the solution to the optimization problem
- H_0 is the initial Hamiltonian whose ground state is easy to prepare.
- Prepare a quantum system to be in the ground state of H_0 and evolve the system using the following time-dependent Hamiltonian,

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- The system will remain to its ground state at all times for a large T , which means for $t=T$, the system will be in the ground state of H_p , our problem Hamiltonian.
- D-wave has built Quantum Annealing that solves optimization problem by transferring the original optimization to a hardware, that allows nearest neighbor interaction of qubits.
- Compared to the processing time of $O(2^n)$ with the simplest but a robust brute-force scanning algorithm with a classical computer, a quantum annealer can have an enormous advantage in the computation complexity as $T_{\text{QUBO}}(n) \sim \mathcal{O}(n^2) \ll \mathcal{O}(2^n)$

Adiabatic Theorem

- **Adiabatic theorem:** A physical system remains in its instantaneous eigenstate, if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum. (Max Born and Vladimir Folk 1928)
- Under a slowly changing Hamiltonian $H(t)$ with instantaneous eigenstate $|n(t)\rangle$ and the corresponding energy $E(t)$, a quantum system evolves from initial state

$$|\psi(0)\rangle = \sum_n c_n(0) |n(0)\rangle \text{ to final state } |\psi(t)\rangle = \sum_n c_n(t) |n(t)\rangle \text{ where}$$

$$c_n(t) = c_n(0) e^{i\theta_n(t)} e^{i\gamma_n(t)} \text{ with the dynamical phase } \theta_n(t) = -\frac{1}{\hbar} \int_0^t E_n(t') dt' \text{ and}$$

$$\text{geometrical phase } \gamma(t) = i \int_0^t \langle n(t') | \dot{n}(t') \rangle dt'$$

- **Adiabatic approximation:** the rate of change of Hamiltonian $\dot{H}(t)$ is small and there is finite gap $E_m(t) - E_n(t) \neq 0$ between energies for $m \neq n \rightarrow$

$$\langle n(t') | \dot{n}(t') \rangle = -\frac{\langle m(t) | \dot{H}(t) | n(t) \rangle}{E_m(t) - E_n(t)} \rightarrow 0$$

- $|c_n(t)|^2 = |c_n(0)|^2$ so if the system begins in an eigenstate of $H(0)$, it remains in an eigenstate of $H(t)$ during the evolution with a change of phase only.

Adiabatic Theorem

$$H(t) |n(t)\rangle = E_n(t) |n(t)\rangle$$

$|n(t)\rangle$: is eigenstates of Hamiltonian, basis

$$|\psi(t)\rangle = \sum_n c_n(t) |n(t)\rangle$$

satisfies time-dependent
Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

$\frac{d}{dt}$:

$$\dot{H}(t) |n(t)\rangle + H(t) |\dot{n}(t)\rangle = \dot{E}_n(t) |n(t)\rangle + E_n(t) |\dot{n}(t)\rangle$$

Assume $m \neq n$ and perform inner product with $|m(t)\rangle$: $H(t) |m(t)\rangle = E_m(t) |m(t)\rangle$

$$\langle m(t) | n(t) \rangle = \delta_{mn},$$

$$\langle m(t) | \dot{H}(t) | n(t) \rangle + \langle m(t) | H(t) | \dot{n}(t) \rangle = \dot{E}_n(t) \langle m(t) | n(t) \rangle + E_n(t) \langle m(t) | \dot{n}(t) \rangle$$

$$\langle m(t) | \dot{H}(t) | n(t) \rangle + E_m(t) \langle m(t) | \dot{n}(t) \rangle = E_n(t) \langle m(t) | \dot{n}(t) \rangle \rightarrow \langle m(t) | \dot{n}(t) \rangle = - \frac{\langle m(t) | \dot{H}(t) | n(t) \rangle}{E_m(t) - E_n(t)}$$

Adiabatic approximation: the rate of change in Hamiltonian $\dot{H}(t)$ is small and there is finite gap $E_m(t) - E_n(t) \neq 0$ between energies $\rightarrow \langle m(t) | \dot{n}(t) \rangle \approx 0$.

Adiabatic Theorem

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(t) |\psi(t)\rangle \quad \rightarrow \quad i\hbar \sum_n \dot{c}_n(t) |n(t)\rangle + c_n(t) |\dot{n}(t)\rangle = \sum_n E_n(t) c_n(t) |n(t)\rangle$$

$$|\psi(t)\rangle = \sum_n c_n(t) |n(t)\rangle$$

Inner product with $|m(t)\rangle$: $\langle m(t) | \left[i\hbar \sum_n \dot{c}_n(t) |n(t)\rangle + c_n(t) |\dot{n}(t)\rangle = \sum_n E_n(t) c_n(t) |n(t)\rangle \right]$

Using $\langle m(t) | n(t) \rangle = \delta_{mn}$, we obtain $i\hbar \dot{c}_m(t) + i\hbar \sum_n c_n(t) \langle m(t) | \dot{n}(t) \rangle = c_m(t) E_m(t)$

In the adiabatic limit,
 $\langle m(t) | \dot{n}(t) \rangle \approx 0$ for $m \neq n$

$$i\hbar \dot{c}_m(t) + i\hbar c_m(t) \langle m(t) | \dot{n}(t) \rangle = c_m(t) E_m(t)$$

$$i \dot{c}_m(t) = \left(\frac{E_m(t)}{\hbar} - i \langle m(t) | \dot{n}(t) \rangle \right) c_m(t) \quad \rightarrow \quad \dot{c}_m(t) = i \left(-\frac{E_m(t)}{\hbar} + i \langle m(t) | \dot{n}(t) \rangle \right) c_m(t)$$

$$\frac{d}{dt} \ln c_m(t) = \frac{1}{c_m(t)} \frac{dc_m(t)}{dt} = \frac{\dot{c}_m(t)}{c_m(t)} = -\frac{i}{\hbar} E_m(t) + i \langle m(t) | \dot{n}(t) \rangle$$

$$c_m(t) = c_m(0) e^{i\theta_m(t)} e^{i\gamma_m(t)} \quad \theta_m(t) = -\frac{1}{\hbar} \int_0^t E_m(t') dt' \quad \gamma(t) = i \int_0^t \langle m(t') | \dot{n}(t') \rangle dt'$$

dynamical phase,
real, function of E

geometrical phase,
pure real

Adiabatic Theorem

$$c_m(t) = c_m(0)e^{i\theta_m(t)}e^{i\gamma_m(t)} \quad \theta_m(t) = -\frac{1}{\hbar} \int_0^t E_m(t') dt' \quad \gamma(t) = i \int_0^t \langle m(t') | \dot{m}(t') \rangle dt'$$

dynamical phase,
real, function of E

geometrical phase,
pure real,
Has something to do with
direction in the Hilbert space

$$\begin{aligned} 0 &= \frac{d}{dt} \langle m(t) | m(t) \rangle = \langle \dot{m}(t) | m(t) \rangle + \langle m(t) | \dot{m}(t) \rangle \\ &= \langle m(t) | \dot{m}(t) \rangle^* + \langle m(t) | \dot{m}(t) \rangle \\ &= 2 \operatorname{Re} \langle m(t) | \dot{m}(t) \rangle \quad \rightarrow \quad \gamma_m(t) : \text{ pure real} \end{aligned}$$

$$\langle \phi | \psi \rangle^* = \langle \psi | \phi \rangle$$

Adiabatic Theorem

- Schrodinger equation: $i\hbar \frac{d\psi(t)}{dt} = H(t) \psi(t)$
- Instantaneous eigenstate: $H(t) \psi_n(t) = E_n(t) \psi_n(t)$
- Initial condition: $\psi(t = 0) = \psi_0$
- If evolution is slow enough, $\psi(t) \approx e^{i\theta(t)} \psi_0$

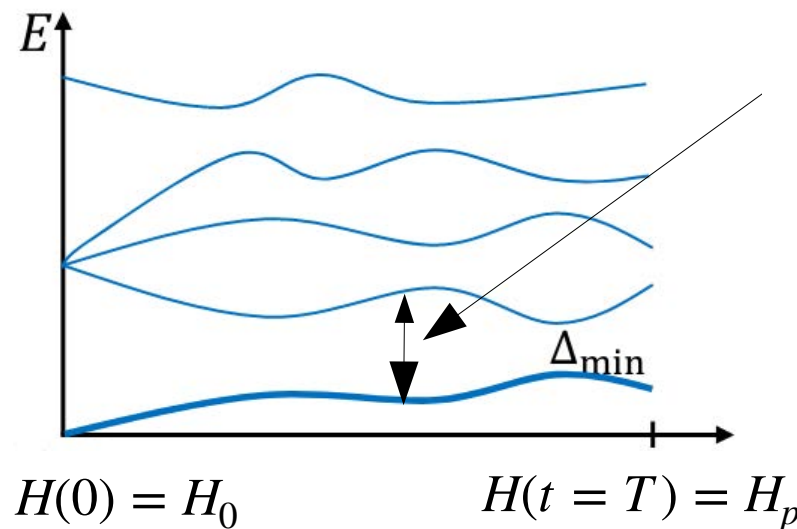
Born and Folk 1928

$$\psi(t) = U(t) \psi(0)$$

$$U_I(t) = \sum_{q=0}^{\infty} (-i)^q \int_0^t dt_q \cdots \int_0^{t_2} dt_1 H_I(t_q) \cdots H_I(t_1)$$

Limitation of Quantum Annealing

- Performance of quantum annealing are governed by the size of the gap.



$$\Delta t \gg \max_{0 \leq s \leq 1} \left| \frac{\langle 1(s) | \frac{dH(s)}{ds} | 0(s) \rangle}{\Delta(s)^2} \right|$$

$$T = \Delta s, \quad 0 \leq s \leq 1$$

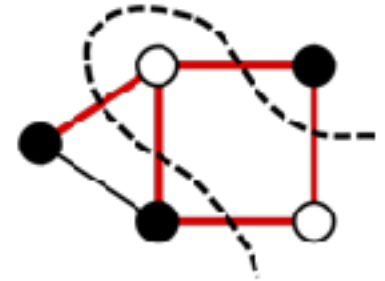
$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_p$$

- If the energy gap b/w the ground state and 1st excited state is small, T must be very large $\rightarrow$ computationally difficult.
- Performance is poor, when eigenvalues are degenerate.

Why should we care about QAOA?

- Solve optimization problems
 - Solves quadratic unconstrained problems with binary variables
- Near-term algorithm
 - Algorithm runs on small quantum computers
 - Low depth, robust to errors
 - Requires relatively few physical qubits to get to interesting practical problem sizes
- Adaptable algorithm
 - In principle, we can easily model the objective function that we are trying to solve
- **Expected** to be faster than classical
 - Classical approaches move through the search space one solution at a time
 - In quantum, we can create a superposition of states and operate in all states in parallel.

QUBO example: Max-cut Problem



- The cost function to be maximized:

$$C(x) = \sum_{(i,j) \in E} (x_i + x_j - 2x_i x_j) \quad \text{where } x_i \in \{0,1\}$$

$x_i + x_j - 2x_i x_j = 1$, if x_i and x_j belong in different sets .

$x_i + x_j - 2x_i x_j = 0$, if x_i and x_j belong in the same set .

$$s_i \in Z_2 = \{-1,1\}$$

- Introducing $x_i = \frac{s_i + 1}{2}$, the cost function can be rewritten

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} (1 - s_i s_j) \longrightarrow C(s) = \frac{1}{2} \sum_{(i,j) \in E} (1 - \sigma_i^z \sigma_j^z) \quad \begin{array}{l} (i,j) : \text{ the edge index} \\ i : \text{ vertex index} \end{array}$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{array}{l} \sigma^z |0\rangle = +1 |0\rangle \\ \sigma^z |1\rangle = -1 |1\rangle \end{array}$$

σ_i^z : Pauli's Z matrix acting on the i^{th} vertex

σ_j^z : Pauli's Z matrix acting on the j^{th} vertex

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrices = linear operators = observables

Eigenvalues = what are actually measured in experiments

Quantum Approximate Optimization Algorithm (QAOA)

Farhi et al 2014

$$H_P = C(s) = H_C = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right) : \text{Problem Hamiltonian}$$

(i, j) : the edge index

i : vertex index

$$H_M = B = H_B = \sum_j \sigma_j^X : \text{Mixer Hamiltonian (Initial Hamiltonian)}$$

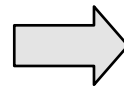
Full Hamiltonian: $H(t) = \left(1 - \frac{t}{T} \right) H_M + \frac{t}{T} H_P$

$$|\psi\rangle = \exp \left[-i \int_0^t H(t') dt' \right] |\psi_0\rangle = \exp \left[-i \sum_{j=1}^p H(j\Delta t) \Delta t \right] |\psi_0\rangle$$

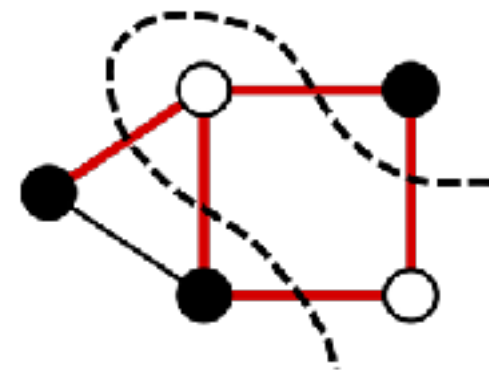
$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left[\left(1 - \frac{j\Delta t}{T} \right) H_M + \frac{j\Delta t}{T} H_P \right] \right] |\psi_0\rangle$$

$$\approx \prod_{j=1}^p \exp \left[-i\Delta t \left(1 - \frac{j\Delta t}{T} \right) H_M \right] \exp \left[-i\Delta t \frac{j\Delta t}{T} H_P \right] |\psi_0\rangle$$

$$= \prod_{j=1}^p \underbrace{\exp \left[-i\beta_j H_M \right]}_{U(H_M, \beta_j)} \underbrace{\exp \left[-i\gamma_j H_P \right]}_{U(H_P, \gamma_j)} |\psi_0\rangle$$



$$|\gamma, \beta\rangle = \prod_{j=1}^p U(H_M, \beta_j) U(H_P, \gamma_j) |+\rangle^{\otimes n}$$



Undirected Graph: $G = (V, E)$

V : set of nodes

E : set of edges

Works in the adiabatic limit or $p \rightarrow \infty$

Quantum Approximate Optimization Algorithm (QAOA)

Farhi et al 2014

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right), \quad B = \sum_j \sigma_j^X$$

(i, j) : the edge index

i : vertex index

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

σ_i^z : Pauli's Z matrix acting on the i^{th} vertex

σ_j^z : Pauli's Z matrix acting on the j^{th} vertex

$$\sigma^z |0\rangle = +1 |0\rangle \quad \sigma^z |1\rangle = -1 |1\rangle \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U(C, \gamma) = e^{-i\gamma C} = \prod_{(i,j) \in E} e^{-i\gamma C_{ij}}, \quad U(B, \beta) = e^{-i\beta B} = \prod_{j=1}^n e^{-i\beta B_j}$$

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = \left[\prod_{i=1}^p U(B, \beta_i) U(C, \gamma_i) \right] H^{\otimes n} |0\rangle$$

$$e^{-i\beta \sigma_j^X} = \cos \beta - i \sigma_j^X \sin \beta$$

$$= U(B, \beta_p) U(C, \gamma_p) \cdots U(B, \beta_1) U(C, \gamma_1) \frac{1}{\sqrt{2}^n} \sum_{i=1}^{2^n-1} |i\rangle$$

2p angles (parameters): $\vec{\gamma} = (\gamma_1, \gamma_2, \dots, \gamma_p)$, and $\vec{\beta} = (\beta_1, \beta_2, \dots, \beta_p)$

Goal is to find minimum/maximum over angles:

$$M_p = \max_{\vec{\gamma}, \vec{\beta}} \langle \psi(\vec{\gamma}, \vec{\beta}) | C | \psi(\vec{\gamma}, \vec{\beta}) \rangle$$

Quantum Approximate Optimization Algorithm (QAOA)

$$H|\psi\rangle = i\frac{\partial}{\partial t}|\psi\rangle \quad \longrightarrow \quad |\psi\rangle = e^{-iHt}|\psi_0\rangle \quad \text{Farhi et al 2014}$$

$$H = C(s) + B$$

Problem Hamiltonian

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} \left(1 - \sigma_i^z \sigma_j^z \right), \quad B = \sum_j \sigma_j^x$$

Initial Hamiltonian

$$U(C, \gamma) = e^{-i\gamma C} = \prod_{(i,j) \in E} e^{-i\gamma C_{ij}}, \quad U(B, \beta) = e^{-i\beta B} = \prod_{j=1}^n e^{-i\beta B_j}$$

(i, j) : the edge index
 i : vertex index

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = \left[\prod_{i=1}^p U(B, \beta_i) U(C, \gamma_i) \right] H^{\otimes n} |0\rangle \quad \text{For a large } p$$

minimum/maximum over angles:

$$M_p = \max_{\vec{\gamma}, \vec{\beta}} \langle \psi(\vec{\gamma}, \vec{\beta}) | C | \psi(\vec{\gamma}, \vec{\beta}) \rangle$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

σ_i^z : Pauli's Z matrix acting on the i^{th} vertex

σ_j^z : Pauli's Z matrix acting on the j^{th} vertex

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

$$\sigma^z |0\rangle = +1 |0\rangle$$

$$\sigma^z |1\rangle = -1 |1\rangle$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Quantum Approximate Optimization Algorithm (QAOA)

- How do $U(C, \gamma)$ and $U(B, \beta)$ operate on $|\psi\rangle$?

$$\begin{aligned}
 U(C, \gamma) H^{\otimes n} |0 \dots 0\rangle &= e^{-i\gamma C} H^{\otimes n} |0 \dots 0\rangle = \exp\left[-i\gamma \frac{1}{2} \sum_{(i,j) \in E} (1 - \sigma_i^Z \sigma_j^Z)\right] H^{\otimes n} |0 \dots 0\rangle \\
 &= \prod_{(i,j) \in E} \exp\left[-i\gamma \frac{1}{2} (1 - \sigma_i^Z \sigma_j^Z)\right] H^{\otimes n} |0 \dots 0\rangle
 \end{aligned}$$

$$\exp\left[-i\gamma \frac{1}{2} (1 - \sigma_i^Z \sigma_j^Z)\right] H^{\otimes n} |0 \dots 0\rangle = \exp\left(-i\frac{\gamma}{2}\right) \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) H^{\otimes n} |0 \dots 0\rangle$$

Four different possibilities:

$$\begin{aligned}
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots 0 \dots 0 \dots\rangle &= \exp\left(+i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 0 \dots 0 \dots\rangle \\
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots 0 \dots 1 \dots\rangle &= \exp\left(-i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 0 \dots 1 \dots\rangle \\
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots 1 \dots 0 \dots\rangle &= \exp\left(-i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 1 \dots 0 \dots\rangle \\
 \exp\left(+i\frac{\gamma}{2} \sigma_i^Z \sigma_j^Z\right) |\dots 1 \dots 1 \dots\rangle &= \exp\left(+i\frac{\gamma}{2} 1 \cdot 1\right) |\dots 1 \dots 1 \dots\rangle
 \end{aligned}$$

Quantum Approximate Optimization Algorithm (QAOA)

- If bits i and j are the same, $\exp\left[-i\gamma\frac{1}{2}\left(1 - \sigma_i^Z\sigma_j^Z\right)\right]|\dots\rangle = +1|\dots\rangle$
- If bits i and j are different, $\exp\left[-i\gamma\frac{1}{2}\left(1 - \sigma_i^Z\sigma_j^Z\right)\right]|\dots\rangle = e^{-i\gamma}|\dots\rangle$

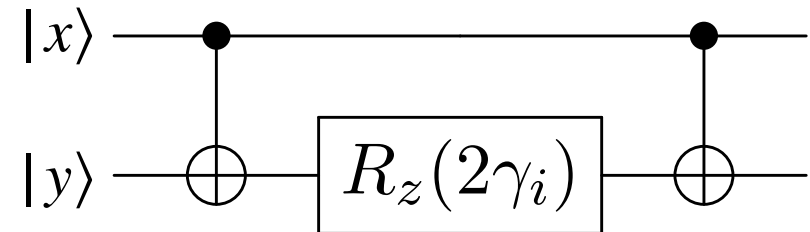
➡ • If bits i and j are different, rotate the output state around z axis by an angle γ .

$$\sigma_Z|a\rangle = (-1)^a|a\rangle \quad R_Z(\theta) = \exp\left(-i\frac{\theta}{2}\sigma_Z\right) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{+i\theta/2} \end{pmatrix}$$

In circuit: CNOT $|x y\rangle = |x x \oplus y\rangle$

$$I \otimes R_z(2\gamma) |x x \oplus y\rangle = \exp\left(-i\gamma(-1)^{x \oplus y}\right) |x x \oplus y\rangle$$

$$\text{For } U(B, \beta) = e^{-i\beta B} = \prod_{j=1}^n e^{-i\beta \sigma_j^X} = \prod_{j=1}^n R_x^j(2\beta)$$



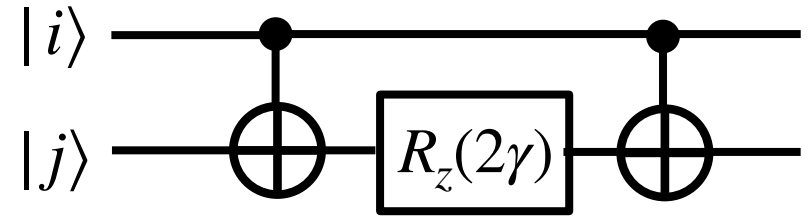
- Rotation of all n -qubits about x -axis with angle 2β

Quantum Approximate Optimization Algorithm (QAOA)

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = U(B, \beta_p) U(C, \gamma_p) \cdots U(B, \beta_1) U(C, \gamma_1) \frac{1}{\sqrt{2}^n} \sum_{i=1}^{2^n-1} |i\rangle$$

Farhi et al 2014

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle = \left[\prod_{i=1}^p U(B, \beta_i) U(C, \gamma_i) \right] H^{\otimes n} |0\rangle$$



$$e^{-i\beta\sigma_j^x} = \cos \beta - i \sin \beta \sigma_j^x$$

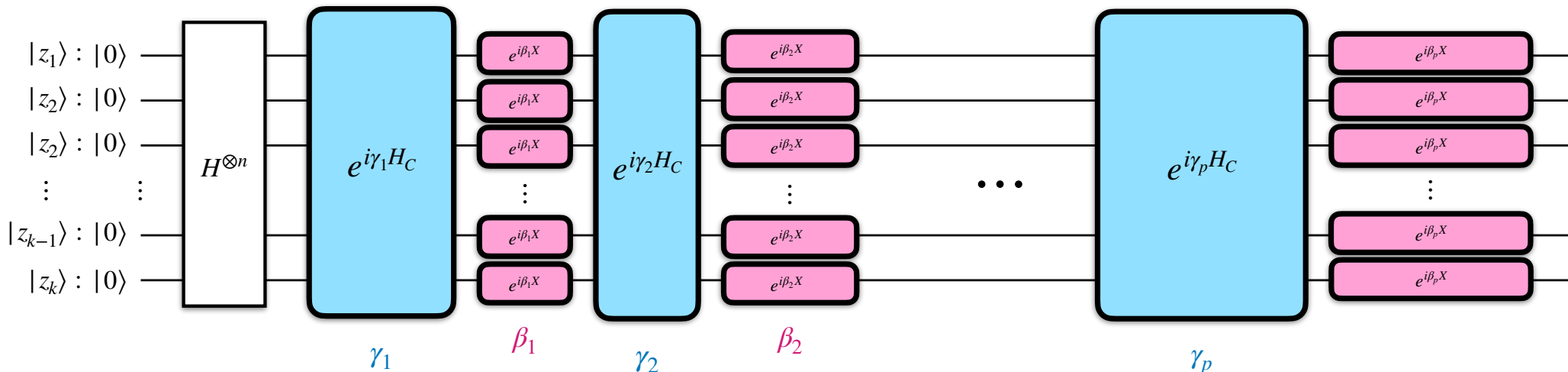
Rotate qubit j around x-axis by β

$$C(s) = \frac{1}{2} \sum_{(i,j) \in E} (1 - \sigma_i^z \sigma_j^z), \quad B = \sum_j \sigma_j^x$$

C=problem Hamiltonian

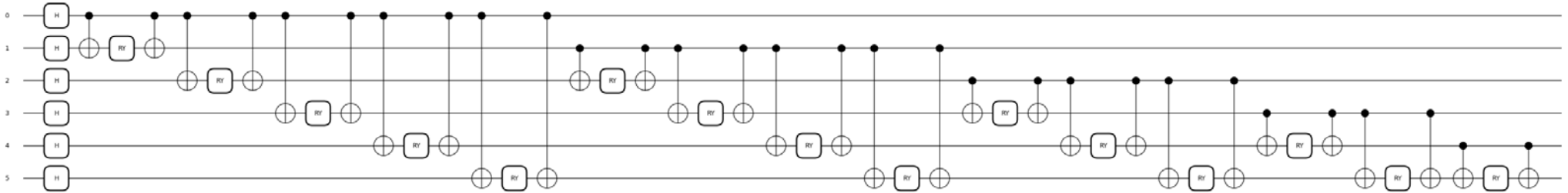
B = Initial Hamiltonian

$$H|x\rangle = \frac{1}{\sqrt{2}} (|0\rangle + (-1)^x |1\rangle)$$

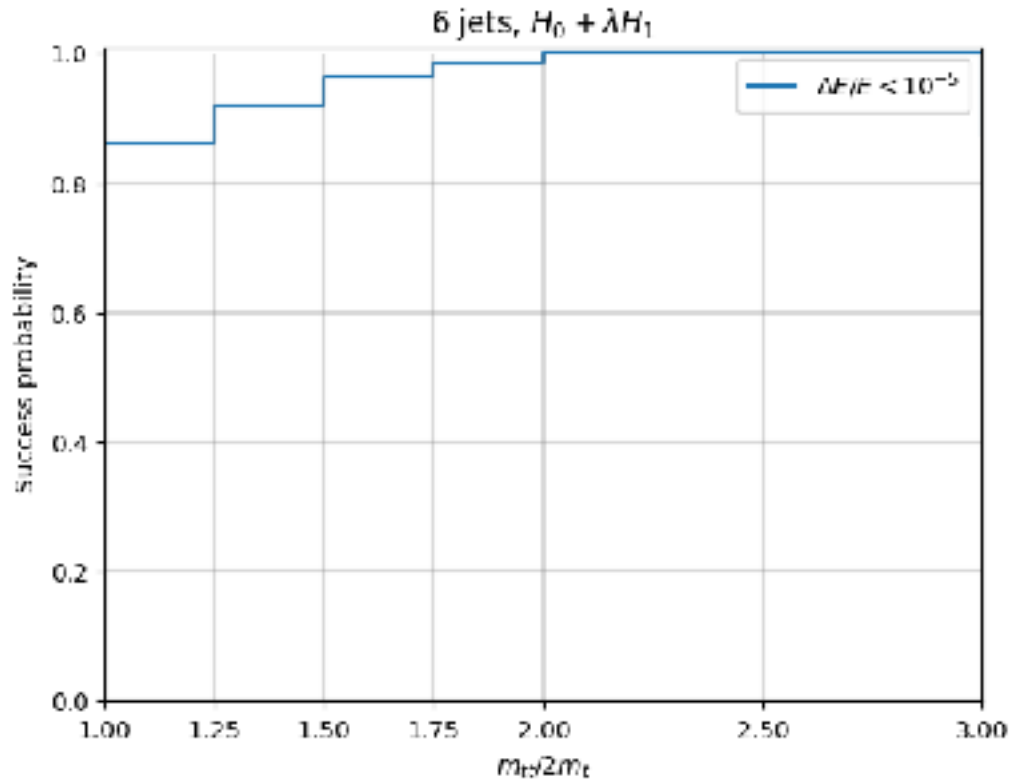


VarQITE Ansatz (with IonQ)

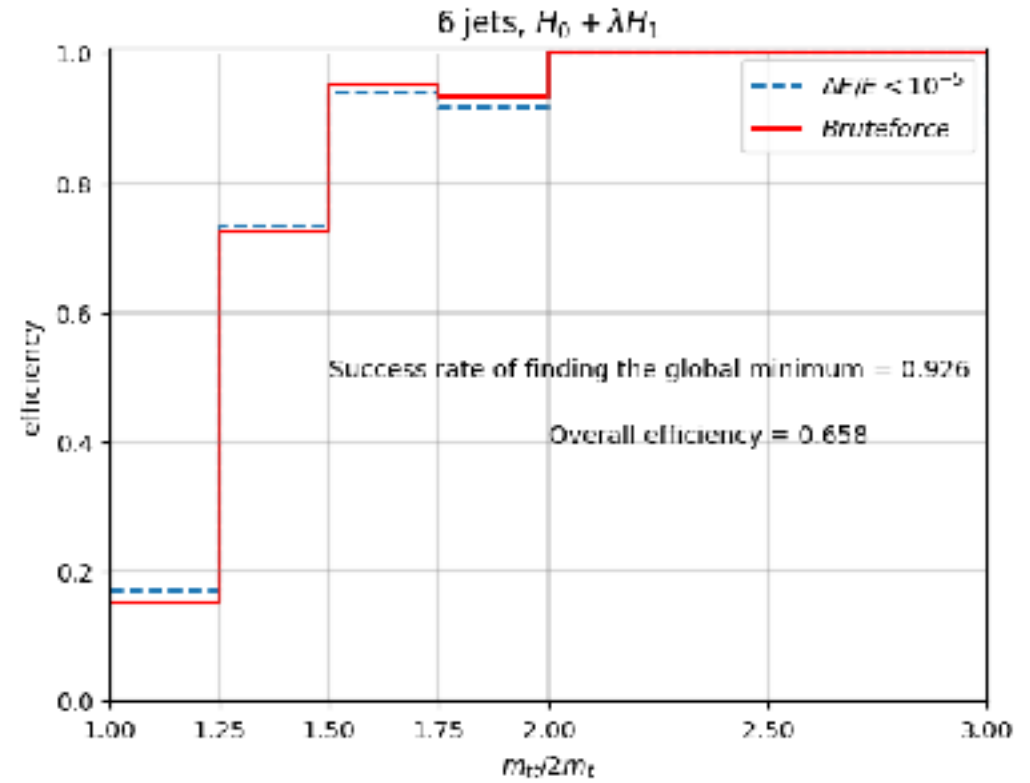
15 parameters
no classical optimizers needed



Performant near-term quantum combinatorial optimization : 2404.16135



Success probability of
finding the global minimum



Efficiency of resolving the
combinatorial problem

Feedback-based ALgorithm for Quantum Optimization (FALQON)

Feedback-based ALgorithm Quantum Optimization (FALQON)

- 2103.08619, https://pennylane.ai/qml/demos/tutorial_falqon.html
- Consider a quantum system whose dynamics is governed by

$$i \frac{d}{dt} |\psi(t)\rangle = (H_p + H_d \beta(t)) |\psi(t)\rangle$$

- Goal is to minimize: $\langle H_p \rangle = \langle \psi(t) | H_p | \psi(t) \rangle$

H_p : drift Hamiltonian (Problem H) $\beta(t)$: time – dependent control function

H_d : control Hamiltonian

- One can minimize $\langle H_p \rangle$ by designing $\beta(t)$ such that

$$\frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle(t) \leq 0, \quad \forall t \geq 0$$

$$\begin{aligned} \frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle &= i \langle \psi(t) | (H_p + H_d \beta(t)) H_p | \psi(t) \rangle - i \langle \psi(t) | H_p (H_p + H_d \beta(t)) | \psi(t) \rangle \\ &= \langle \psi(t) | i [H_d, H_p] | \psi(t) \rangle \beta(t) \equiv A(t) \beta(t) \end{aligned}$$

Feedback-based ALgorithm Quantum Optimization (FALQON)

$$\frac{d}{dt} \langle \psi(t) | H_p | \psi(t) \rangle = \langle \psi(t) | i [H_d, H_p] | \psi(t) \rangle \beta(t) \equiv A(t) \beta(t)$$

- We can choose any $\beta(t)$.
- Consider $\beta(t) = -w f(t, A(t))$ for $w > 0$, where $f(t, A(t))$ is any continuous function with $f(t, 0) = 0$ and $A(t)f(t, A(t)) > 0$ for all $A(t) \neq 0$.
- Take $w = 1$ and $f(t, A(t)) = A(t)$ such that $\beta(t) = -A(t)$ for simplicity.
- Consider alternating (rather than concurrent) applications of H_p and H_d , leading to a time evolution:

$$U = U_d(\beta_\ell) U_p \cdots U_d(\beta_1) U_p$$

$$U_p = e^{-iH_p \Delta t} \quad k = 1, 2, \dots, \ell \quad \beta_k = \beta(k\tau - \Delta t)$$

$$U_d(\beta_k) = e^{-i\beta_k H_d \Delta t} \quad \tau = 2\Delta t \quad = \beta((k-1)\Delta t)$$

- For small Δt , this unitary evolution yields Trotterized approximation to the continuous time evolution of the system.

Feedback-based ALgorithm Quantum Optimization (FALQON)

- During the time evolution when H_p is applied, $\frac{d}{dt}\langle H_p \rangle = 0$, but eigenstate of H_p accumulates phase changes. (H_p is time-independent.)
- For the time evolution when H_d is applied, we recover $\frac{d}{dt}\langle H_p \rangle = A(t)\beta(t)$
- Set $\beta_{k+1} = -A_k$, where $A_k = \langle \psi_k | i [H_d, H_p] | \psi_k \rangle$
- In this setting, it is always possible to choose Δt small enough such that $\frac{d}{dt}\langle \psi(t) | H_p | \psi(t) \rangle \leq 0$. If Δt is chosen to be too large, the inequality will be violated.
- FALQON is a constructive, optimization free procedure for assigning values to each β_k according to a feedback law.
- By design, the quality of the solution to the combinatorial optimization problem improves monotonically with respect to depth of the circuit, k .

FALQON vs QAOA

- Circuits used in QAOA has the same alternative structure as those in FALQON with additional parameters $\vec{\gamma} = (\gamma_1, \dots, \gamma_\ell)$ that enter into U_p such that $U_{QAOA} = U_d(\beta_\ell)U_p(\gamma_\ell)\cdots U_d(\beta_1)U_p(\gamma_1)$.
- Solution to the original combinatorial optimization is found by minimizing $\langle \psi(\vec{\gamma}, \vec{\beta}) | H_p | \psi(\vec{\gamma}, \vec{\beta}) \rangle$ over 2ℓ parameters, using classical optimization.
 $(|\psi(\vec{\gamma}, \vec{\beta})\rangle = U_{QAOA} |\psi_0\rangle)$
- FALQON minimizes $\langle H_p \rangle$ over a sequence of quantum circuit layers, guided by qubit measurement-based feed back without classical optimization.

FALQON for MaxCut problem

- MaxCut: $H_p = - \sum_{(i,j) \in E} \frac{1}{2} (1 - Z_i Z_j)$ and $H_d = \sum_{j=1}^n X_j$
- $i [H_d, H_p] = \sum_{(i,j) \in E} Y_i Z_j + Z_i Y_j$ where X_j, Y_j and Z_j are Pauli's matrices.

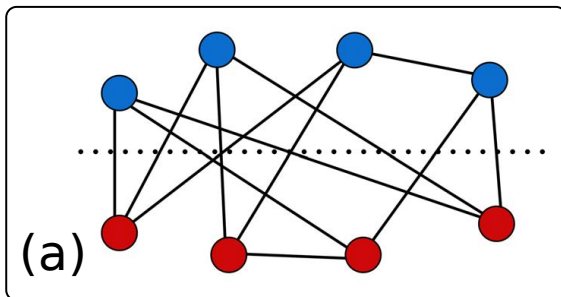
FALQON for MaxCut problem

Approximation ration:

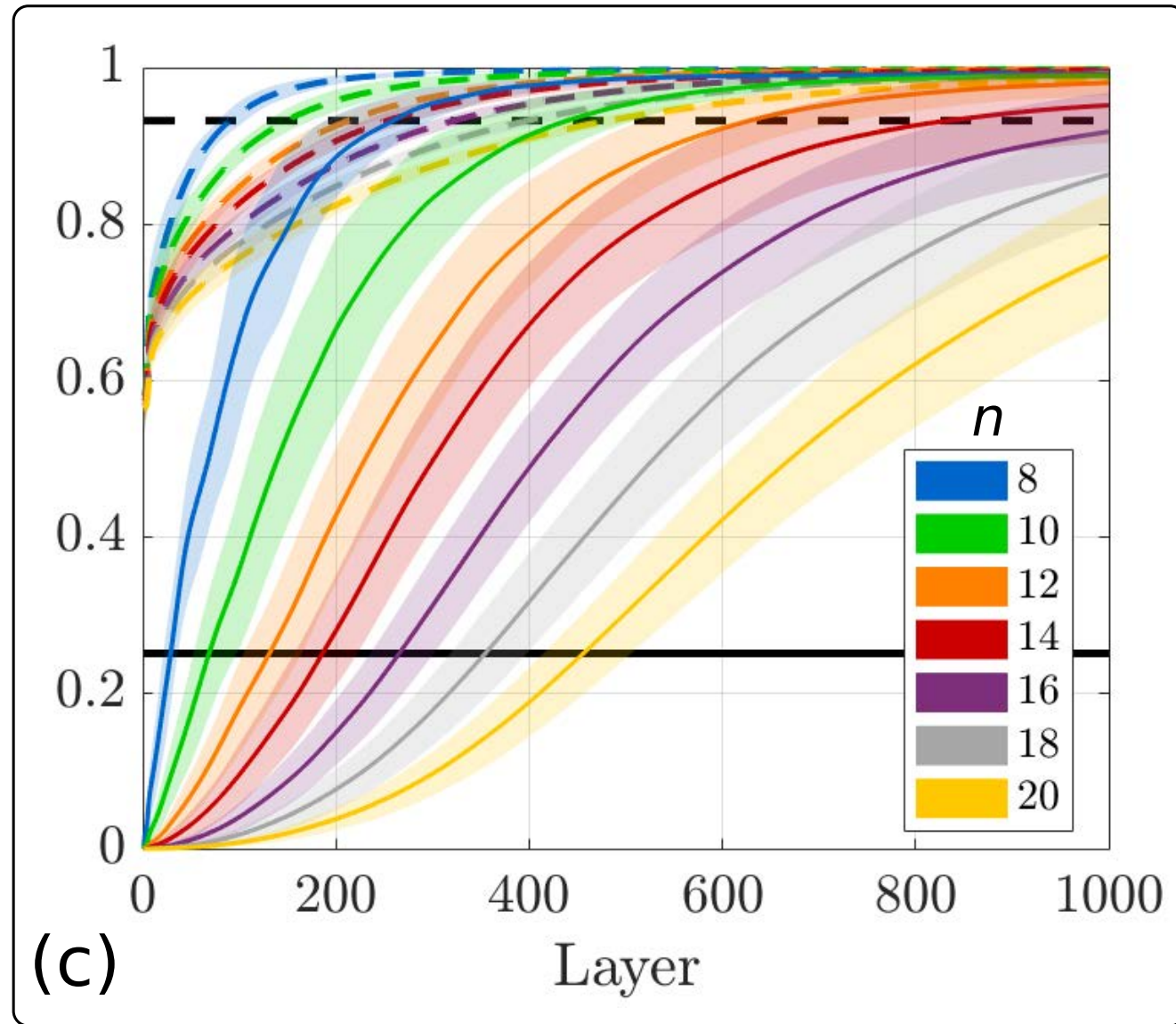
$$r_A = \langle H_p \rangle / \langle H_p \rangle_{\min}$$

The largest known approximation ratio $r_A = 0.932$ by algorithm of Goemans and Williamson.

approximation ratio (dashed curves)
and the success probability of
measuring the degenerate ground
state (solid curves)

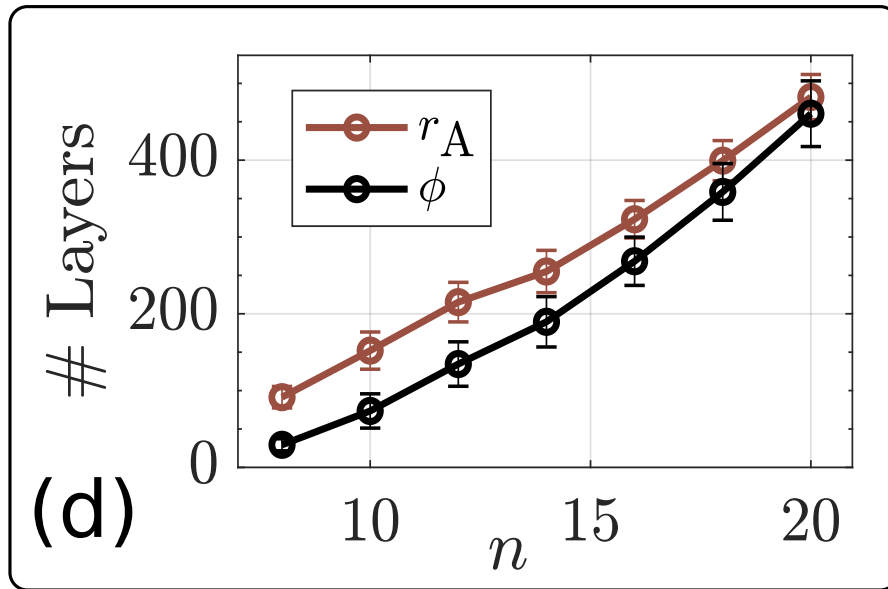


Pictorial representation of
MaxCut on a 3-regular
graph with 8 vertices.

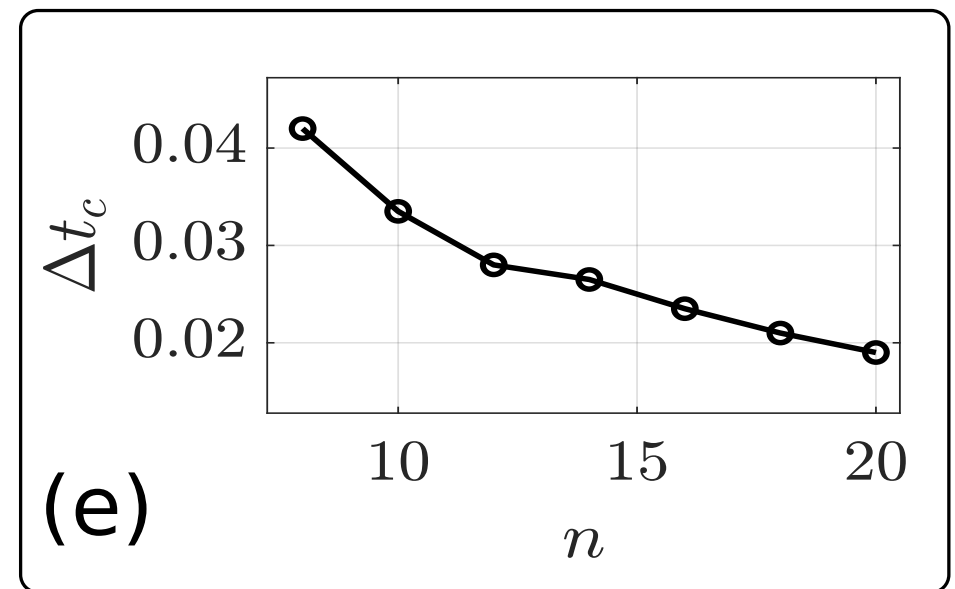


n = the number of vertices

FALQON for MaxCut problem



The mean number of layers needed to achieve the reference values of $r_A = 0.932$ (dashed curve) and $\phi = 0.25$ (solid curve) is shown; error bars report the associated standard deviation.

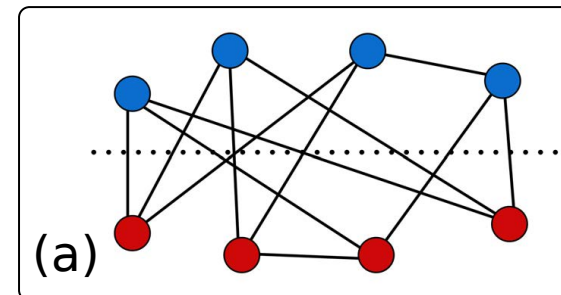


The critical Δt values for different problem sizes are plotted.

The only free parameter is time step Δt , which is tuned to be as large as possible.

$$r_A = \langle H_p \rangle / \langle H_p \rangle_{\min} = \text{approximation ratio}$$

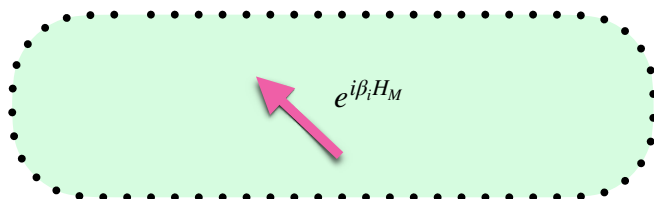
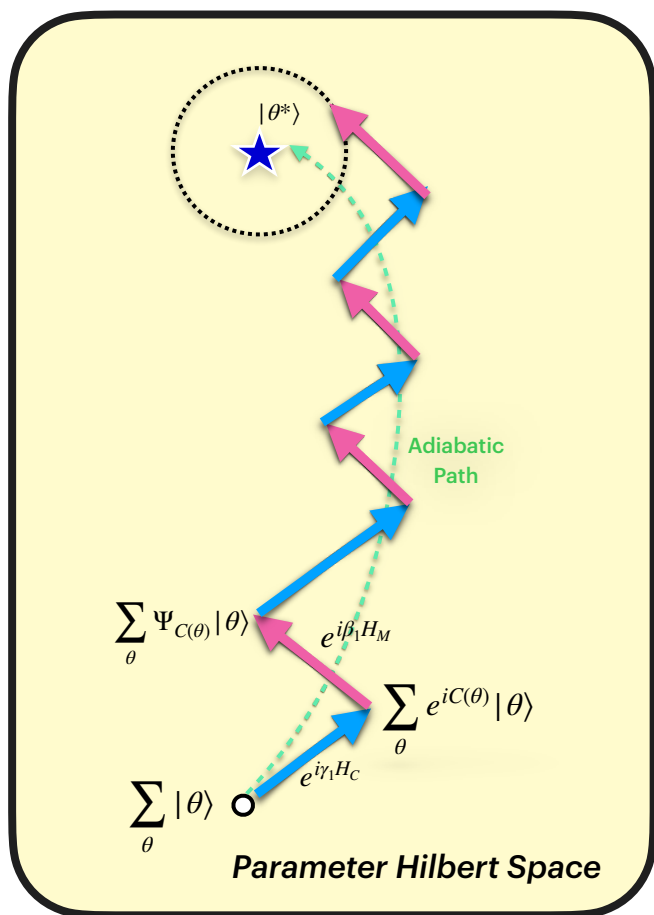
$$\phi = \sum_i |\langle \psi | q_{0,i} \rangle|^2 = \text{the success probability of measuring the (potentially degenerate) ground state(s) } \{ |q_{0,i}\rangle \},$$



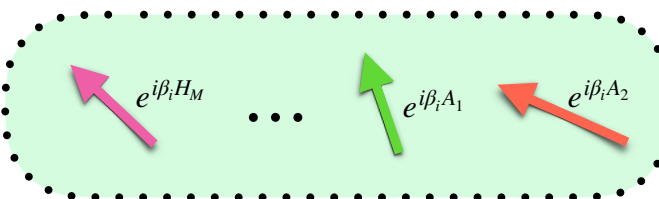
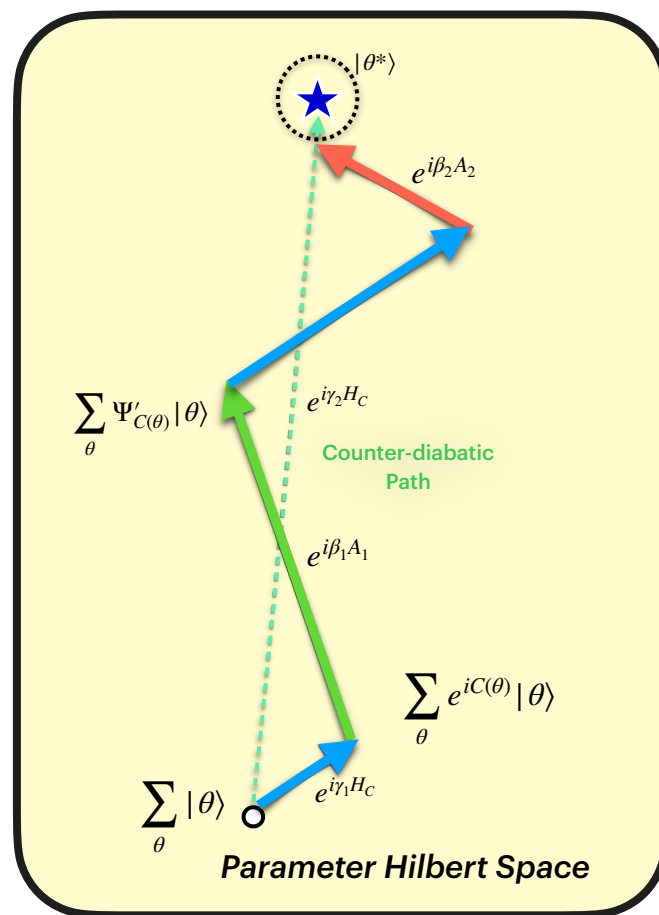
Pictorial representation of MaxCut on a 3-regular graph with 8 vertices.

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)

QAOA



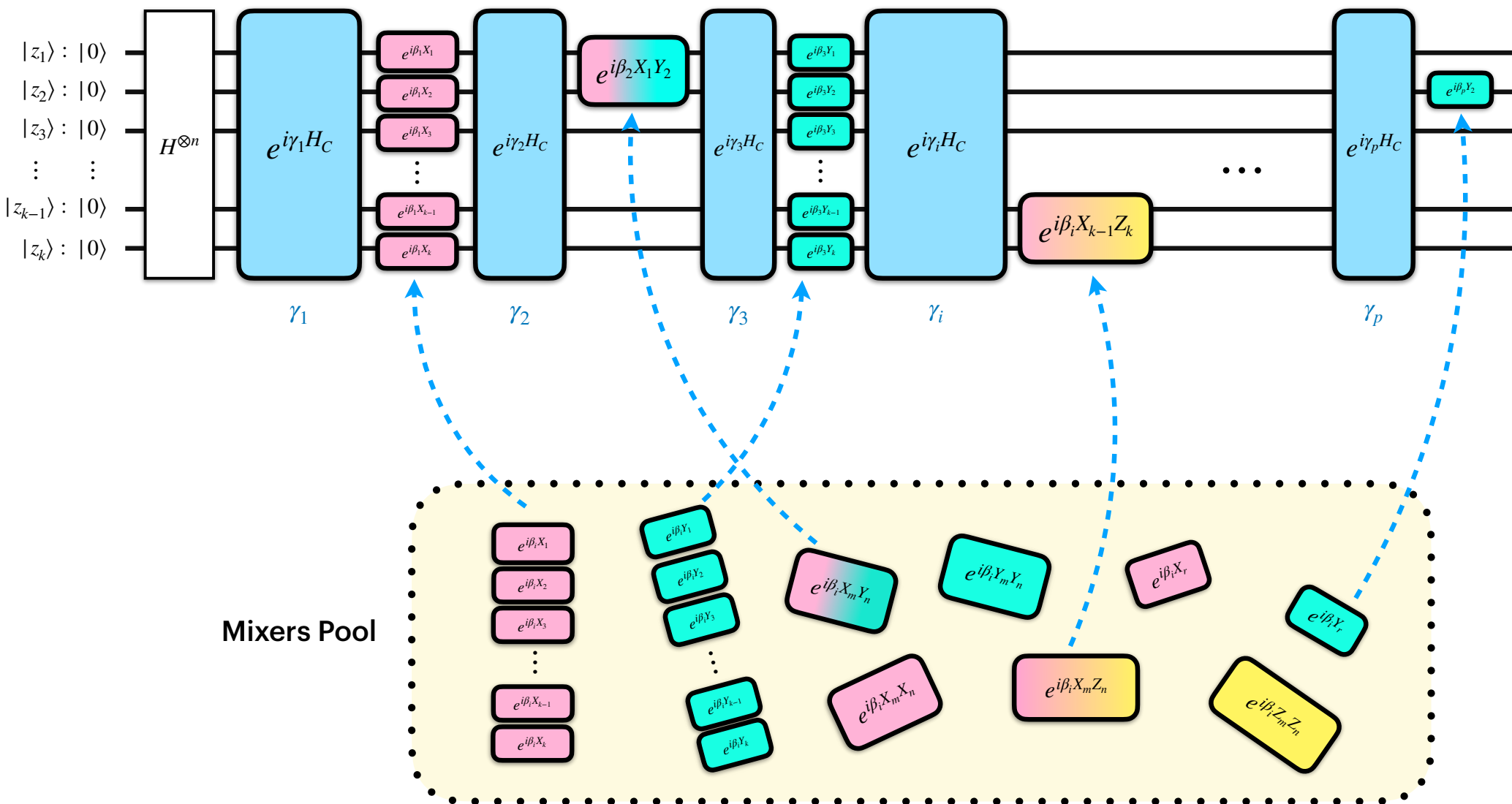
Adaptive QAOA



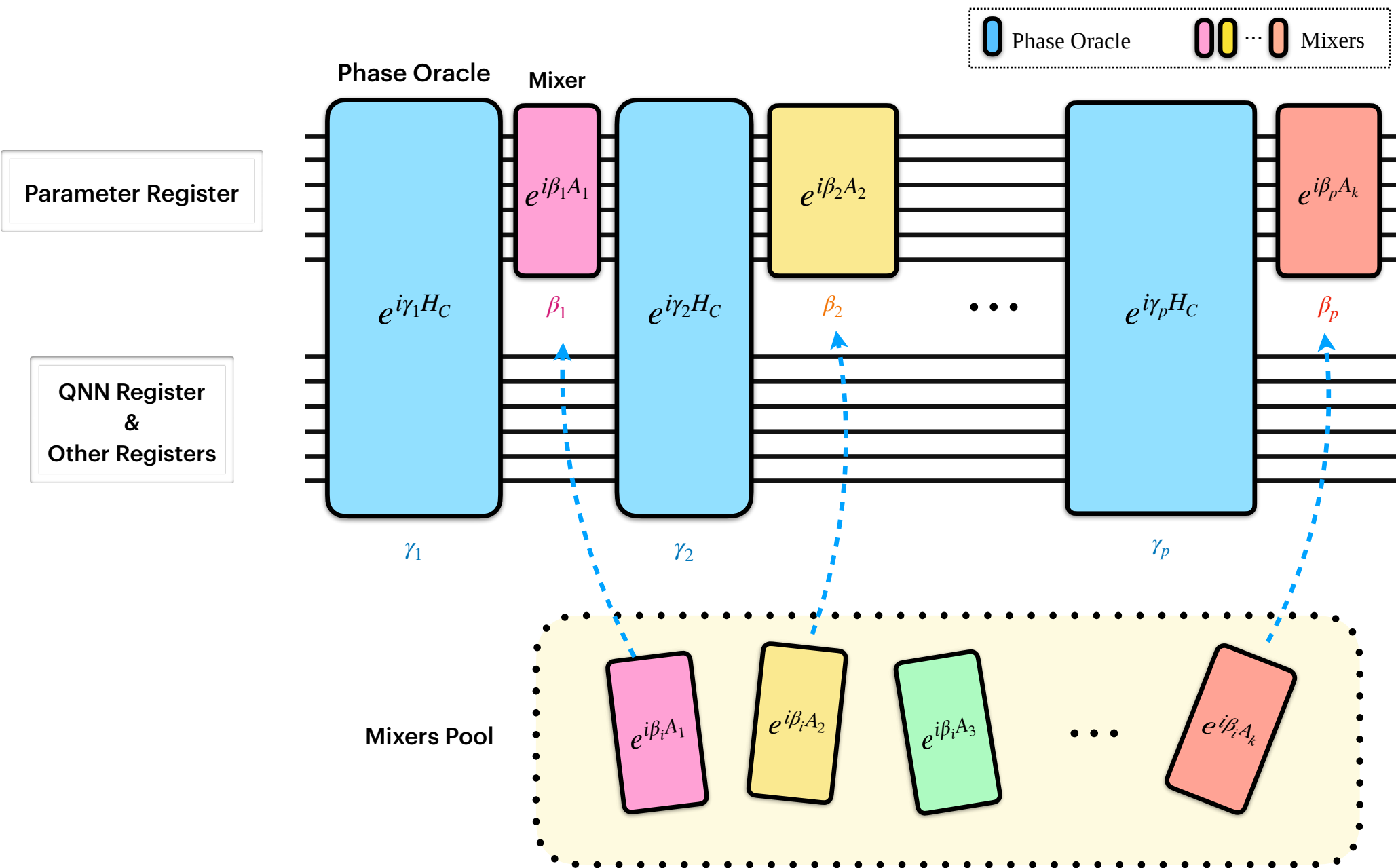
Mixers Pool

ADAPT-QAOA

Zhu et al 2020



Quantum Neural Networks



Adaptive Derivative Assembled Problem Tailored - Quantum Approximate Optimization Algorithm (ADAPT-QAOA)

- <https://arxiv.org/pdf/2005.10258.pdf>

ADAPT-QAOA

$$\left| \psi_p(\vec{\gamma}, \vec{\beta}) \right\rangle = \left(\prod_{k=1}^p \left[e^{-iH_M \beta_k} e^{-iH_C \gamma_k} \right] \right) \left| \psi_{\text{ref}} \right\rangle \Rightarrow \left| \psi_p(\vec{\gamma}, \vec{\beta}) \right\rangle = \left(\prod_{k=1}^p \left[e^{-iA_k \beta_k} e^{-iH_C \gamma_k} \right] \right) \left| \psi_{\text{ref}} \right\rangle$$

$$\left| \psi^{(0)} \right\rangle = \left| \psi_{\text{ref}} \right\rangle = \left| + \right\rangle^{\otimes n}$$

n=number of qubits

mixer pool = set of A_j : $\{A_j\}$

Algorithm 1 ADAPT-QAOA

Initial state: $\left| \psi^{(0)} \right\rangle = \left| \psi_{\text{ref}} \right\rangle = \left| + \right\rangle^{\otimes n}$
 Predefined: Number of layers p ; Cost Hamiltonian H_C ;
 Initial parameter for optimization: γ_0 ; Operator pool with
 m operators A_j , $j \in [1, m]$
for $k = 1 \dots p$ **do**
 //From operator pool select operator
 for $j = 1 \dots m$ **do**
 //Get max measured gradient operator $A_{\text{max}}^{(k)}$:
 Set $\gamma_k = \gamma_0$
 Define $\left| \psi^{(k)} \right\rangle_t = e^{-iH_C \gamma_k} \left| \psi^{(k-1)} \right\rangle$
 $A_{\text{max}}^{(k)} = \text{argmax} \left(-i {}_t \langle \psi^{(k)} | [H_C, A_j] | \psi^{(k)} \rangle_t \right)$
 end for
 //Add $A_{\text{max}}^{(k)}$ to current ansatz:
 $\left| \psi^{(k)} \right\rangle = e^{-iA_{\text{max}}^{(k)} \beta_k} e^{-iH_C \gamma_k} \left| \psi^{(k-1)} \right\rangle$
 // Optimization
 $\min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle \rightarrow \vec{\beta}, \vec{\gamma}$
 output.add($\vec{\beta}, \vec{\gamma}, A_{\text{max}}^{(k)}, \min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle$)
end for
 return output

ADAPT-QAOA

$$|\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iH_M \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle \Rightarrow |\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iA_k \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle$$

$$|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$$

n=number of qubits

mixer pool = set of A_j : $\{A_j\}$

Define $|\psi^{(k)}\rangle_t = e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$
 $A_{\text{max}}^{(k)} = \text{argmax} \left(-i {}_t\langle \psi^{(k)} | [H_C, A_j] | \psi^{(k)} \rangle_t \right)$

end for
 //Add $A_{\text{max}}^{(k)}$ to current ansatz:
 $|\psi^{(k)}\rangle = e^{-iA_{\text{max}}^{(k)} \beta_k} e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$
 // Optimization
 $\min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle \rightarrow \vec{\beta}, \vec{\gamma}$
 output.add($\vec{\beta}, \vec{\gamma}, A_{\text{max}}^{(k)}, \min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle$)
end for
 return output

$$\begin{aligned} \left. \frac{\partial}{\partial \beta_k} \langle \psi_k | H_P | \psi_k \rangle \right|_{\beta_k=0} &= \left. \frac{\partial}{\partial \beta_k} \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} H_P e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \right|_{\beta_k=0} \\ &= \left. \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} (iA_j H_P - iH_P A_j) e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \right|_{\beta_k=0} \\ H_P &= H_C \\ &= -i \langle \psi_{k-1} | e^{i\gamma_k H_P} [H_P, A_j] e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \end{aligned}$$

ADAPT-QAOA

$$|\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iA_k\beta_k} e^{-iH_C\gamma_k}] \right) |\psi_{\text{ref}}\rangle$$

$$|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$$

mixer pool = set of $A_j : \{A_j\}$

$\mathcal{O}(1)$ elements $P_{\text{QAOA}} = \left\{ \sum_{i \in Q} X_i \right\}$

$\mathcal{O}(n)$ elements $P_{\text{single}} = \cup_{i \in Q} \{X_i, Y_i\} \cup \left\{ \sum_{i \in Q} Y_i \right\} \cup P_{\text{QAOA}}$

$\mathcal{O}(n^2)$ elements $P_{\text{multi}} = \cup_{i,j \in Q \times Q} \{B_i C_j | B_i, C_j \in \{X, Y, Z\}\} \cup P_{\text{single}}$

$$P_{\text{QAOA}} \subset P_{\text{single}} \subset P_{\text{multi}} \quad \longrightarrow \quad \text{Best performance}$$

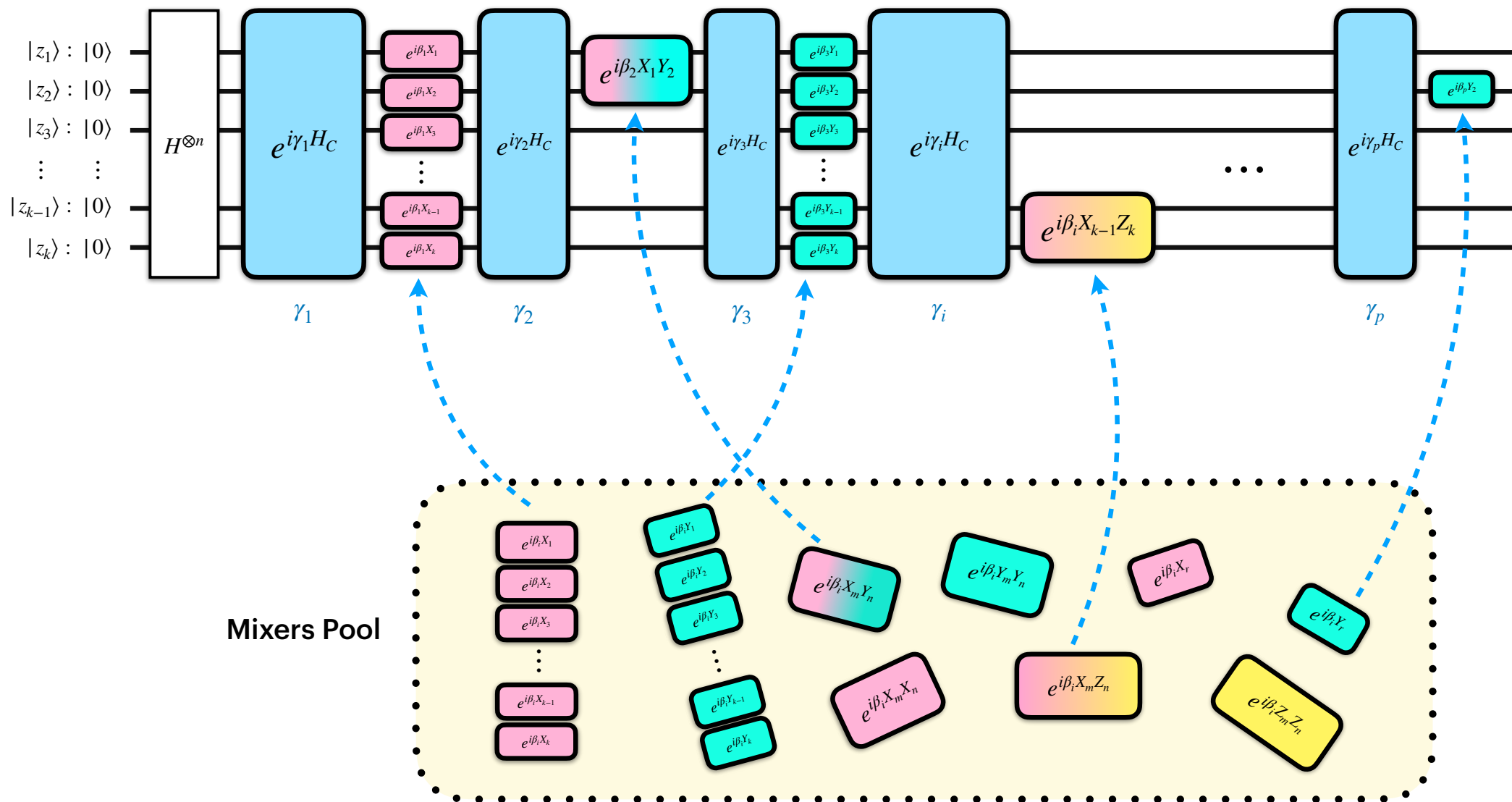
$$H_C = -\frac{1}{2} \sum_{i,j} w_{i,j} (I - Z_i Z_j)$$

- $H_C = H_P$ has a Z_2 symmetry associated with the operator $F = \otimes_i X_i$. Since $[F, H_C] = 0$, one can show that the gradient is only nonzero for $[F, A_j] \neq 0$. The A_j that commutes with F are Pauli strings that have an even number of Y or Z operators.

ADAPT-QAOA

2005.10258

Zhu et al 2020



ADAPT-QAOA applied to Mascot

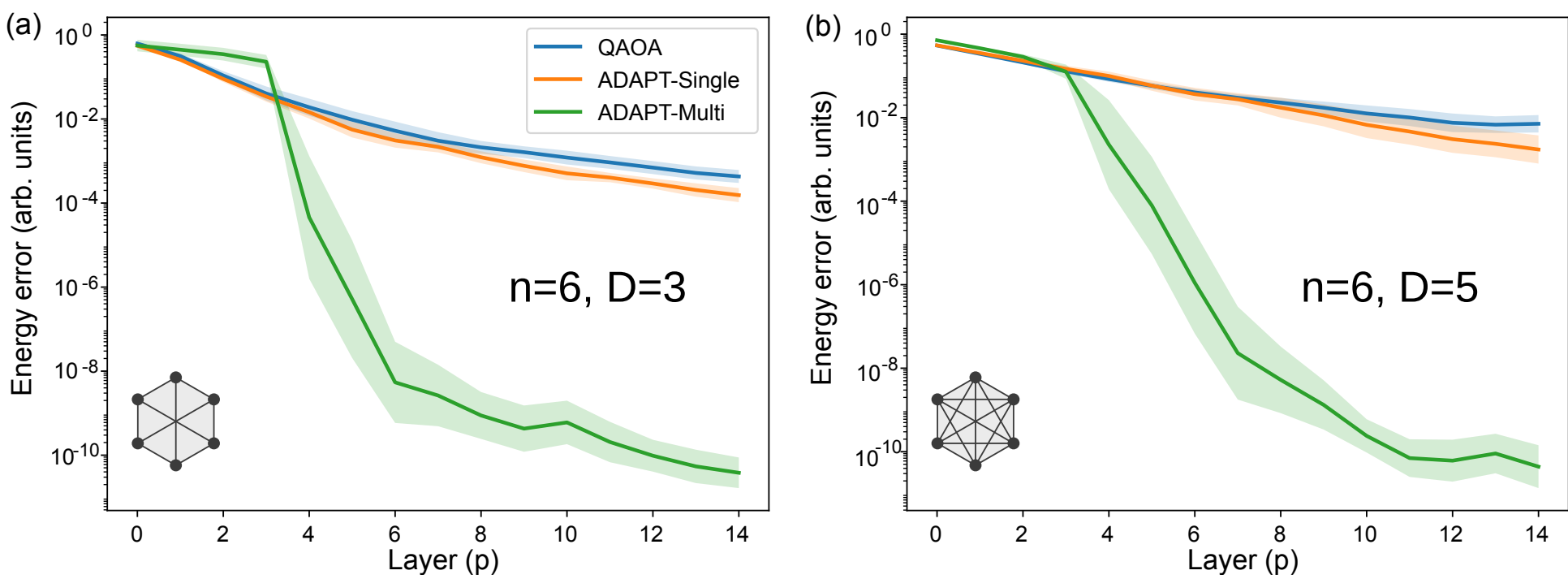


FIG. 1. Comparison of the performance of standard QAOA (blue) with ADAPT-QAOA for the single-qubit (orange) and multi-qubit (green) pools. The algorithms are run on the Max-Cut problem for the regular graphs shown in the figure, which have $n=6$ vertices and are of degree $D=3$ (a) and $D=5$ (b). The energy error (the difference between the energy estimate obtained by the algorithm and the exact ground state energy of H_C) is shown as a function of the number of layers in the ansatz. Results are shown for 20 different instances of edge weights, which are randomly sampled from the uniform distribution $U(0, 1)$. The shaded regions indicate 95% confidence intervals.

$\gamma_0 = 0.01$

- Nelder-Mead for optimization
- = downhill simplex method
- = amoeba method
- = polytope method

- How much does the ADAPT-QAOA ansatz differ from the standard QAOA ansatz?
- When the single-qubit mixer pool is used, the single-qubit operators X_i are chosen instead of the standard mixer approximately **36.6%** of the time for $n=6, D=3$ graphs and **25%** of the time for $n=6, D=5$ graphs.
- For the multi-qubit mixer pool, the algorithm chooses operators other than the standard mixer approximately **75%** of the time for $n=6, D=3$ graphs and **80%** of the time for $n=6, D=5$ graphs.
- This trend supports the intuitive idea that a more connected graph requires more entanglement for a rapid convergence to the solution.

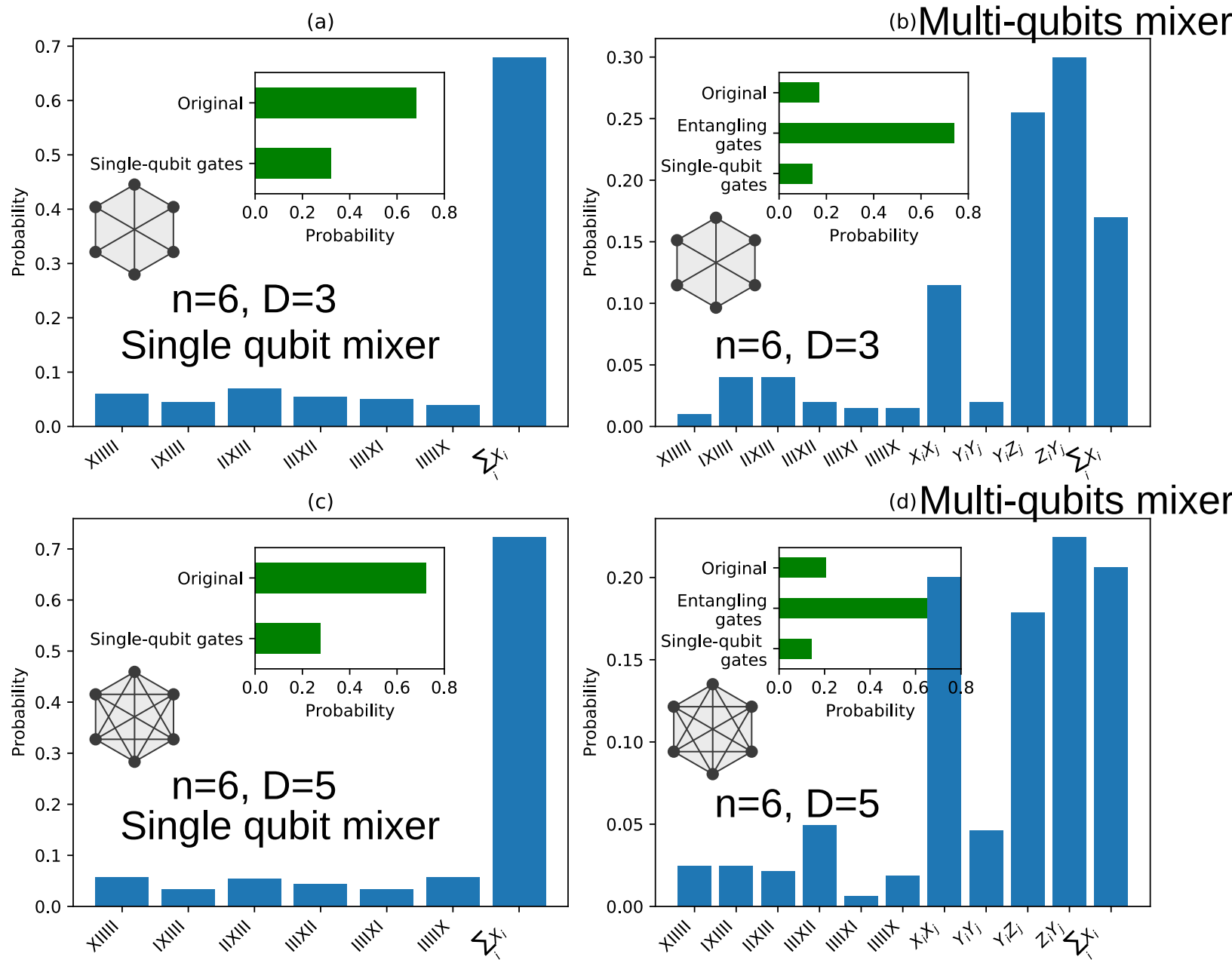
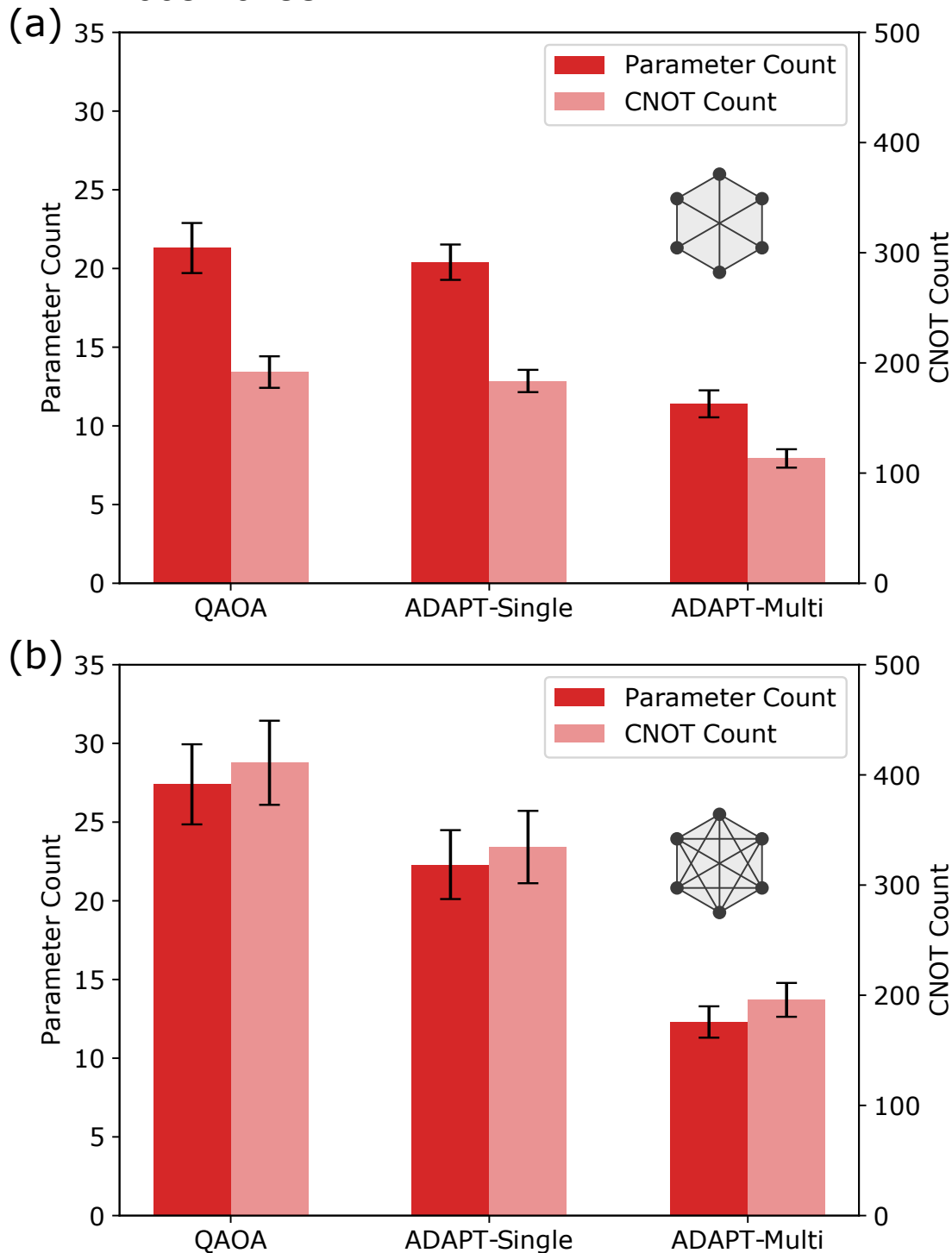


FIG. 6. Probability of operators picked by the original QAOA, ADAPT-QAOA with the single-qubit mixer and ADAPT-QAOA with multi-qubit pool for the Max-Cut problem on regular graphs with $n=6$ vertices with degree $D=3$ (a)(b) and $D=5$ (c)(d) with random edge weights sampled from a uniform distribution $U(0, 1)$. The blue bars show the probability of each particular operator used for ansatz, and green bars show the probability of the original mixer, sum over all single-qubit gates and sum over all entangling gates used in ansatz. The results from 20 instances of random edge weights.

2005.10258



- ADAPT-QAOA provides a systematic way to both improve performance and reduce the number of parameters and CNOTs.

FIG. 2. Resource comparison of the standard QAOA, ADAPT-QAOA with the single-qubit mixer pool, and ADAPT-QAOA with the multi-qubit mixer pool for the Max-Cut problem on regular graphs with $n=6$ vertices and random edge weights. Panels (a) and (b) show the comparison for graphs of degree $D=3$ and $D=5$, respectively. For all cases except the standard QAOA applied to $D=5$ graphs, we count the number of parameters and CNOTs needed to reach an energy error of $\delta E = 10^{-3}$. As standard QAOA for $D=5$ graphs never reaches this error threshold, we instead count the CNOT gates and parameters at the end of the simulation (15 layers). The dark (light) red bars show variational parameter (CNOT gate) counts. The error bars show variances obtained by sampling over 20 different instances of edge weights.

Why ADAPT-QAOA performs better?

- Considering that the standard QAOA ansatz has a structure dictated by the adiabatic theorem, a possible explanation is related to **Shortcuts to adiabaticity (STA)**.
- **STA (counter-diabatic or transitionless driving)** was introduced by Demirplak and Rice and later, independently, by Berry.
- If we want to drive a system such that it remains in the instantaneous ground state at all times, then by adding a certain term H_{CD} to the Hamiltonian, we can achieve this without paying the price of a slow evolution.
- Although the instantaneous eigenstates of the original Hamiltonian only solve the time-dependent Schrodinger equation in the adiabatic limit, they become exact solutions when the Hamiltonian is updated to include H_{CD} .
- The advantage of STA is that the evolution can be achieved non-adiabatically.

Gauge transformation in electrodynamics

$$\nabla \cdot \vec{E} = \rho \quad \text{Gauss' law}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{Gauss' law}$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \text{Faraday's law}$$

$$\nabla \times \vec{E} - \frac{\partial \vec{E}}{\partial t} = \vec{J} \quad \text{Ampere-Maxwell's law}$$

$$\vec{F}_{\text{Lorentz}} = q \left(\vec{E} + \vec{v} \times \vec{B} \right) \quad \text{Lorentz force}$$

$$\vec{F} = m\vec{a} \quad \text{Newton's equation of motion}$$

$$\frac{\partial \rho(\vec{x}, t)}{\partial t} + \nabla \cdot \vec{J}(\vec{x}, t) = 0 \quad \text{Continuity equation}$$

- The electromagnetic fields $\vec{E} = -\nabla\phi + \frac{\partial \vec{A}}{\partial t}$ and $\vec{B} = \nabla \times \vec{A}$ are invariant under the following Gauge transformation

$$\phi \longrightarrow \phi - \frac{\partial \Lambda(\vec{x}, t)}{\partial t}$$

$$\vec{A} \longrightarrow \vec{A} + \nabla \Lambda(\vec{x}, t)$$

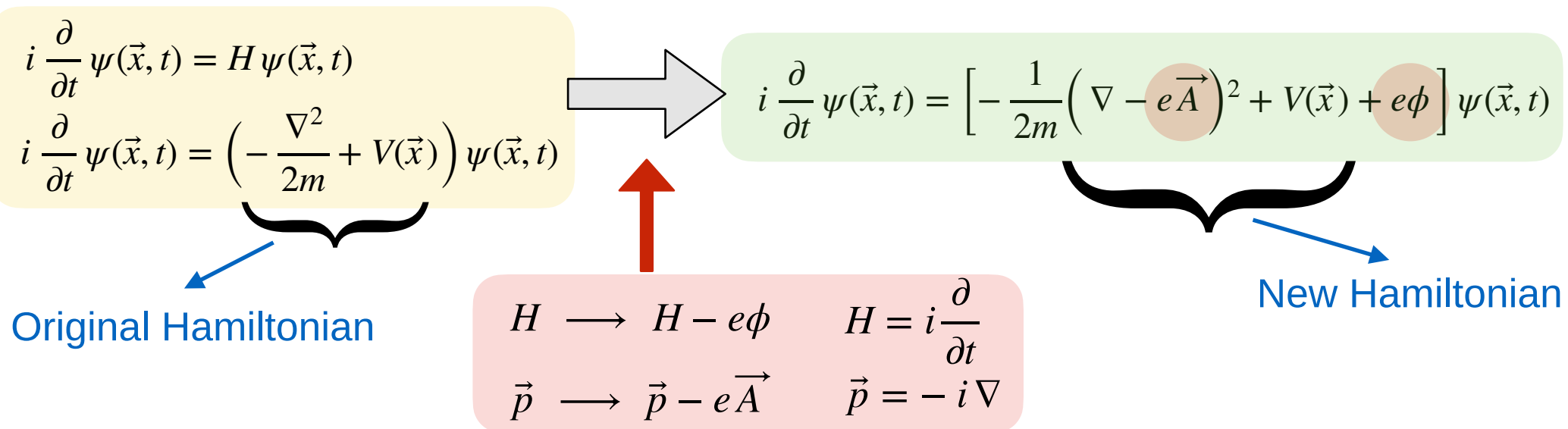
- Therefore the Lorentz force and the corresponding trajectory of an object under the electromagnetic remain the same.
- The gauge symmetry implies the conservation of electric charge.
- Perhaps $(\phi, \vec{A})$ may be **more fundamental** physical quantities than the electromagnetic fields $(\vec{E}, \vec{B})$. $A_\mu = (\phi, \vec{A})$ is called photon field.

Gauge transformation in electrodynamics

- The electromagnetic field $\vec{E} = -\nabla\phi + \frac{\partial\vec{A}}{\partial t}$ and $\vec{B} = \nabla \times \vec{A}$ are invariant (and therefore the classical trajectory of an object) under the following Gauge transformation

$$\phi \longrightarrow \phi - \frac{\partial\Lambda(\vec{x}, t)}{\partial t} \qquad \vec{A} \longrightarrow \vec{A} + \nabla\Lambda(\vec{x}, t)$$

- In quantum mechanics, we need to modify Hamiltonian (for action of quantum state) to be invariant under the gauge transformation.



- Quanta of electromagnetic fields are photons, $A_\mu = (\phi, \vec{A})$

Shortcuts to Adiabaticity (transitionless driving protocols)

$$i \partial_t |\psi\rangle = H(\theta(t)) |\psi\rangle \quad \longrightarrow \quad i \partial_t |\tilde{\psi}\rangle = (\tilde{H} - \dot{\theta} \tilde{A}_\theta) |\tilde{\psi}\rangle$$

$$\begin{aligned} |\psi\rangle &\longrightarrow |\tilde{\psi}\rangle = U^\dagger |\psi\rangle & H_{CD} &= \dot{\theta} A_\theta \\ H &\longrightarrow \tilde{H} = U^\dagger H U & \tilde{A}_\theta &= i U^\dagger \partial_\theta U \\ i \partial_t &\longrightarrow i \partial_t - \dot{\theta} \tilde{A}_\theta & A_\theta &= U \tilde{A}_\theta U^\dagger \end{aligned}$$

- Suppose that we consider a unitary transformation $U(\theta(t))$ to move the Hamiltonian $H(\theta(t))$ from the initial basis to its instantaneous eigenbasis, where $\tilde{H}(\theta) = U^\dagger(\theta) H(\theta) U(\theta)$ is diagonal at all times.
- The Schrodinger equation in the instantaneous eigenbasis is $i \partial_t |\tilde{\psi}\rangle = (\tilde{H} - \dot{\theta} \tilde{A}_\theta) |\tilde{\psi}\rangle$, where $\tilde{A}_\theta = i U^\dagger A_\theta U$ is the **adiabatic gauge potential** in the rotated frame. It is evident that the term $-\dot{\theta} \tilde{A}_\theta$ drives transitions between the energy levels of the original Hamiltonian H . Therefore, one can add the counterdiabatic term $H_{CD} = \dot{\theta} A_\theta$ to $H(\theta)$, with $A_\theta = U \tilde{A}_\theta U^\dagger$, to eliminate such transitions in the rotated frame. **This is the core of transitionless driving protocols.**
- Ref. [40] proposes an approximate gauge potential:

$$\mathcal{A}_\theta^{(p)} = i \sum_{k=1}^p a_k [\mathcal{H}, \partial_\theta \mathcal{H}]_{2k-1} \quad [X, Y]_{k+1} = [X, [X, Y]]_k$$

Connection between ADAPT-QAOA and STA

- Apply the above formalism using the Hamiltonian

$$H = \frac{t}{T} H_C + \left(1 - \frac{t}{T}\right) \sum_{i=1}^n X_i, \text{ and set}$$

$\theta = t$. T is the duration of the evolution from the initial state $|\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$ to the ground state of the cost Hamiltonian H_C .

- In all cases, the mixer operator at the first layer is also an element of the set $\mathcal{O}_{CD}$.
- Going to higher order in the H_{CD} approximation increases the probability of finding the mixers in the set $\mathcal{O}_{CD}$.
- ADAPT-QAOA finds the appropriate rotation axes in Hilbert space for faster convergence to the solution, and that these axes may in some sense be universal across all possible choices of $H(t)$ that interpolate between the initial and target states. This suggests that STA can be used as a tool to construct operator pools for ADAPT-QAOA.

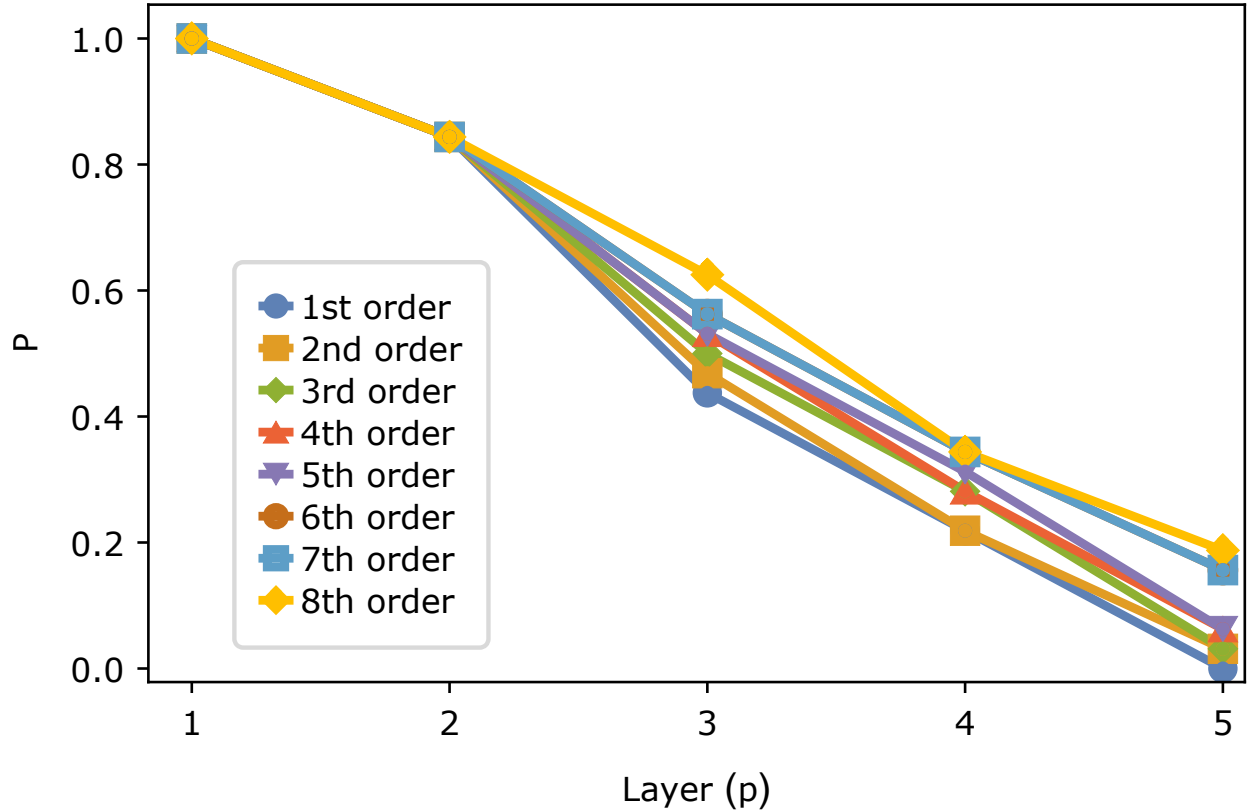
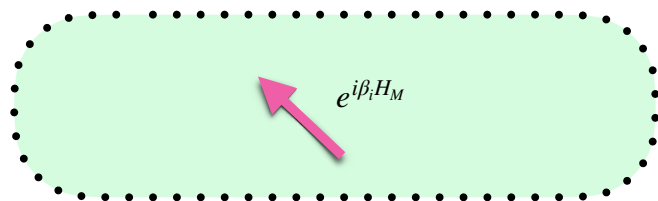
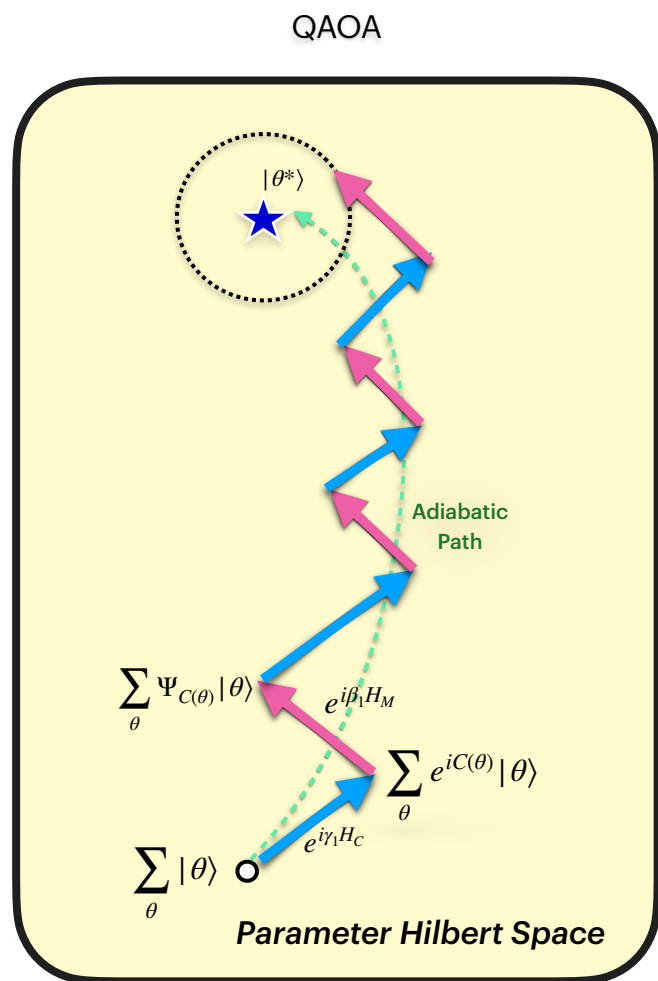
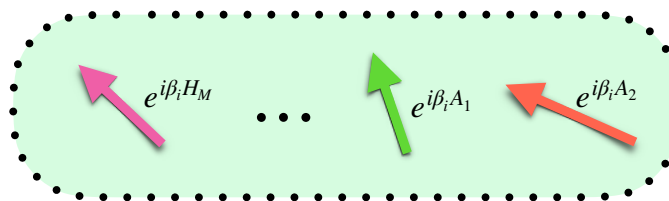
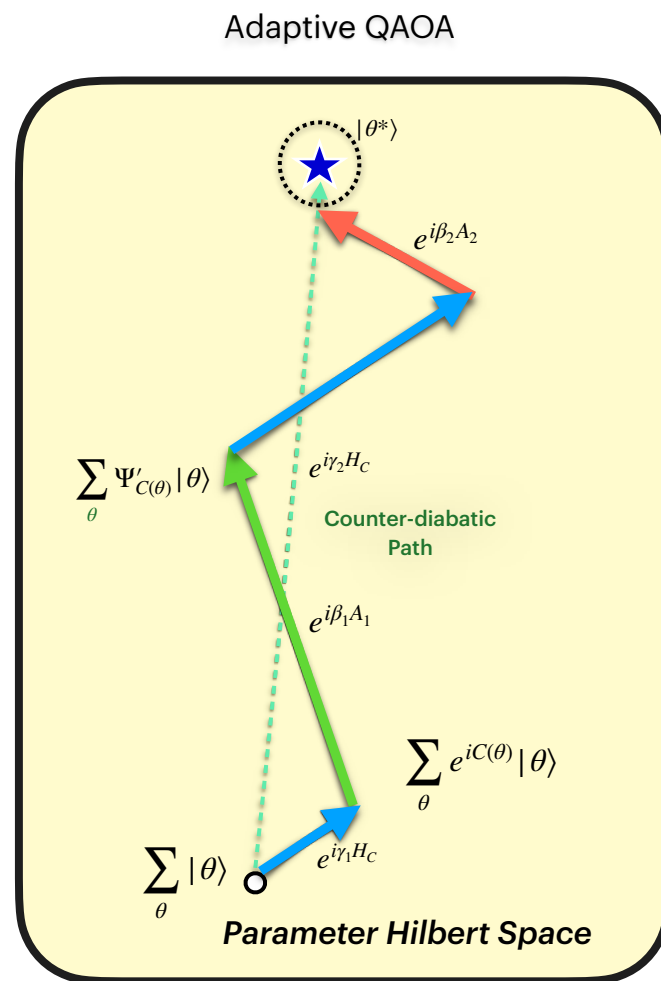


FIG. 3. Probability P of the operator at layer p of the ADAPT-QAOA ansatz to be among the Pauli strings with the largest coefficient in $\mathcal{H}_{CD}$ averaged over 32 graphs with $n = 6$, $D = 3$. The different curves correspond to different orders of the approximation.

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)



Mixers Pool



Combinatorial problems at the LHC

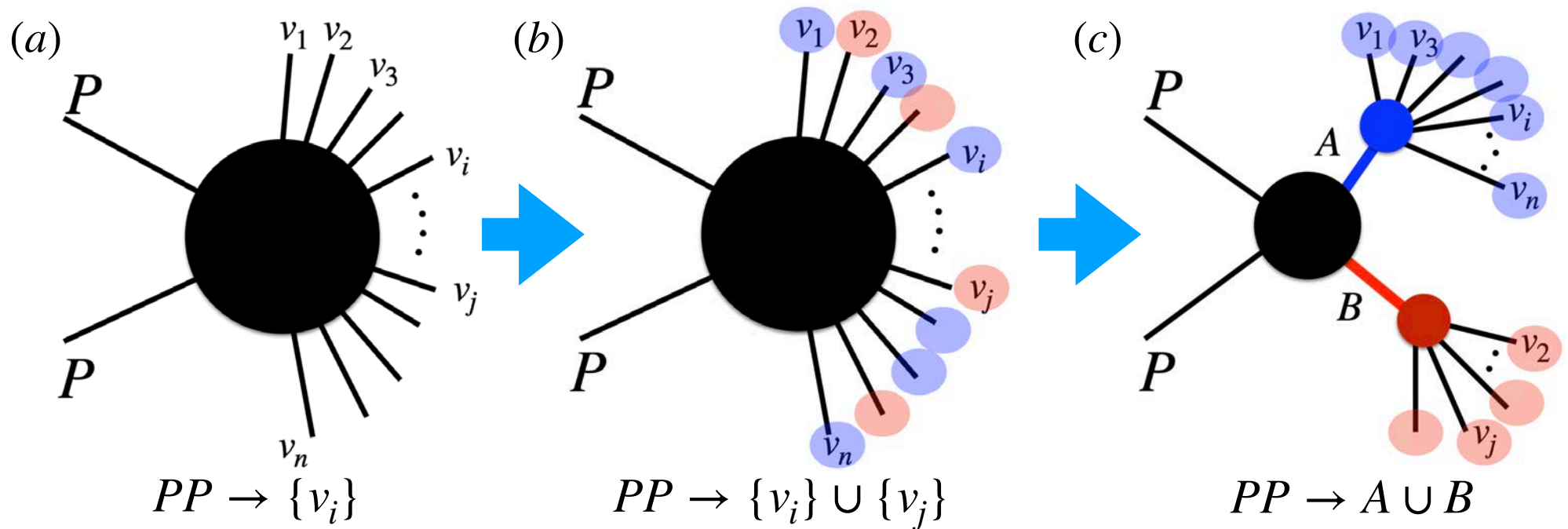


FIG. 1. (a) n -observed particles (b) Dividing n particles into two groups for $2 \rightarrow 2$ process (c) Identified event-topology with A and B .



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The Nobel Prize in Physics 2022



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Alain Aspect

Prize share: 1/3



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John F. Clauser

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Anton Zeilinger

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The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Yuri Milner founded the Breakthrough prize in 2012.

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BB84, quantum cryptography, factoring algorithms, Deutsch's algorithm

Basic Group Theory

- **Group:** a set G with an associative binary operation $\bullet$ satisfying
 - For any two elements g_1 and $g_2 \in G$, $g_1 \bullet g_2 \in G$ (closure)
 - $\exists e \in G$ such that $e \bullet g = g \bullet e = g$ for $\forall g \in G$ (identity)
 - $\exists g^{-1} \in G$ such that $g^{-1} \bullet g = g \bullet g^{-1} = e$ for $\forall g \in G$ (inverse)
 - $g_1 \bullet (g_2 \bullet g_3) = (g_1 \bullet g_2) \bullet g_3$ (associativity)
- $(\mathbb{Z}_n, + \pmod n) = \{0, 1, 2, \dots, n-1\}$ forms a group under addition modulo n .
- Set of k -bit string, $\mathbb{Z}_2^k$ forms a group under bitwise addition modulo 2.
- For a prime p , $\{1, 2, \dots, p-1\}$ forms a group $\mathbb{Z}_p^*$ under multiplication modulo p .
- $U(n)$: all unitary operators on an n -dimensional vector space V .
- Order = # of elements = $|G|$
- Finite group if $|G| < \infty$. Otherwise G is an infinite group.
- The order of an element g = the size of the subgroup of G that it generates.
 - The order of an element g must divide the order of the group.
- A set of generators of a group G is a subset of G such that all elements of G can be written as a finite product of the generators and their inverse.

Basic Group Theory

- A set of generators of a group is independent, if no generator can be written as a product of the other generators.
- A group is finitely generated if a finite group of generators exists.
- If a group can be generated by a single element, it is cyclic.
- The centralizer $Z(H)$ of a subgroup H of G is the set of elements of G that commute with all elements of H : $Z(H) = \{g \in G \mid gh = hg \text{ for all } h \in H\}$.
- For $H < G$, $Z(H)$ is a subgroup of G .
- If $g_1 \bullet g_2 = g_2 \bullet g_1$, G is Abelian or commutative.
- Every finite Abelian group is isomorphic to a product of one or more cyclic groups, $\mathbb{Z}_n$.
- If $n = pq$ and p and q are relatively prime, $\mathbb{Z}_n \cong \mathbb{Z}_p \times \mathbb{Z}_q$ ($\mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_q$).
- Any Abelian group A has the unique decomposition (up to ordering of factors) into cyclic groups of prime power order: $A \cong \mathbb{Z}_{c_1} \times \mathbb{Z}_{c_2} \times \cdots \times \mathbb{Z}_{c_K}$
 - $|A| = \prod_i c_i$ (prime factorization) where $c_i = p_i^{s_i}$ and p_i are distinct primes
- Product group $G \times H$ with operations $\bullet$ and $\circ = \{(g, h) \mid g \in G, h \in H\}$ with $(g_1, h_1) \star (g_2, h_2) = (g_1 \bullet g_2, h_1 \circ h_2)$.

Discrete Logarithm Problem

- All standard public key encryption system and digital signature schemes are based on either factoring or discrete logarithm problem.
- $\mathbb{Z}_p^*$: group of integers $\{1, 2, \dots, p - 1\}$ under multiplication modulo p .
 - b : generator of $\mathbb{Z}_p^*$ (any b relatively prime to $p - 1$ will work)
 - The discrete logarithm of $y \in \mathbb{Z}_p^*$ with respect to base b is the element $x \in \mathbb{Z}_p^*$ such that $b^x = y \pmod{p}$.
- **Discrete logarithm problem**: Given a prime p , a base $b \in \mathbb{Z}_p^*$ and an arbitrary element $y \in \mathbb{Z}_p^*$, find an $x \in \mathbb{Z}_p^*$ such that $b^x = y \pmod{p}$
 - Find the discrete logarithm of $y \in \mathbb{Z}_p^*$ with respect to base b such that $b^x = y \pmod{p}$
 - For a large p , this problem is computationally difficult to solve.
 - It is a special case of Abelian hidden subgroup problem.
 - Can be generalized to arbitrary finite cyclic groups.

Hidden Subgroup Problem

- **Hidden subgroup problem:** Let G be a group. Suppose a subgroup $H < G$ is implicitly defined by a function f on G in that f is constant and distinct on every coset of H . Find a set of generators of H .
 - Aim is to find an algorithm that computes a set of generators for H in $\mathcal{O}((\log |G|)^k)$ steps for some integer k .
- **Finite Abelian hidden subgroup problem:** Let G be a finite Abelian group with cyclic decomposition $G = \mathbb{Z}_{n_0} \times \cdots \times \mathbb{Z}_{n_L}$. Suppose G contains a subgroup $H < G$ that is implicitly defined by a function f on G that is constant and distinct on every coset of H . Find a set of generators.
- **Period finding as a hidden subgroup problem:** f is a periodic function on $\mathbb{Z}_N$ with period r that divides N . The subgroup $H < \mathbb{Z}_N$ generated by r is the hidden subgroup. Once a generator h for H has been found, the period r can be found by taking the greatest common divisor of h and N .
- H is a subgroup of G . The left coset is defined as $gH = \{gh \mid h \in H, \text{ for all } g \in G\}$

Ansatz	Main Idea	Enhancement & Applications
ma-QAOA [77] 2021	Multi-angle ansatz with a unique parameter for each element of cost and mixer Hamiltonians	Improves approximation ratio for MaxCut while reducing circuit depth
QAOA+ [78]	Augments traditional QAOA with an additional multi-parameter problem-independent layer	Higher approximation ratios for MaxCut on random regular graphs
DC-QAOA [79, 80]	Adds a problem-dependent counterdiabatic driving term to the QAOA ansatz	Improves the convergence rate of the approximation ratio while reducing circuit depth
ab-QAOA [81]	Incorporates local fields into the operators to reduce computation time	Computation time reduction for combinatorial optimization
ADAPT-QAOA [82] 2020	Iterative version of QAOA with systematic selection of mixers based on gradient criterion	Can be problem-specific and addresses hardware constraints
Recursive QAOA [83]	Non-local variant of QAOA that iteratively reduces problem size by eliminating qubits	Overcomes locality constraints and achieves better performance
QAOAnsatz [84]	Extends the original formulation with broader families of operators and allows for encoding of constraints	Adaptable to a wider range of optimization problems with hard and soft constraints
GM-QAOA [85]	Uses Grover-like selective phase shift mixing operators	Solves k -Vertex Cover, Traveling Salesperson Problem, Discrete Portfolio Rebalancing
2306.09198	A Review on Quantum Approximate Optimization Algorithm and its Variants	

Th-QAOA [86]	Replaces standard phase separator with a threshold function	Solves MaxCut, Max k -Vertex Cover, Max Bisection
Constraint Preserving Mixers [87]	Constructs mixers that enforce hard constraints	Solves optimization problems with hard constraints
WS-QAOA [88]	Modifies the initial state and mixer Hamiltonian based on the optimal solution to the relaxed QUBO problem	Solutions guaranteed to retain the GW bound for the MaxCut problem
FALQON [66]	Uses qubit measurements for feedback-based quantum optimization, avoiding classical optimizers	Produces monotonically improving approximate solutions as circuit depth grows while bypassing classical optimization loops
FALQON+ [89]	Combines FALQON's initialization with QAOA for better parameter initialization	Improves initialization of standard QAOA for non-isomorphic graphs with 8 to 14 vertices
FQAOA [90]	Utilizes fermion particle number preservation to intrinsically impose constraints in QAOA process	Improves performance in portfolio optimization, applicable to Grover adaptive search and quantum phase estimation
Quantum Dropout [91]	Selectively drops out clauses defining the quantum circuit while keeping the cost function intact	Improves QAOA performance on hard cases of combinatorial optimization problems
ST-QAOA [92]	Uses an approximate classical solution to construct a problem instance-specific circuit	Achieves same performance guarantee as the classical algorithm, outperforms QAOA at low depths for MaxCut problem
Modified QAOA [31]	Modifies cost Hamiltonian with conditional rotations	Improves approximation ratio for MaxCut at $p = 1$

2021

ADAPT-QAOA

$$|\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iH_M \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle \Rightarrow |\psi_p(\vec{\gamma}, \vec{\beta})\rangle = \left(\prod_{k=1}^p [e^{-iA_k \beta_k} e^{-iH_C \gamma_k}] \right) |\psi_{\text{ref}}\rangle$$

$$|\psi^{(0)}\rangle = |\psi_{\text{ref}}\rangle = |+\rangle^{\otimes n}$$

n=number of qubits

mixer pool = set of A_j : $\{A_j\}$

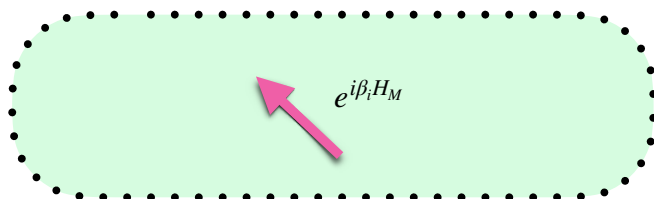
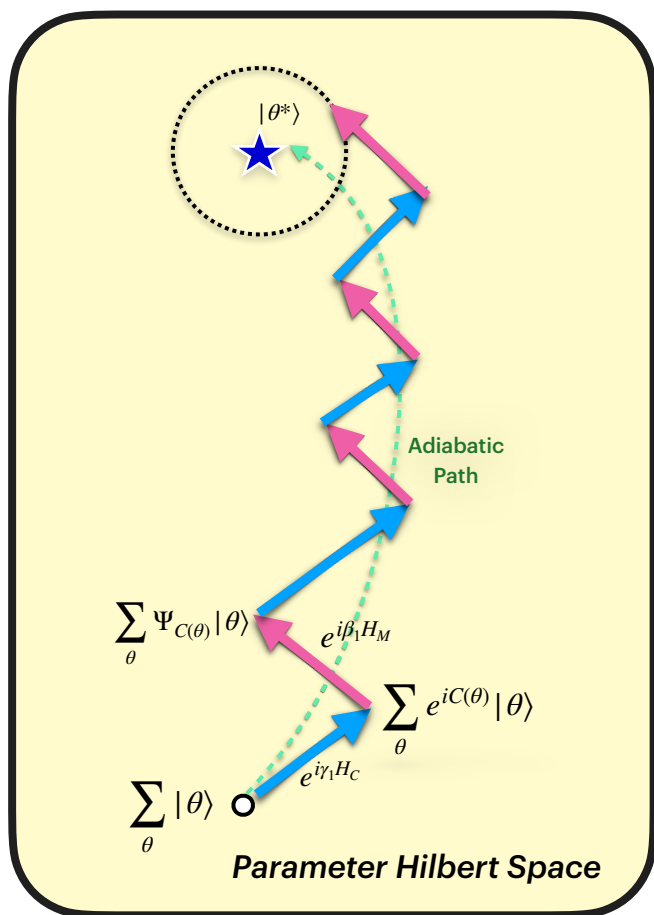
Define $|\psi^{(k)}\rangle_t = e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$
 $A_{\text{max}}^{(k)} = \text{argmax} \left(-i {}_t\langle \psi^{(k)} | [H_C, A_j] | \psi^{(k)} \rangle_t \right)$

end for
 //Add $A_{\text{max}}^{(k)}$ to current ansatz:
 $|\psi^{(k)}\rangle = e^{-iA_{\text{max}}^{(k)} \beta_k} e^{-iH_C \gamma_k} |\psi^{(k-1)}\rangle$
 // Optimization
 $\min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle \rightarrow \vec{\beta}, \vec{\gamma}$
 output.add($\vec{\beta}, \vec{\gamma}, A_{\text{max}}^{(k)}, \min \langle \psi^{(k)} | H_C | \psi^{(k)} \rangle$)
end for
 return output

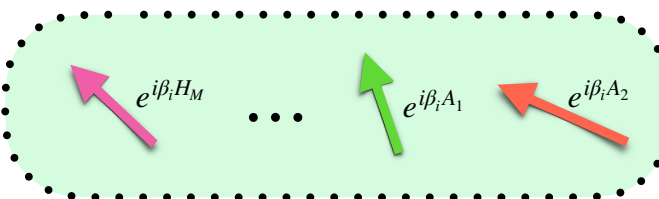
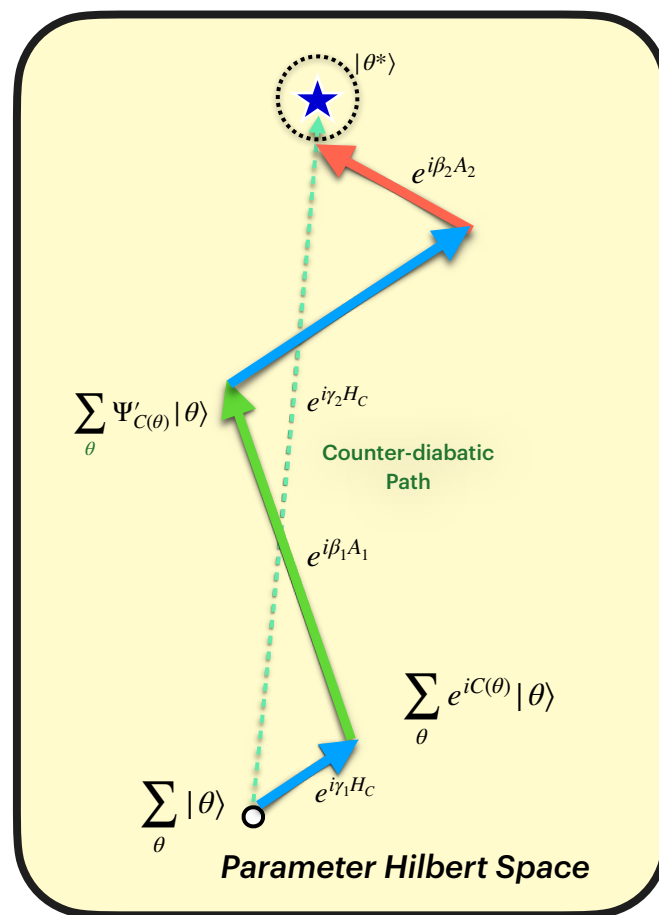
$$\begin{aligned} \left. \frac{\partial}{\partial \beta_k} \langle \psi_k | H_P | \psi_k \rangle \right|_{\beta_k=0} &= \left. \frac{\partial}{\partial \beta_k} \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} H_P e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \right|_{\beta_k=0} \\ &= \left. \langle \psi_{k-1} | e^{i\gamma_k H_P} e^{i\beta_k A_j} (iA_j H_P - iH_P A_j) e^{-i\beta_k A_j} e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \right|_{\beta_k=0} \\ H_P &= H_C \\ &= -i \langle \psi_{k-1} | e^{i\gamma_k H_P} [H_P, A_j] e^{-i\gamma_k H_P} | \psi_{k-1} \rangle \end{aligned}$$

Adaptive Derivative Assembled Problem Tailored QAOA (ADAPT-QAOA)

QAOA

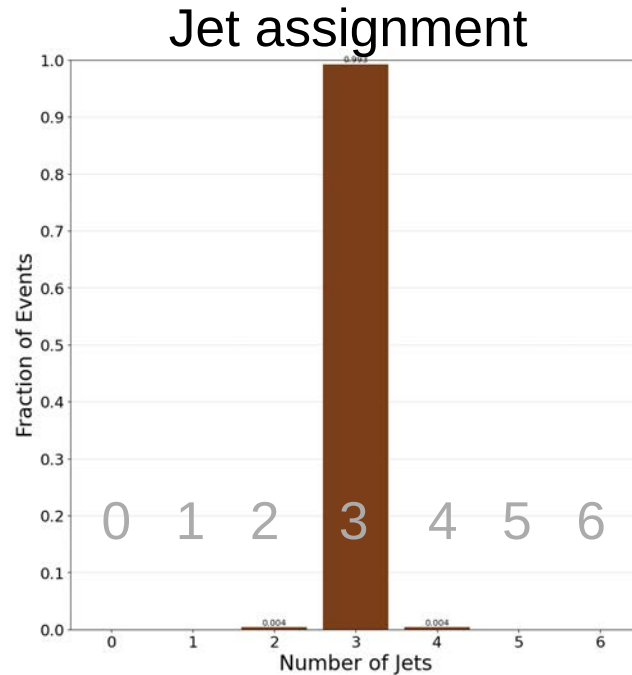
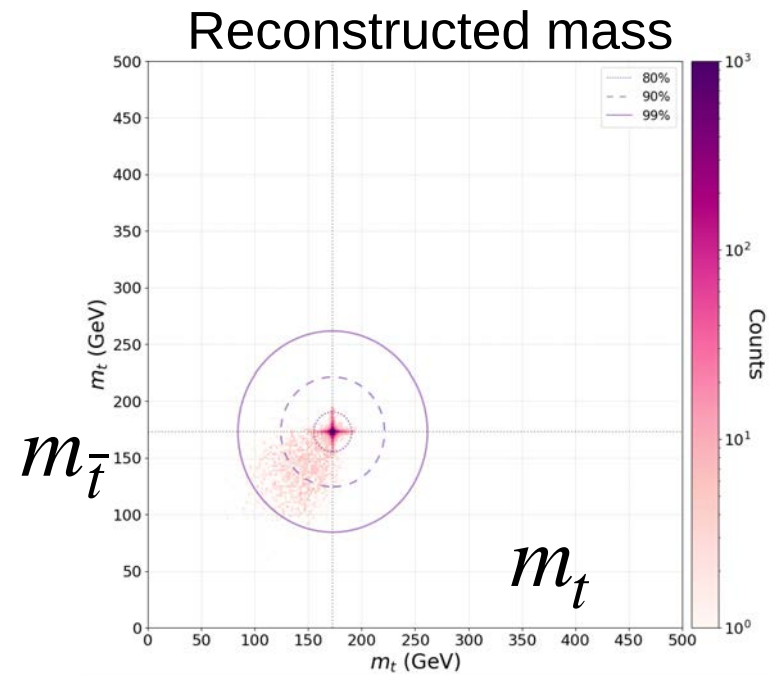


Adaptive QAOA



Mixers Pool

Parton-level truth and hemisphere method

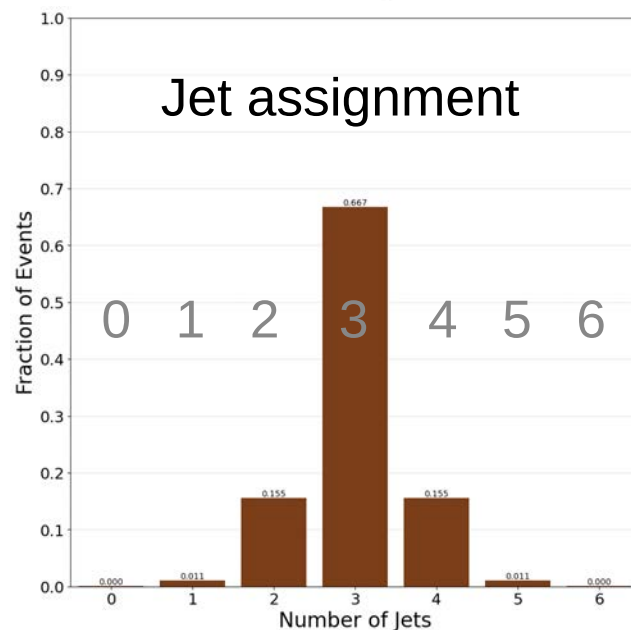
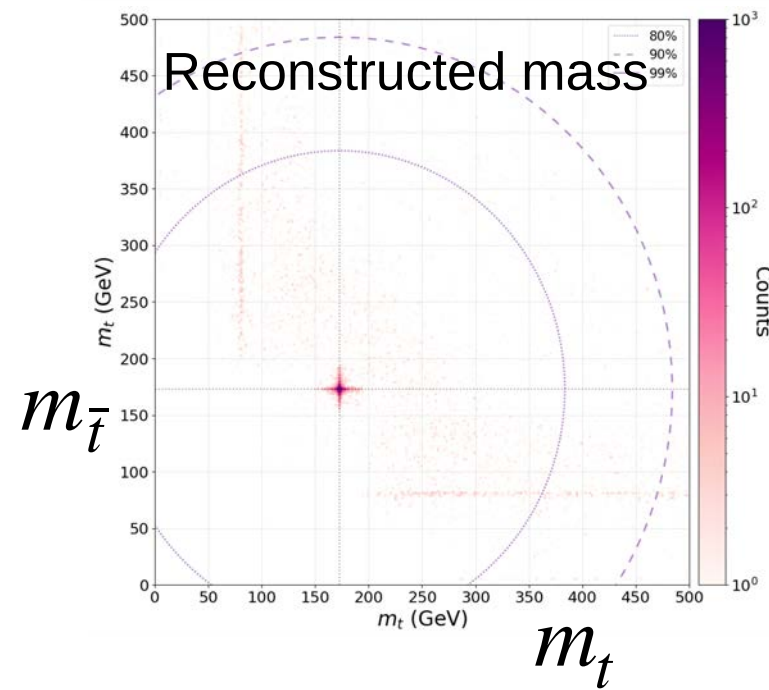


parton-level truth

79% overall efficiency

No mass information
used.

Hybrid quantum-classical
approach for combinatorial
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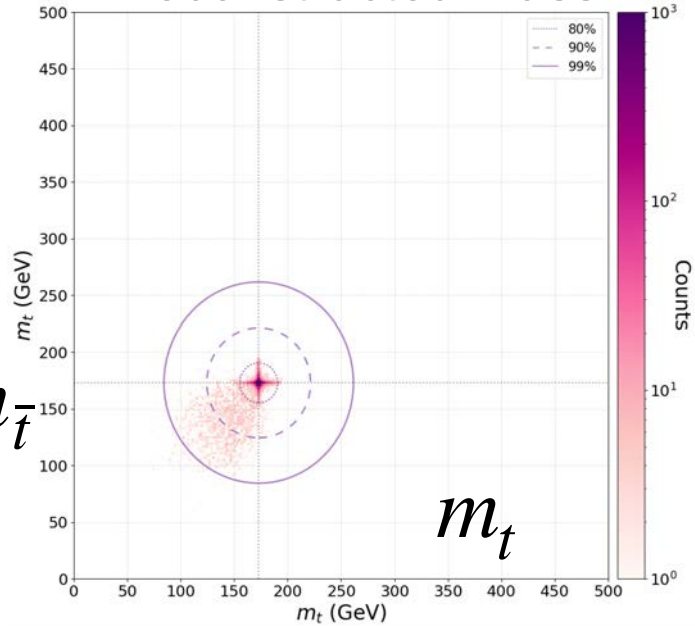
Hemisphere method

50% overall efficiency

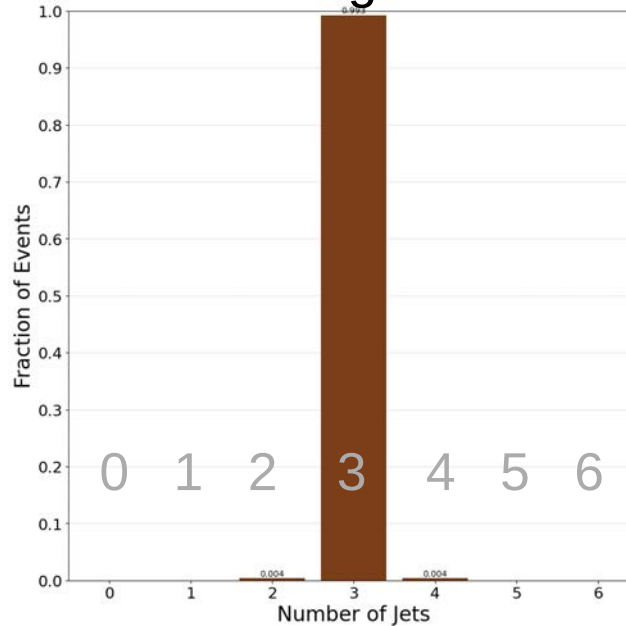
No mass information
used.

QAOA method

Reconstructed mass



Jet assignment



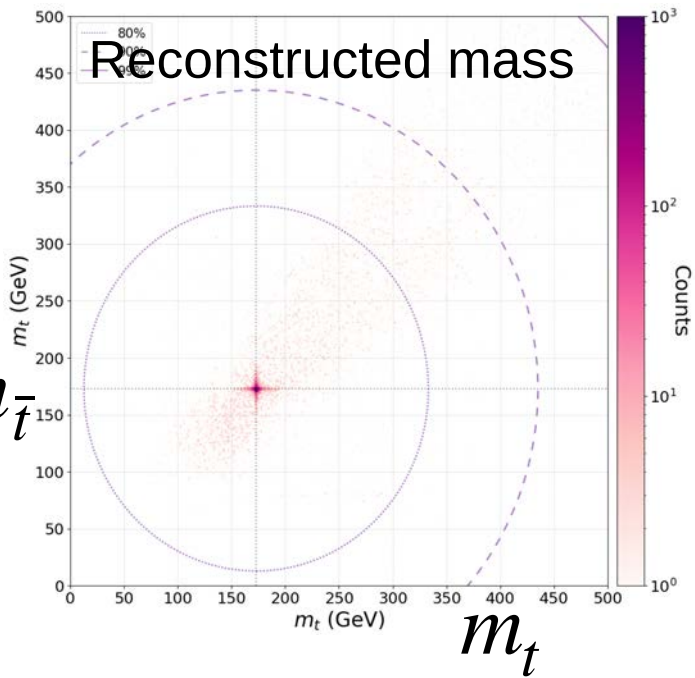
parton-level truth

79% overall efficiency

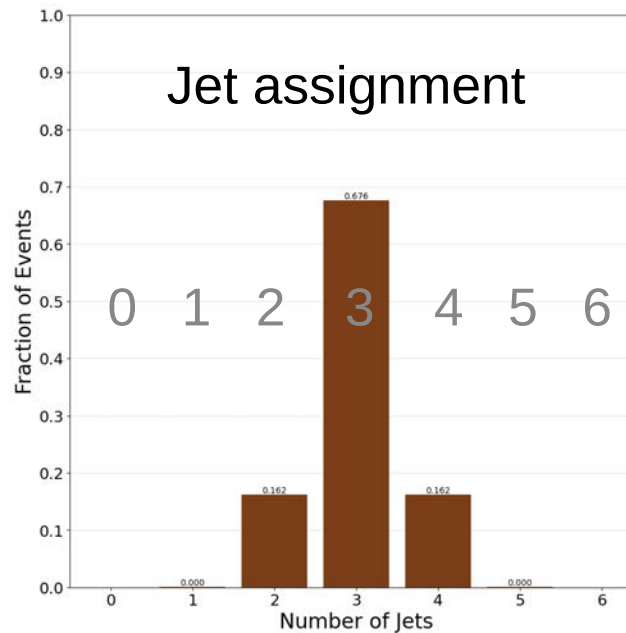
No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

Reconstructed mass



Jet assignment



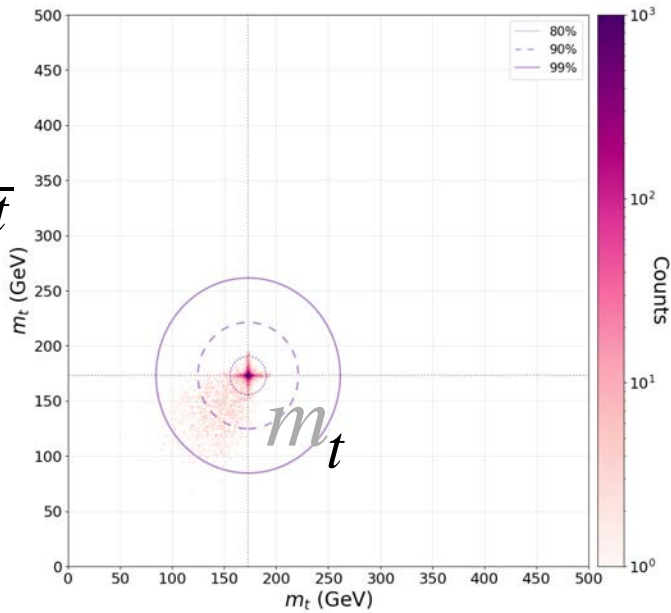
QAOA

43% overall efficiency

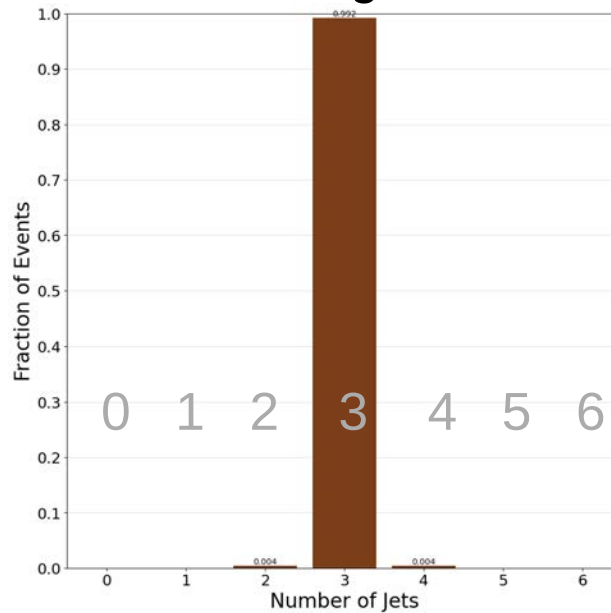
No mass information used.

ma-QAOA and FALQON

Reconstructed mass



Jet assignment



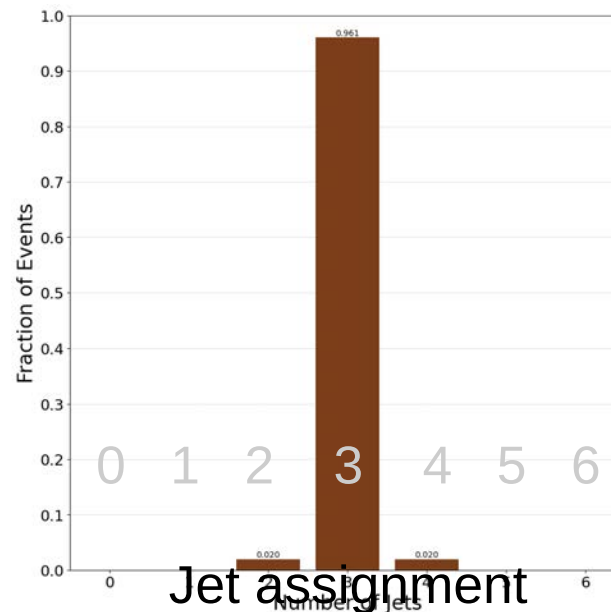
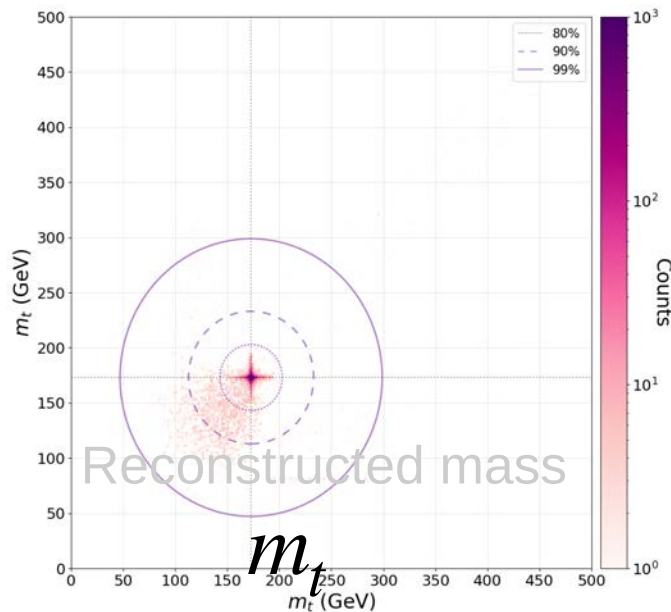
ma-QAOA / VarQITE

79% overall efficiency

No mass information used.

Hybrid quantum-classical approach for combinatorial problems at hadron colliders: 2410.22417

$m_{\bar{t}}$

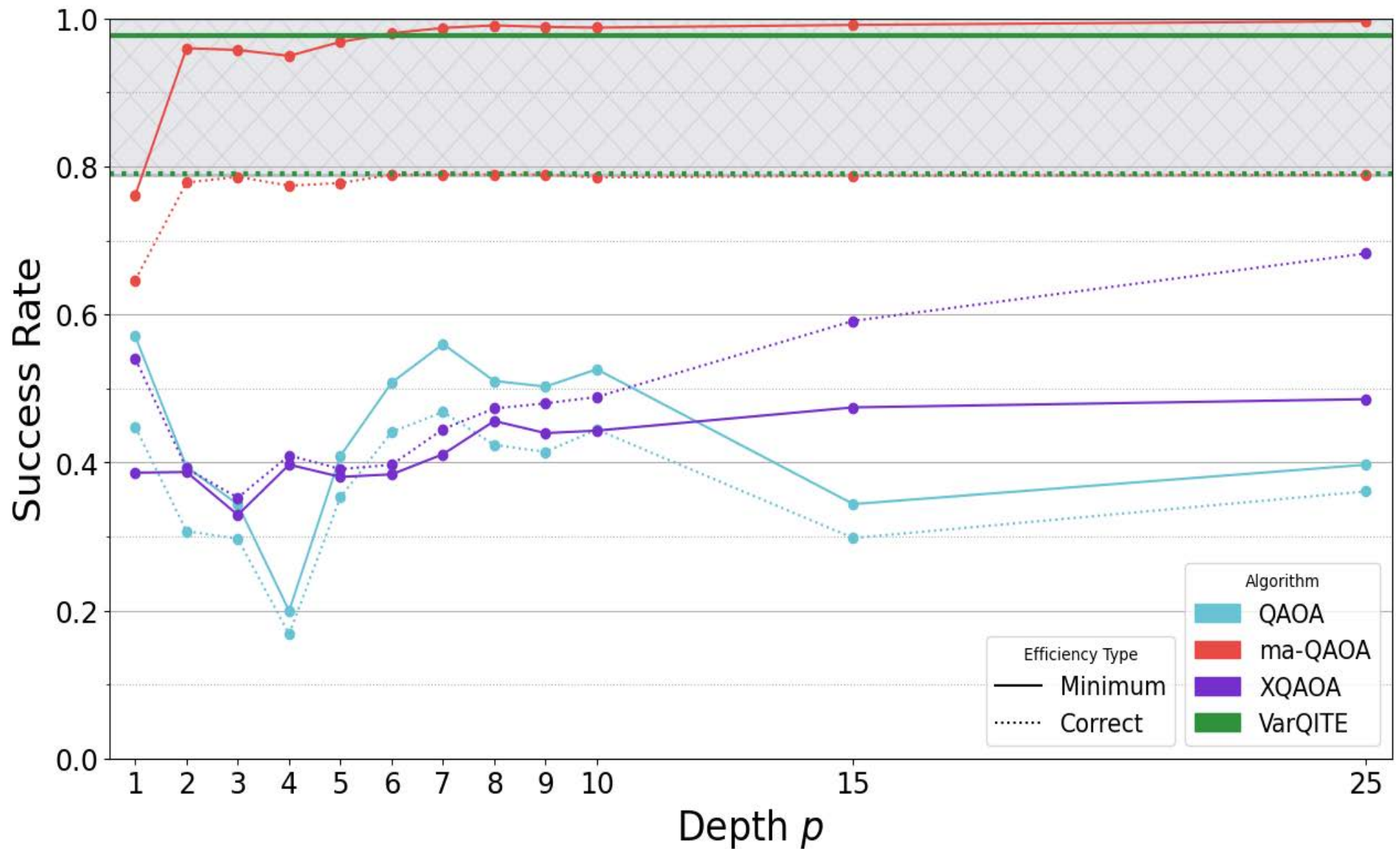


FALQON

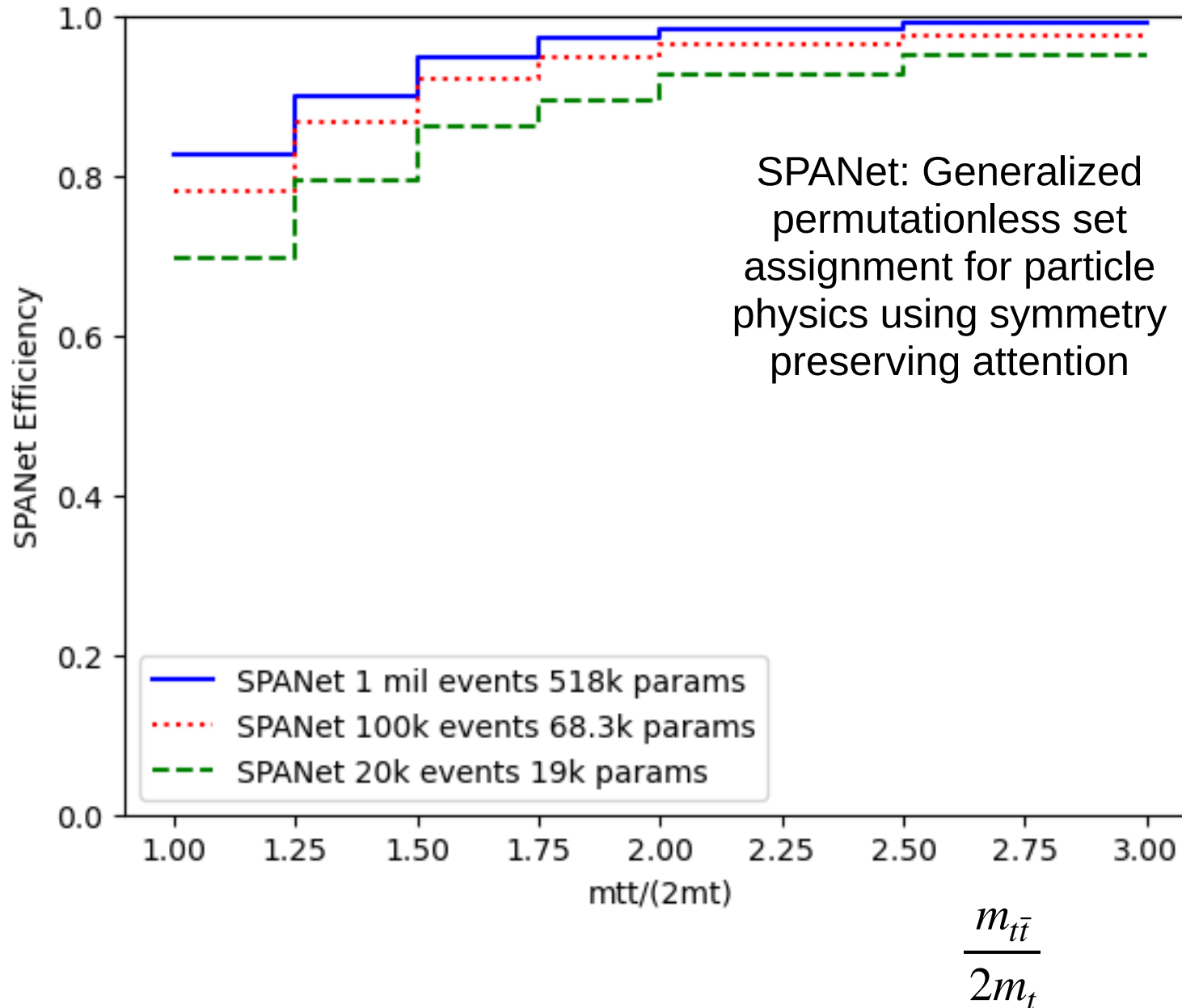
78% overall efficiency

No mass information used.

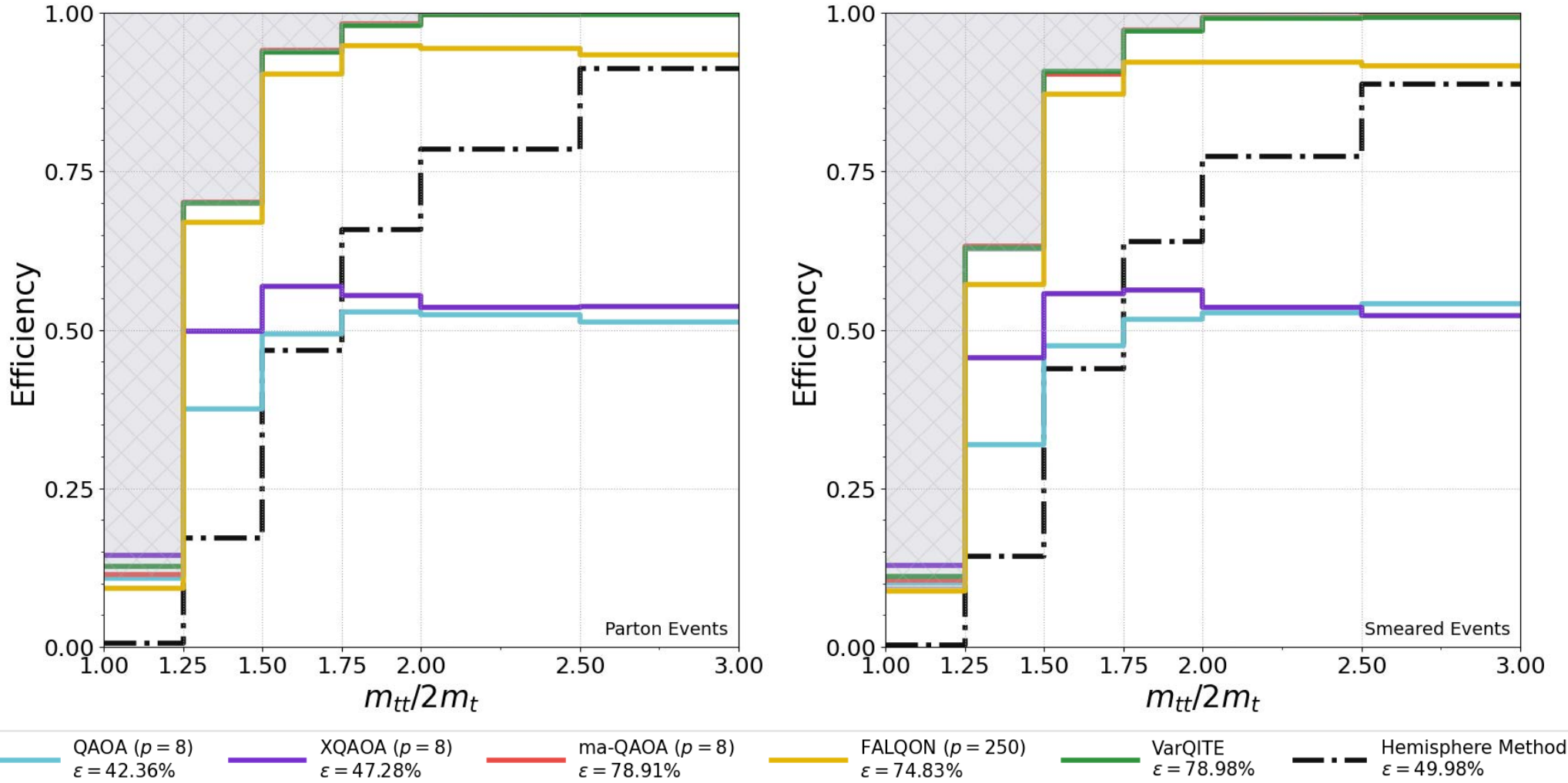
Combinatorial problems in the top quark production



Combinatorial problems in the top quark production



Detector effects

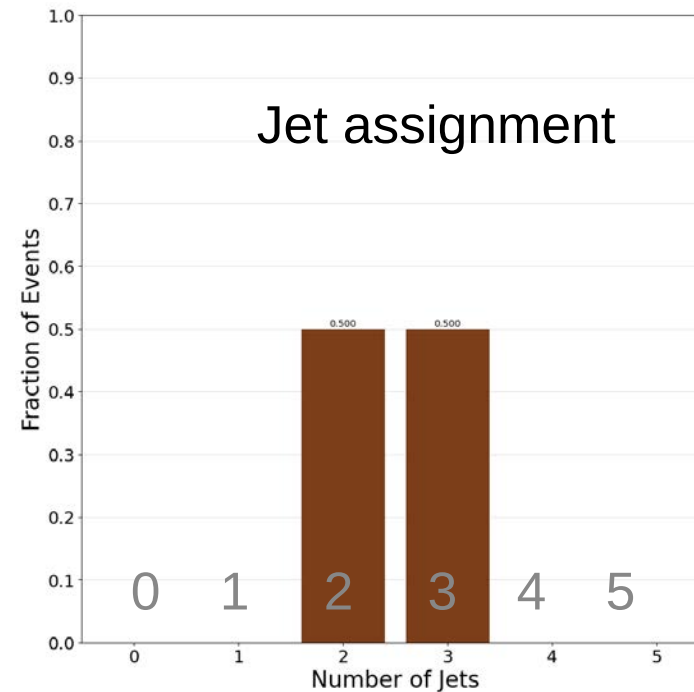
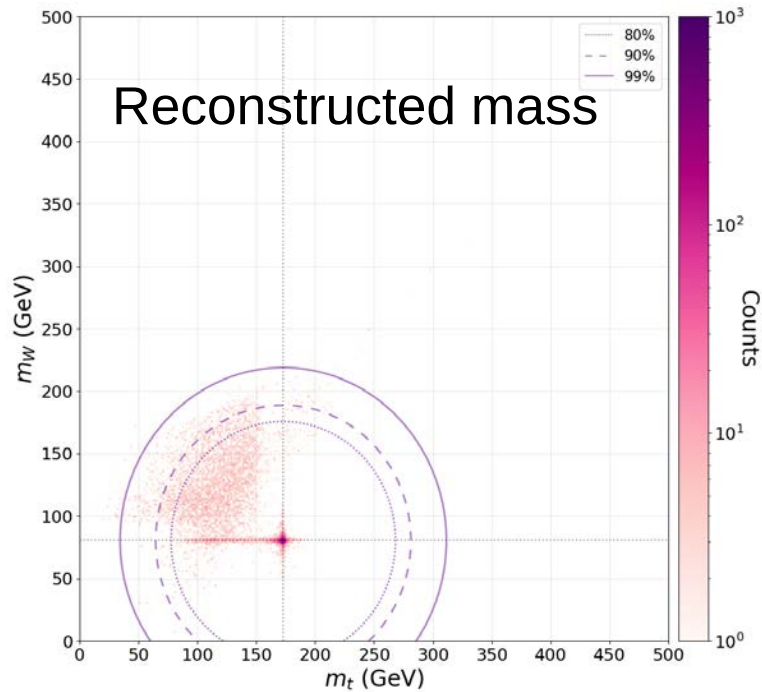


$$\frac{\sigma}{E} = \sqrt{\left(\frac{N}{E}\right)^2 + \left(\frac{S}{\sqrt{E}}\right)^2 + C^2},$$

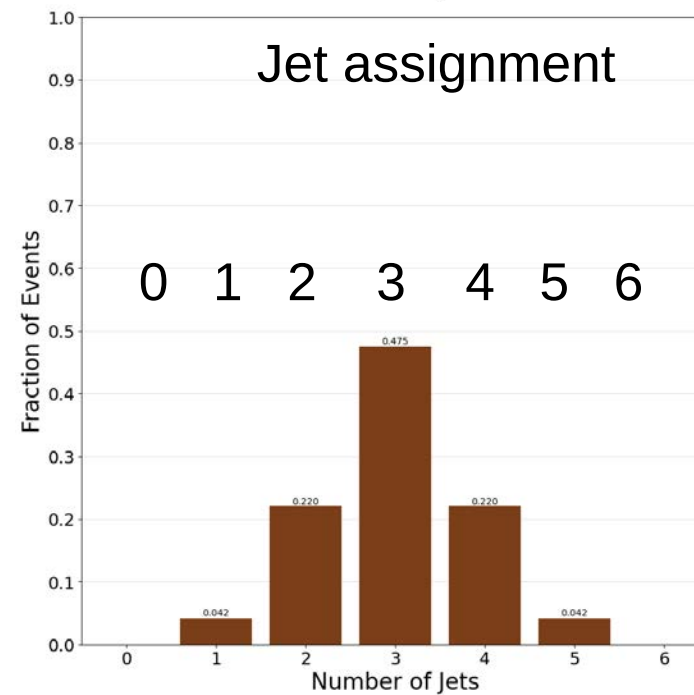
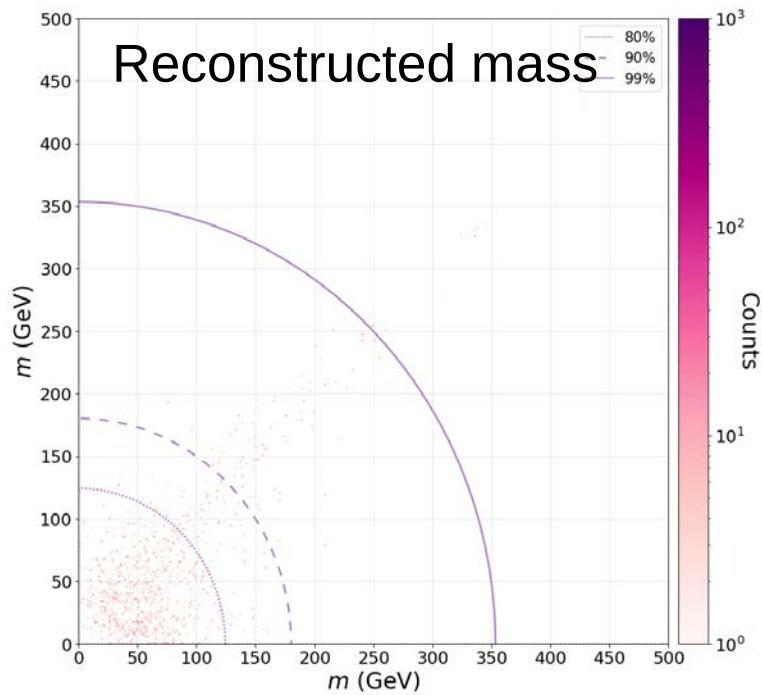
$$N=5.3, S=0.74, C=0.05$$

tW and multi-jet production

ma-QAOA with $p=8$
parton-level events

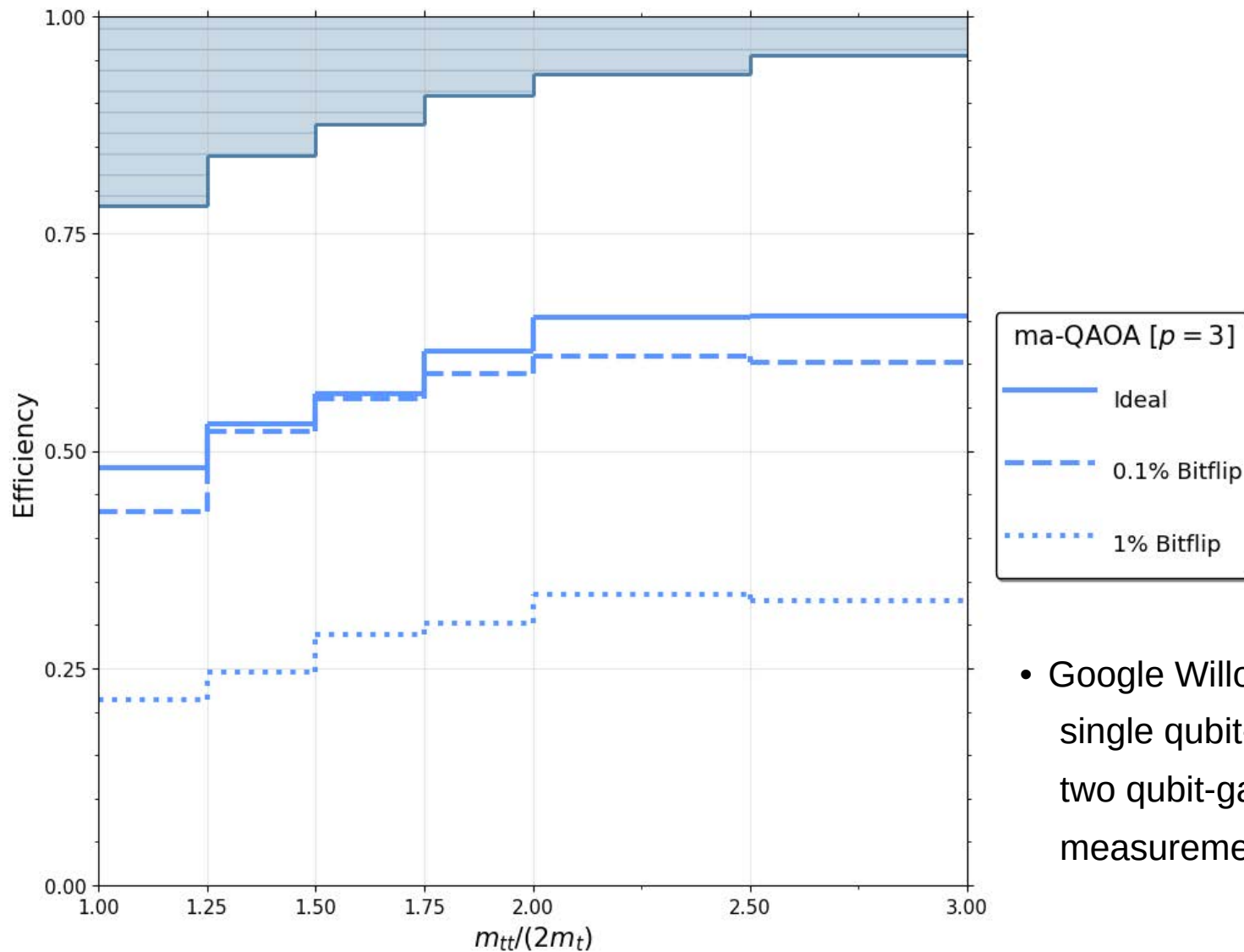


tW (5 jets)



multi-jet (6 jets)

Quantum noises



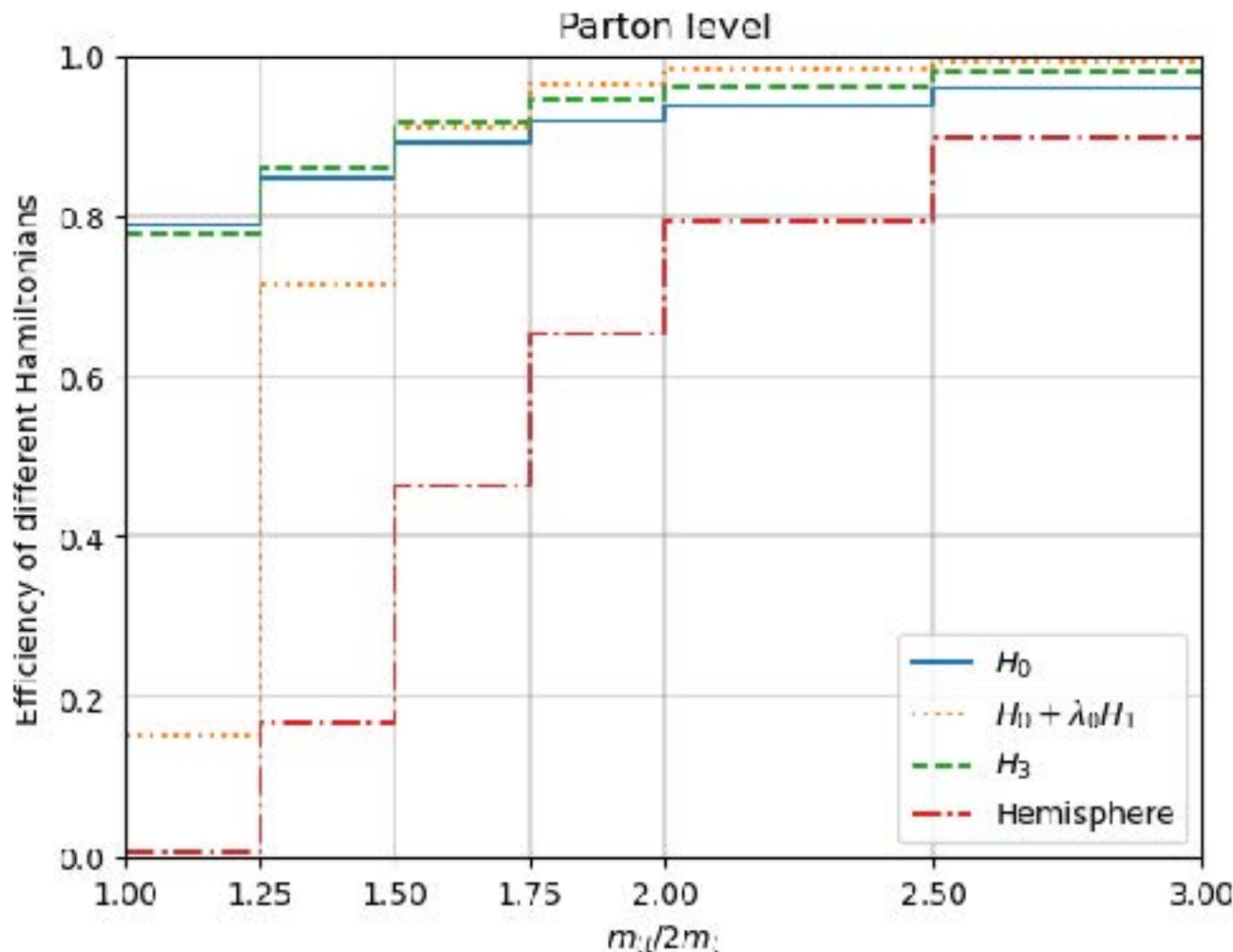
- Google Willow
 - single qubit-gate error $\sim 3 \times 10^{-4}$
 - two qubit-gate error $\sim 3 \times 10^{-3}$
 - measurement error $\sim 7 \times 10^{-3}$

Discussion

- Real hardware run: IonQ with $n=6$
- Scalability: $pp \rightarrow AB \rightarrow j_1 \cdots j_n, n > 6$
- Can we do this with hadrons ($n \sim O(100)$)?
- Full error correction
- Combinatorial problems with ISR
 - Naive extension requires twice more qubits for n particles (two binary variables, x_i and y_i for $i = 1, \dots, n$)
 - Qubits with efficient embedding: $2 \otimes 2 = 3 \oplus 1$
 - Qutrits
- Combinatorial problems with missing transverse momentum
 - Multi-jets + MET
- Higher order terms

Different Hamiltonian Ansatz

$$\begin{aligned}
 H_3 &= H_0 H_1 \\
 &= (P_1^2 - P_2^2)^2 \cdot (P_1^2 + P_2^2) \\
 &= \frac{1}{2} \left[\left(\sum_{k,\ell} P_{k\ell} \right) \sum_{i,j} J_{ij} s_i s_j + \sum_{i,jk\ell} J_{ij} P_{k\ell} s_i s_j s_k s_\ell \right]
 \end{aligned}$$



- QUBO is extended to a problem with higher order (quartic) terms.
- 85% of the time, the ground state of H_3 resolves the combinatorial problem.
- What quantum algorithm can simulate this Hamiltonian?
- QAOA is not an efficient algorithm due to many combination of quartic terms.

Quantum Enhancement of the Signal Rate

U_{DM} induces pure phase rotation of its eigenstates

E.g. for $\alpha = 0$: $U_{\text{DM}} \simeq \begin{pmatrix} 1 & i\delta \\ i\delta & 1 \end{pmatrix}$ with $\delta \equiv \eta\tau$

$$\Rightarrow U_{\text{DM}}|\pm\rangle = e^{\pm i\delta}|\pm\rangle \quad \text{with} \quad |\pm\rangle \equiv \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$$

$$\Rightarrow U_{\text{DM}}^{\otimes N_q}|+\rangle^{\otimes N_q} = e^{iN_q\delta}|+\rangle^{\otimes N_q}$$

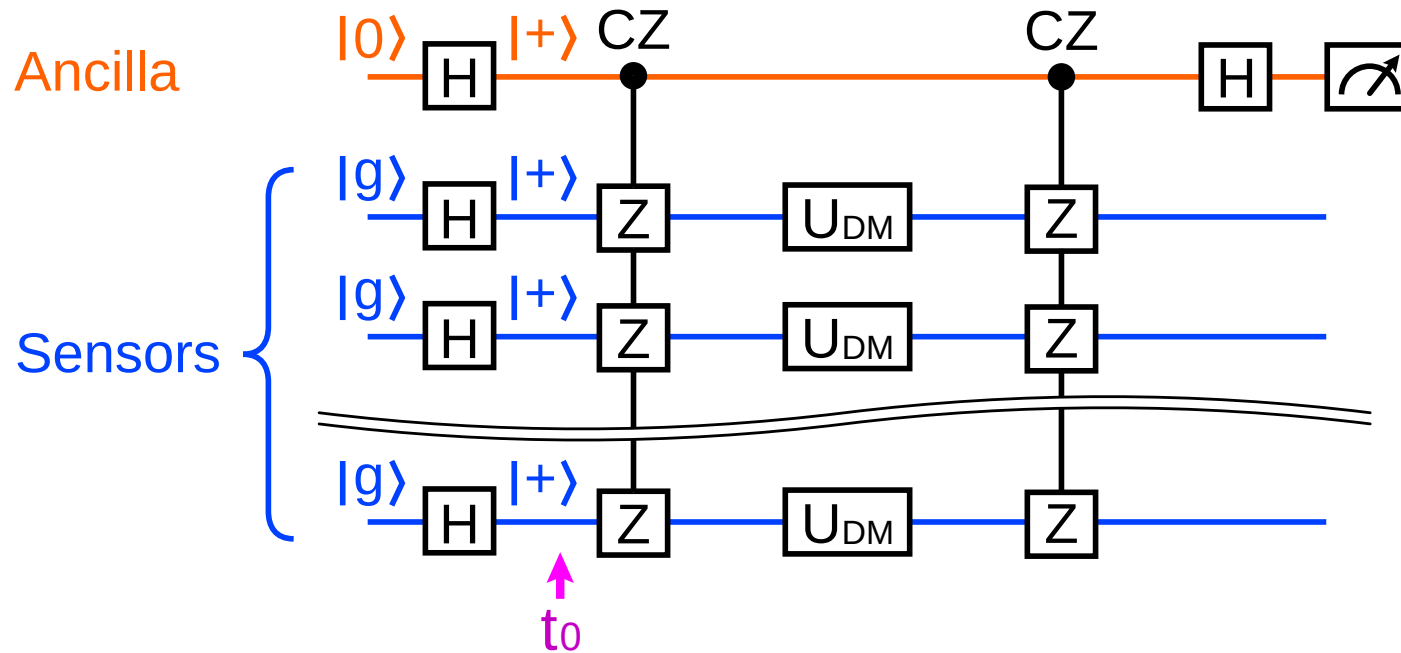
We can design quantum operations to enhance the signal

$\Rightarrow$ Quantum enhanced parameter estimation

[Giovannetti, Lloyd, Maccone ('04)]

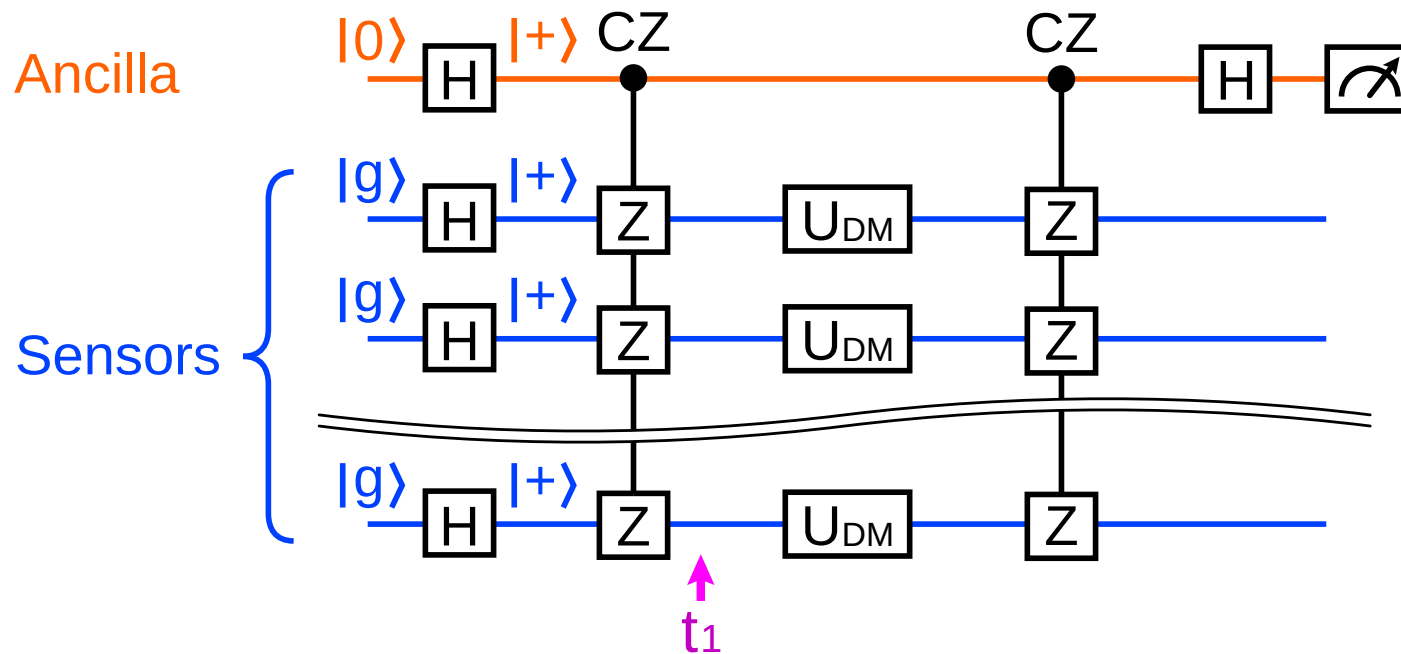
Quantum Enhancement of the Signal Rate

One measurement cycle for the signal enhancement



$$|\Psi(t_0)\rangle = |+\rangle \otimes |+\rangle^{\otimes N_q} = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}}|1\rangle \otimes |+\rangle^{\otimes N_q}$$

One measurement cycle for the signal enhancement



$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle^{\otimes N_q}$$

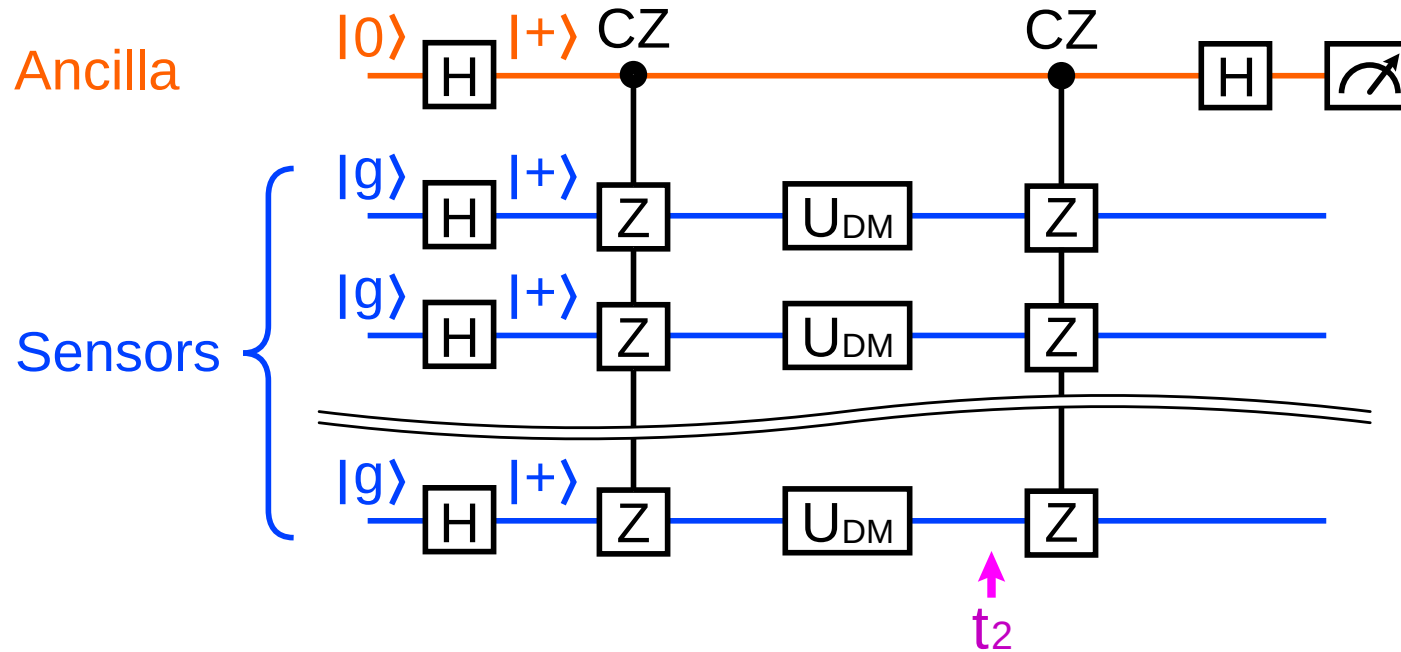
- Controlled Z gate

$$CZ = |0\rangle\langle 0| \otimes \mathbf{1} + |1\rangle\langle 1| \otimes Z$$

$$\Rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |+\rangle \xrightarrow{CZ} \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle$$

Quantum Enhancement of the Signal Rate

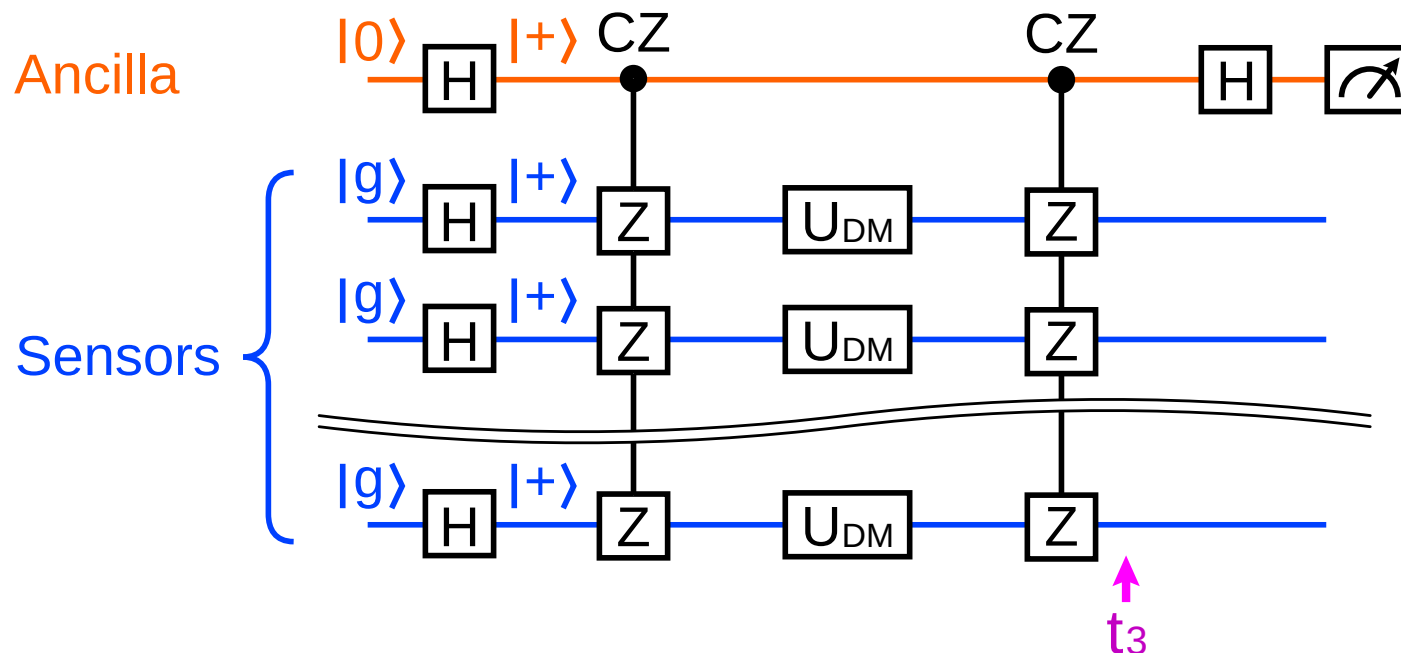
One measurement cycle for the signal enhancement



$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}} e^{iN_q\delta} |0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta} |1\rangle \otimes |-\rangle^{\otimes N_q}$$

Quantum Enhancement of the Signal Rate

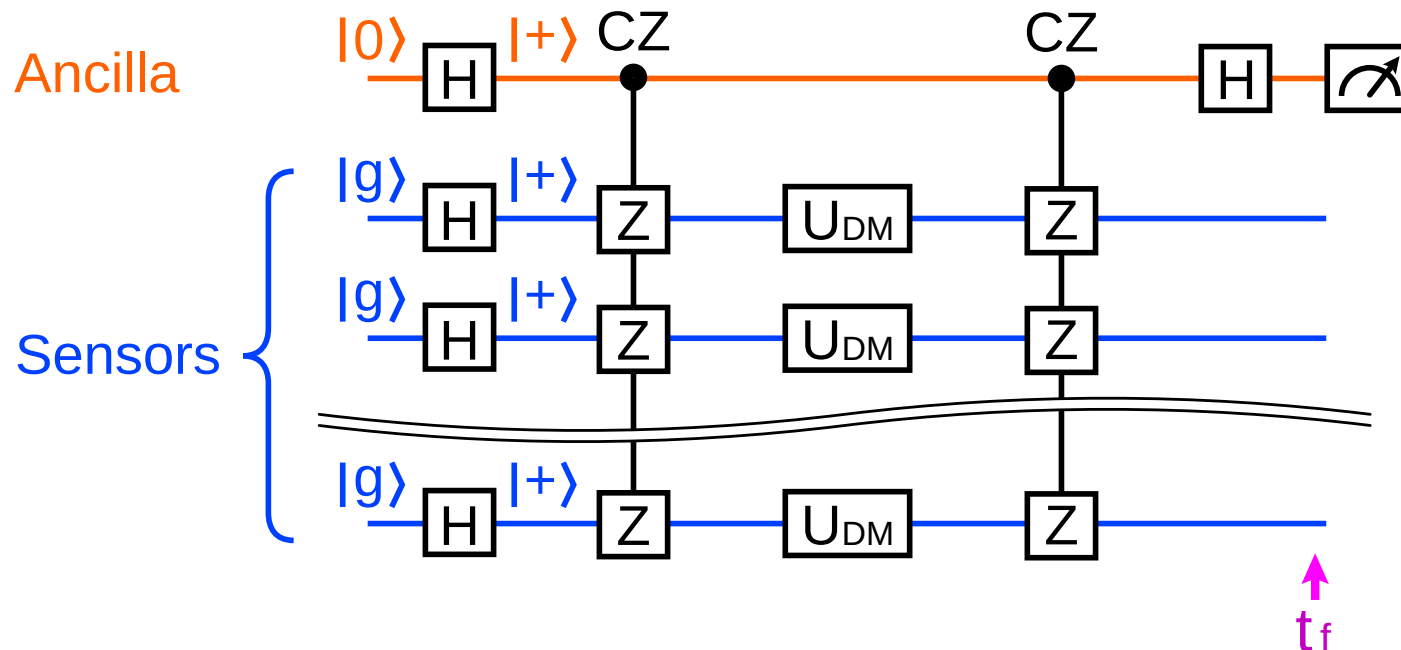
One measurement cycle for the signal enhancement



$$\begin{aligned}
 |\Psi(t_3)\rangle &= \frac{1}{\sqrt{2}} e^{iN_q\delta} |0\rangle \otimes |+\rangle^{\otimes N_q} + \frac{1}{\sqrt{2}} e^{-iN_q\delta} |1\rangle \otimes |+\rangle^{\otimes N_q} \\
 &= \left(\cos N_q\delta |+\rangle + i \sin N_q\delta |-\rangle \right) \otimes |+\rangle^{\otimes N_q}
 \end{aligned}$$

Quantum Enhancement of the Signal Rate

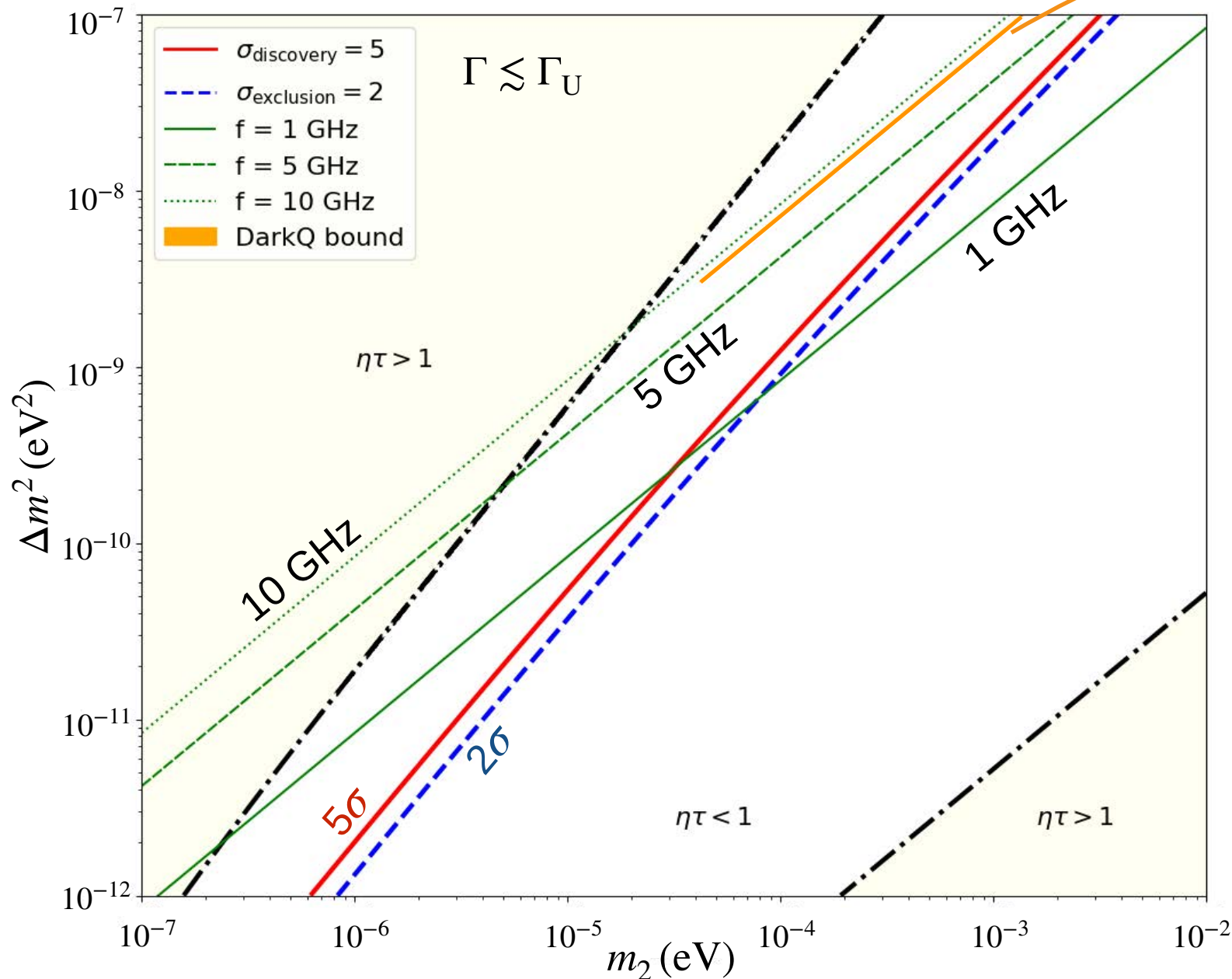
One measurement cycle for the signal enhancement



$$|\Psi(t_f)\rangle = \left(\cos N_q \delta |0\rangle + i \sin N_q \delta |1\rangle \right) \otimes |+\rangle^{\otimes N_q}$$

$\Rightarrow$ Ancilla qubit can be excited: $P_{0 \rightarrow 1} \simeq \sin^2 N_q \delta \simeq N_q^2 \delta^2$

Dark matter decay



DarQ

$$\mu = \mu_{\text{max}}^{\text{Uni}}$$

$$n_q = 1$$

$$\tau = 100 \mu s$$

$$V_{\text{eff}} = 1$$

14 seconds data
collection for
each frequency

DarQ bounds

$$\bar{E}_{(\text{eff})} = \sqrt{\frac{\Gamma \mathcal{T} n_2 \Delta m^2}{2m_2 \epsilon_0}} \quad \rho_{\text{DM}} \approx m_2 n_2$$

$$E_\gamma = \frac{\Delta m^2}{2m_2} = \omega_{\text{res}}^{\text{DarQ}} = 36.1 \mu\text{eV}$$

$$f_{\text{res}}^{\text{DarQ}} = \frac{\omega_{\text{res}}^{\text{DarQ}}}{2\pi} = 8.74 \text{ GHz}$$

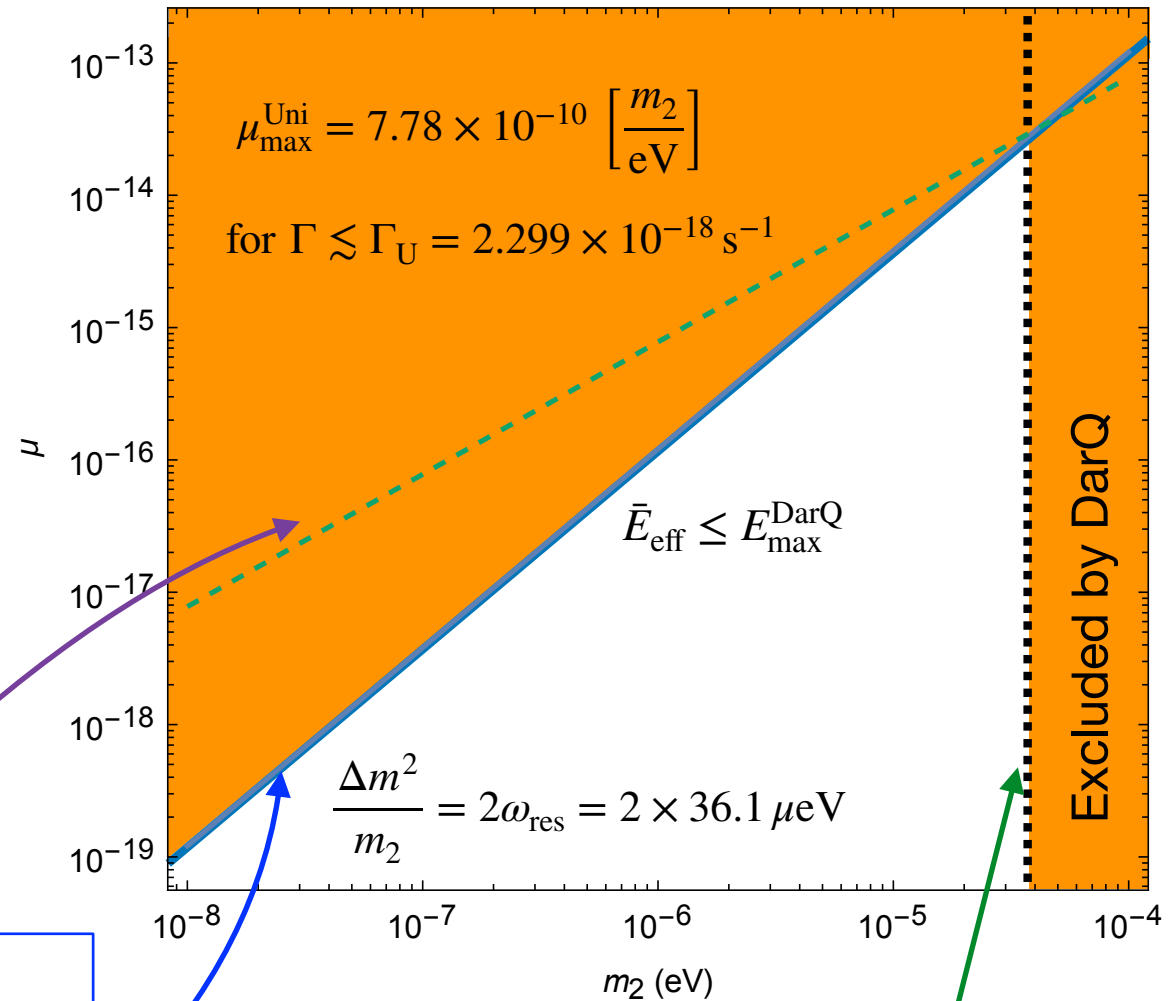
$$\Gamma \lesssim \Gamma_U = 2.299 \times 10^{-18} \text{ s}^{-1}$$

$$(\mu_{\text{max}}^{\text{Uni}})^2 \approx 48\pi m_2^2 \left(\frac{m_2}{\Delta m^2}\right)^3 \Gamma_U$$

$$\mu \leq \mu_{\text{max}}^{\text{Uni}} = 7.78 \times 10^{-10} \times \left(\frac{m_2}{\text{eV}}\right)$$

$$\Gamma(X_2 \rightarrow X_1 + \gamma) \approx \frac{\mu^2}{48\pi} \frac{(\Delta m^2)^3}{m_2^5}$$

$$\mu^2 < \frac{12\pi\epsilon^2\epsilon_0}{(\omega_{\text{res}}^{\text{DarQ}})^4 \mathcal{T}} m_2^3 = 1.46 \times 10^{-14} \times \left(\frac{m_2}{\text{eV}}\right)^3$$



$$\bar{E}_{(\text{eff})} < E_{\text{max}}^{\text{DarQ}} = \epsilon \sqrt{2\rho_{\text{DM}}}$$

$$m_2 = \frac{\Gamma \mathcal{T} \omega_{\text{res}}^{\text{DarQ}}}{2\epsilon_0 \epsilon^2} = 4.15 \times 10^{-5} \text{ eV}$$

$$E = 2.63 \times 10^{-15} \text{ eV}^2 \approx 1.35 \times 10^{-17} \text{ T} \approx 4 \times 10^{-9} \text{ V/m.}$$

Quantum devices (qubits) as sensors

Detecting Hidden Photon Dark Matter Using the Direct Excitation of Transmon Qubits

Shion Chen,¹ Hajime Fukuda,² Toshiaki Inada¹,² Takeo Moroi,^{2,3,*} Tatsumi Nitta,¹ and Thanaporn Sichanugrist²

Quantum Enhancement in Dark Matter Detection with Quantum Computation

Shion Chen,¹ Hajime Fukuda²,² Toshiaki Inada,¹ Takeo Moroi,^{2,3} Tatsumi Nitta,¹ and Thanaporn Sichanugrist^{2,*}

Eliminating Incoherent Noise: A Coherent Quantum Approach in Multi-Sensor Dark Matter Detection

Jing Shu,^{1,2,3} Bin Xu,¹ and Yuan Xu^{4,5}

Search for QCD axion dark matter with transmon qubits and quantum circuit

Shion Chen^(a), Hajime Fukuda^(b), Toshiaki Inada^(c), Takeo Moroi^(b,d),
Tatsumi Nitta^(c), Thanaporn Sichanugrist^{(b)*}

Quantum entanglement of ions for light dark matter detection

Asuka Ito^{1,2}, Ryuichiro Kitano^{2,3}, Wakutaka Nakano² and Ryoto Takai^{2,3}

Directional search for light dark matter with quantum sensors

Hajime Fukuda*

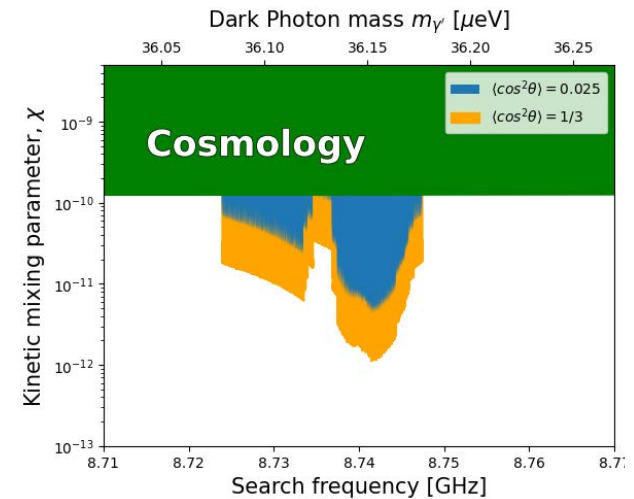
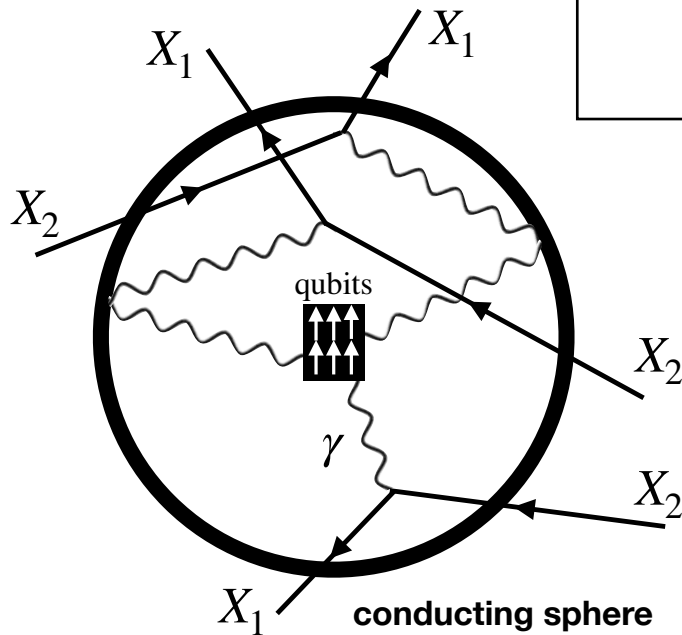
Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan

Ultralight dark matter detection with trapped-ion interferometry

Leonardo Badurina,¹ Diego Blas,^{2,3} John Ellis,^{4,5} and Sebastian A. R. Ellis⁶

Search for Dark Photon Dark Matter of a Mass around $36.1 \mu\text{eV}$ Using a Frequency-tunable Cavity Controlled through a Coupled Superconducting Qubit

K. Nakazono,^{1,*} S. Chen,² H. Fukuda,¹ Y. Iiyama,³ T. Inada,³ T. Moroi,^{1,4} T. Nitta,⁴ A. Noguchi,^{5,6,7} R. Sawada,³ S. Shirai,^{5,6} T. Sichanugrist,¹ K. Terashi,³ and K. Watanabe¹
(DarQ Collaboration)



Quantum Sensing Radiative Decays of Neutrinos and Dark Matter Particles

Zhongtian Dong,^{1,*} Doojin Kim,^{2,†} Kyoungchul Kong,^{1,‡} Myeonghun Park,^{3,4,5,§} and Miguel A. Soto Alcaraz^{1,}

Case of hidden photon X_μ (in mass-eigenstate basis)

$$\mathcal{L} \ni -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu + e \bar{\psi}_e \gamma^\mu \psi_e (A_\mu^{(\text{EM})} + \epsilon X_\mu)$$

ϵ : kinetic-mixing parameter

Oscillating hidden photon can play the role of DM

$$\vec{X} \simeq \bar{X} \vec{n}_X \sin(m_X t + \alpha) \quad \text{with} \quad \rho_{\text{DM}} = \frac{1}{2}m_X^2 \bar{X}^2$$

Hidden photon DM induces effective electric field

$$\vec{E}^{(X)} = -\epsilon \dot{\vec{X}} = -\bar{E}^{(X)} \vec{n}_X \cos(m_X t + \alpha)$$

$$\bar{E}^{(X)} = \epsilon \sqrt{2\rho_{\text{DM}}}$$

Hamiltonian for transmon qubit + hidden photon system

$$H = \omega|e\rangle\langle e| - 2\eta \cos(m_X t + \alpha) (|e\rangle\langle g| + |g\rangle\langle e|)$$

set the ground state energy to zero.

$$\eta \simeq \frac{1}{2\sqrt{2}} d\bar{E}^{(X)} \sqrt{C\omega} = \frac{1}{2} \epsilon d \sqrt{C\omega \rho_{\text{DM}}}$$

Schrödinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle \Rightarrow |\psi(t)\rangle = U_{\text{DM}}(t) |\psi(0)\rangle$$

Resonance limit $\omega = m_X$ (for $\eta t \ll 1$)

$$\begin{pmatrix} \psi_g(t) \\ \psi_e(t) \end{pmatrix} = U_{\text{DM}}(t) \begin{pmatrix} \psi_g(0) \\ \psi_e(0) \end{pmatrix} \simeq \begin{pmatrix} 1 & ie^{-i\alpha}\eta t \\ ie^{i\alpha}\eta t & 1 \end{pmatrix} \begin{pmatrix} \psi_g(0) \\ \psi_e(0) \end{pmatrix}$$

$$|\psi(t)\rangle \equiv \psi_g(t)|g\rangle + e^{-i\omega t} \psi_e(t)|e\rangle$$

$|g\rangle \rightarrow |e\rangle$ transition probability (assuming $|\psi(0)\rangle = |g\rangle$)

$$|\psi_e(t)|^2 \simeq \begin{cases} \eta^2 t^2 & : \omega = m_X \text{ (on-resonance)} \\ \sim \eta^2 (\omega - m_X)^{-2} & : \omega \neq m_X \text{ (off-resonance)} \end{cases}$$

Excitation probability for $\omega = m_X$:

$$P_{ge} \simeq 0.3 \times \left(\frac{\epsilon}{10^{-11}} \right)^2 \left(\frac{m_X}{10 \mu\text{eV}} \right) \left(\frac{C}{0.1 \text{ pF}} \right) \left(\frac{d}{100 \mu\text{m}} \right)^2 \left(\frac{\tau}{100 \mu\text{s}} \right)^2$$

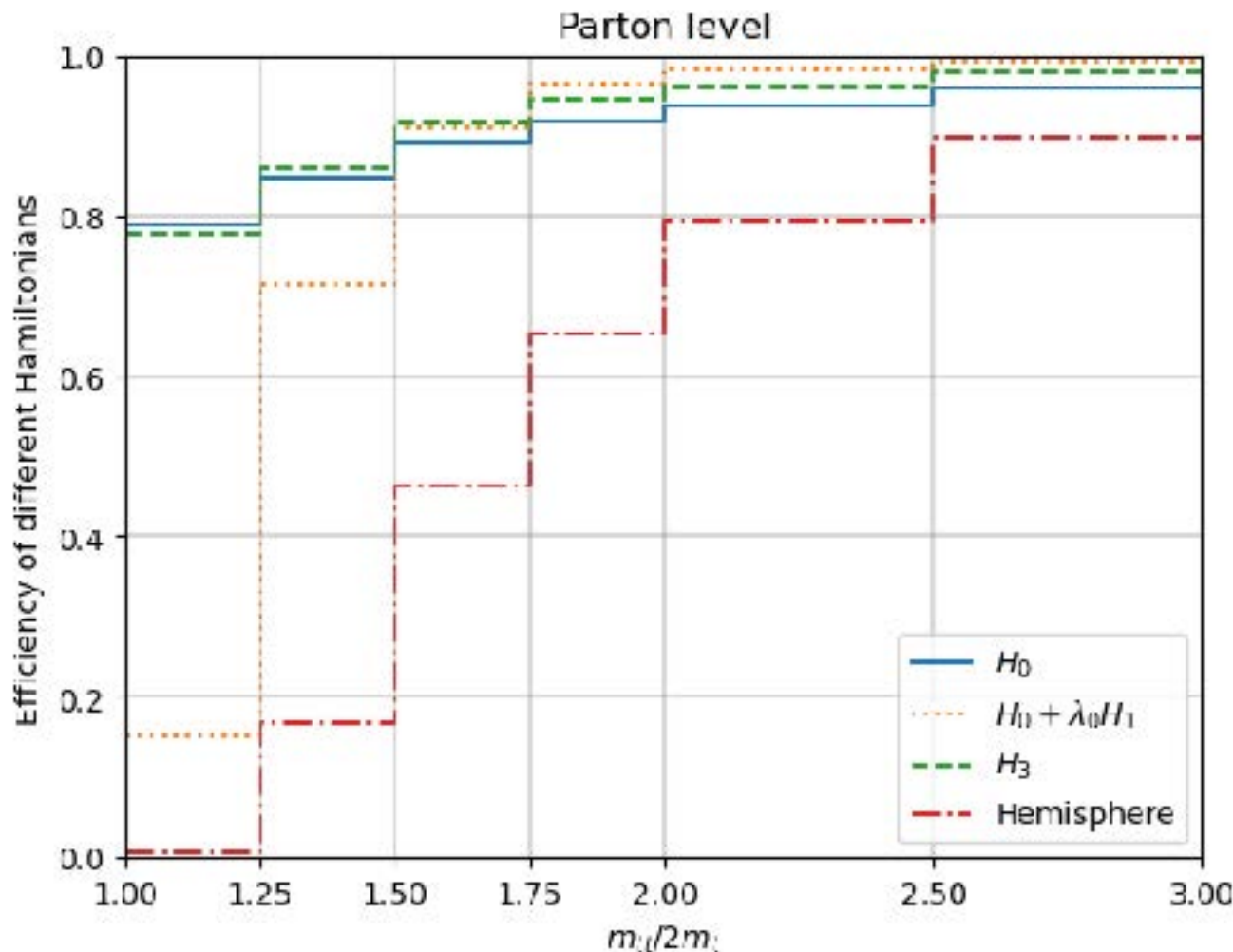
τ = coherence time

Excitation probability can be sizable

$\Rightarrow$ Transmon qubit as a DM detector

Different Hamiltonian Ansatz

$$\begin{aligned}
 H_3 &= H_0 H_1 \\
 &= (P_1^2 - P_2^2)^2 \cdot (P_1^2 + P_2^2) \\
 &= \frac{1}{2} \left[\left(\sum_{k,\ell} P_{k\ell} \right) \sum_{i,j} J_{ij} s_i s_j + \sum_{i,jk\ell} J_{ij} P_{k\ell} s_i s_j s_k s_\ell \right]
 \end{aligned}$$



- QUBO is extended to a problem with higher order (quartic) terms.
- 85% of the time, the ground state of H_3 resolves the combinatorial problem.
- What quantum algorithm can simulate this Hamiltonian?
- QAOA is not an efficient algorithm due to many combination of quartic terms.

Advantages of Quantum Sensing

- Classical sensing: Sufficient for strong signals
- Quantum sensing: Significant advantages for weak signals
- Key quantum features for sensing
 - Fundamental limits from zero-point fluctuations
 - Enhanced sensitivity through entanglement, surpassing classical limits
- Promising applications in fundamental physics, where signals are often extremely weak and changing in time with unknown frequencies.
 - Dark matter searches
 - Neutrino transition magnetic moment
- Main difference between traditional quantum sensors and qubit sensors is that one can run quantum algorithms on qubits

A simplified Setup: signal-sensor interaction

- Core setup for many quantum sensing protocols in HEP (Details vary, but the essential physics is often similar)

- **Sensor: A Two-Level Quantum System**

- A qubit as a simple example
- Two states: Ground $|0\rangle$ and Excited $|1\rangle$
- Can be a physical qubit, a cavity mode ($n = 0$, $n = 1$) etc.

- **Signal: A Weak Classical Field**

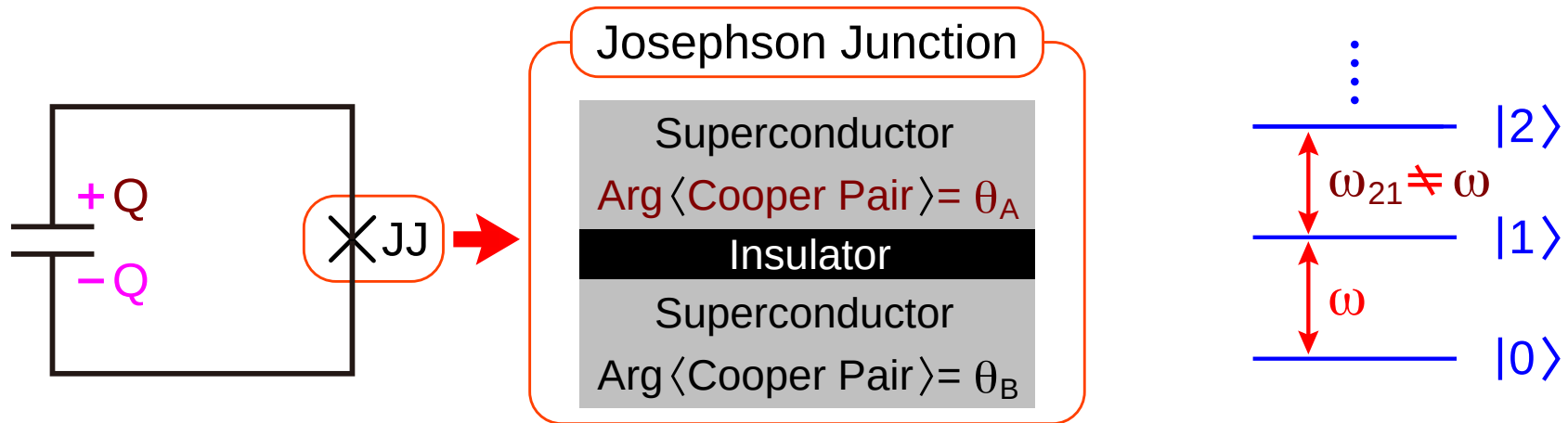
- E.g., wave-like DM, high-frequency GWs
- Drive transitions from $|0\rangle$ to $|1\rangle$
- Interaction Hamiltonian:
$$H_I = \epsilon (\cos \alpha \sigma_X + \sin \alpha \sigma_Y)$$
- ϵ : Signal strength (the quantity to be estimated)
- α : Signal phase (often stochastic/random)

- **The Goal to estimate the signal strength ϵ .**

1. Evolve: System evolves from $|0\rangle$ under H_I . For a weak signal $\epsilon t \ll 1$,
$$|\psi(t)\rangle \approx |0\rangle - ie^{i\alpha} \epsilon t |1\rangle$$

2. Measure: Project onto $|0\rangle$ and $|1\rangle$. Probability of finding $|1\rangle$: $p_1 = \left| \langle 1 | \psi(t) \rangle \right|^2 = \epsilon^2 t^2$

Transmon qubit: Capacitor + Josephson junction (JJ)

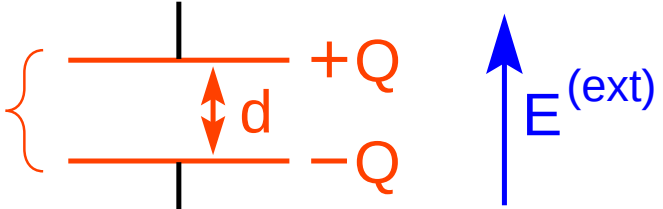


$$H_0 = \frac{1}{2C} Q^2 - J \cos \theta \simeq \frac{1}{2} \frac{C}{(2e)^2} \dot{\theta}^2 - J \cos \theta \quad \text{with } \theta = \theta_B - \theta_A$$

- Transmon qubit has discrete energy levels
 - $|0\rangle$ and $|1\rangle$ can be used as $|g\rangle$ and $|e\rangle$, respectively
 - Transmon qubits are commonly used in today's quantum computers

Transmon qubits

- Transmon qubit couples to external electric field

Capacitor {  $\Leftrightarrow H_{\text{int}} = QdE^{(\text{ext})}$

- Charge operator in the transmon limit: $CJ \gg (2e)^2$

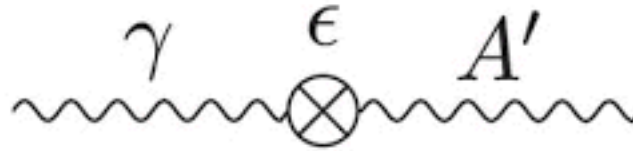
$$Q \simeq \frac{C}{2e} \dot{\theta} \simeq \sqrt{\frac{C\omega}{2}} (|g\rangle\langle e| + |e\rangle\langle g|)$$

- $|g\rangle \leftrightarrow |e\rangle$ transition could occur, **if there is additional electric field!**
 - This is not possible, since qubits are inside some cavity.

Sources of Electric Fields

- $|g\rangle \leftrightarrow |e\rangle$ transition occurs if **DM field** generates electric field

- Hidden photon



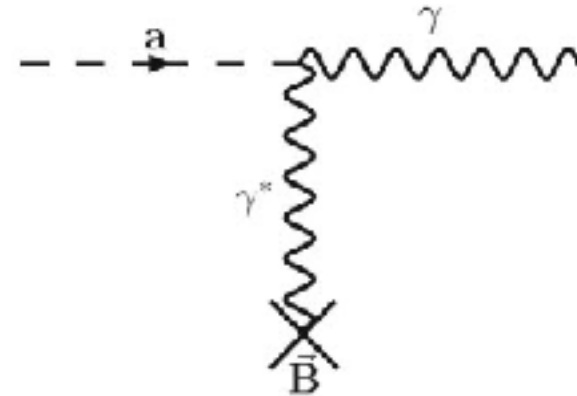
2212.03884, transmon

2507.12860 , Rydberg atom

- Axions (if external magnetic field is given)

2407.19755, transmon

2507.17825, trapped ion



- DM and neutrino decays

2508.09139, transmon, trapped ion

